** You may use an exam booklet if you need extra space for long answer questions **

Physics 200 Exam

December 3, 2008

Name:

Student Number:

46 points available.

Multiple Choice: 22 points

Long Answer: 24 points

#1	#2	#3	#4	#5

#6	#7	#8	#9	#10

#11	#12	#13	#14	#15

#16	#17	#18	#19	#20

#21	#22	
		FREE SPACE

WRITE YOUR MULTIPLE CHOICE ANSWERS HERE!

FORMULA SHEET AT THE BACK

Problem 1

An electron is in a state $\frac{3}{5}|x_1\rangle - \frac{4}{5}|x_2\rangle$. If we do a measurement of position, we are most likely to find the electron at

A) x_1

B) x_2

C) $\frac{3}{5}x_1 - \frac{4}{5}x_2$

D) $\frac{9}{25}x_1 + \frac{16}{25}x_2$

E) All positions between x_1 and x_2 are equally likely.

Problem 2

If we perform the measurement of problem 1 a large number of times on electrons with the same initial state, the average value of the position measurements will be

A) x_1

B) x_2

C) $\frac{3}{5}x_1 - \frac{4}{5}x_2$

D) $\frac{9}{25}x_1 + \frac{16}{25}x_2$

E) $\frac{1}{2}(x_1 + x_2)$.

Problem 3

If we have a polarizer oriented at 0°, which of the following photon states is most likely to pass through?

A) $|30°\rangle$ $\cos 30° = \sqrt{3}/2$ $3/4$

B) $\frac{2}{\sqrt{5}}|0°\rangle + \frac{1}{\sqrt{5}}|90°\rangle$ $4/5$

C) $\frac{1}{\sqrt{2}}|45°\rangle + \frac{1}{\sqrt{2}}|60°\rangle$ LESS THAN A)

D) $|90°\rangle$ 0

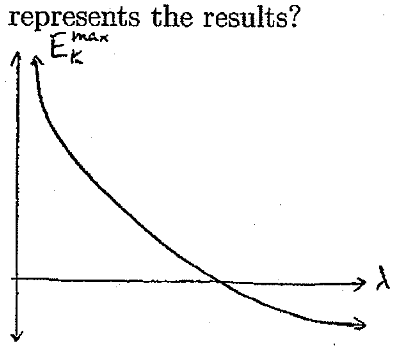
E) There is no way to predict the likelihood before the photon hits the polarizer.

NOTE: STATE C) IS NOT NORMALIZED PROPERLY!

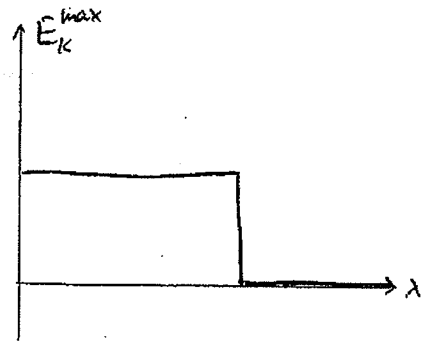
Problem 4

In a photoelectric effect experiment, maximum electron kinetic energy is plotted against wavelength of the incoming photons. Which plot below best represents the results?

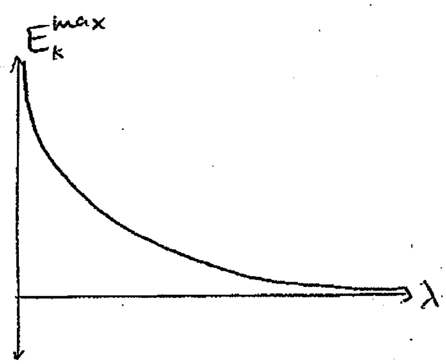
(A)



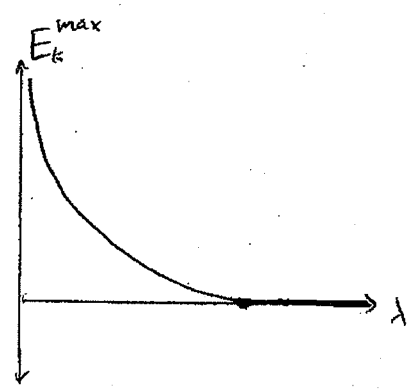
(B)



(C)

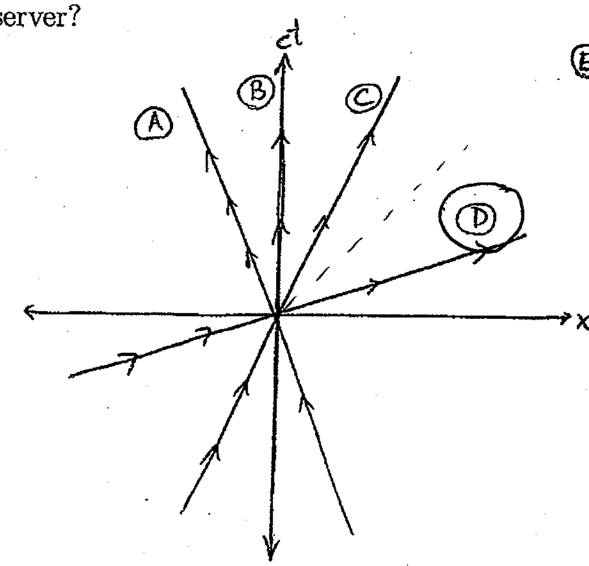


(D)



Problem 5

On the spacetime diagram, which of the lines shown is not a possible trajectory for an observer?



(E) All are possible trajectories

ANSW: D

Problem 6

In a certain exothermic (i.e. releasing energy) nuclear fusion reaction, deuterium (2 neutrons, one proton) and tritium (3 neutrons, one proton) fuse into a Helium nucleus (2 neutrons, 2 protons) and eject a neutron in the process, $D + T \rightarrow He + n$ .For this reaction,

A) $m_D + m_T = m_{He} + m_n$

B) $m_D + m_T > m_{He} + m_n$

C) $m_D + m_T < m_{He} + m_n$

D) The relationship between the masses depends on what frame of reference we measure them in.

Problem 7

A neutron of mass M decays into a proton, an electron, and a neutrino. In the frame of an observer traveling at speed $0.5c$ relative to the original neutron, the total energy of the three final particles will be

A) equal to Mc^2

B) greater than Mc^2

C) less than Mc^2

D) It depends on the direction of the observer's velocity relative to the velocities of the final particles.

Problem 8

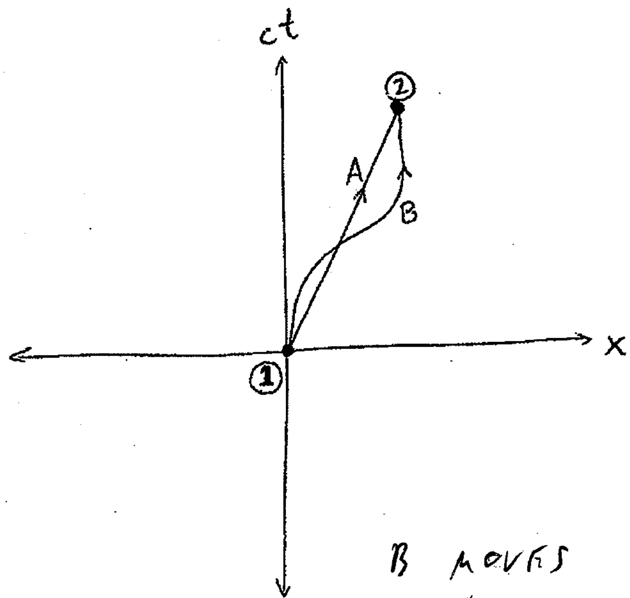
Two ships are traveling towards each other, each with speed 0.5c. One of the ships sends radio waves towards the other ship. In the frame of the other ship, these radio waves will be measured to travel at

A) 2c

B) 1.8c

C) c

D) not enough information, but will be less than c.



B MOVES W.R.T A

$\Rightarrow$ B'S CLOCK SLOWER

$\Rightarrow$ EARLIER TIME

Problem 9

The spacetime diagram shows two observers who are both present at events 1 and 2, but move on different trajectories in between. If both observers synchronize their clocks before they separate, whose clock reads an earlier time when they meet up again?

A) observer A

B) observer B

C) the clocks read the same time

D) It depends on which frame of reference the clocks are observed from.

Problem 10

A rod of proper length 5 meters travels at speed 0.6c relative to an observer. How much time elapses between when the front of the rod passes the observer and when the back of the rod passes the observer?

A) $\frac{125}{12} m/c$

B) $\frac{25}{3} m/c$

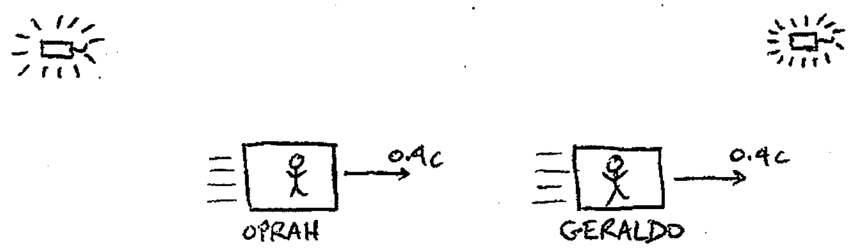
C) $4 m/c$

D) $\frac{20}{3} m/c$

E) $\frac{16}{3} m/c$

$\gamma = \frac{5}{4}$

$\frac{5m/\gamma}{v} = \frac{5 \cdot \frac{4}{5}}{3/5} m/c = \frac{20}{3} m/c$



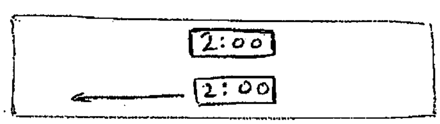
Problem 11 SAME FOR R'!

Oprah and Geraldo are both traveling at velocity $0.4c$ in the positive $\hat{x}$ direction, with Geraldo 1 km ahead of Oprah. In Oprah's frame of reference, two firecrackers separated by 3km along the x direction explode simultaneously. In Geraldo's frame, the firecracker at a larger value of x explodes

A) before the other firecracker.

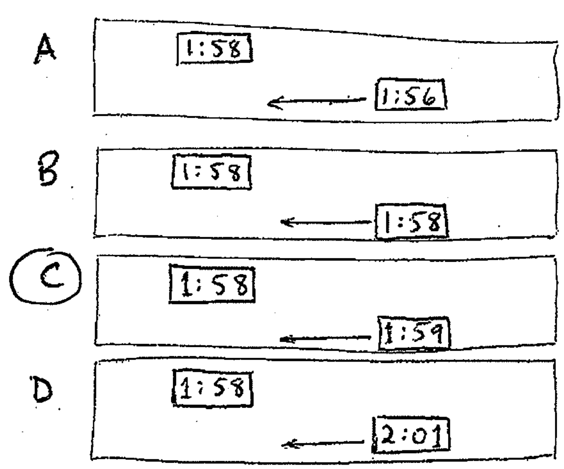
B) after the other firecracker

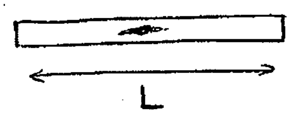
C) at the same time as the other firecracker.



Problem 12

The picture above shows two clocks moving at a large relative velocity. Which of the pictures below represents a possible observation of the clocks at some earlier time (assume the readings on the clocks are exact)?





Problem 13

Consider an electron trapped in a short wire of length L. If we make a measurement of the position of the electron, it puts it in a position eigenstate. For this state

A) There is a definite energy and the probability density will remain constant in time.

B) There is a definite energy but the probability density will change with time.

C) The state does not have a definite energy, but the probability density will remain constant in time.

D) The state does not have a definite energy, and the probability density will change with time.

CORRECTION ?

Problem 14

For the electron in the previous problem, if we instead make a measurement of the ~~position~~ ENERGY of the electron, it puts it in an energy eigenstate. For this state

A) The electron has a definite position and the probability density will remain constant in time.

B) There is a definite position but the probability density will change with time.

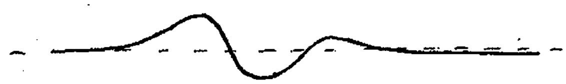
C) The state does not have a definite position, but the probability density will remain constant in time.

D) The state does not have a definite position, and the probability density will change with time.

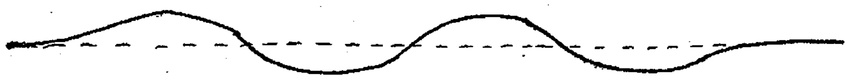
Problem 15

The wavefunction for a traveling electron is described by a wavepacket whose real part is shown above. Which of the following could be the real part of the wavefunction for an electron traveling with half the momentum?

A

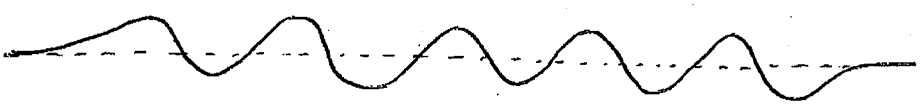


B



C

D



Problem 16

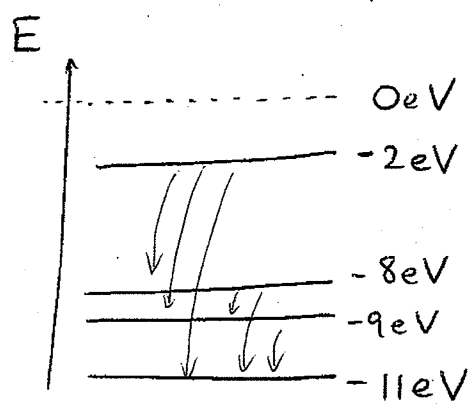
Wavepackets for traveling particles tend to spread out with time, a phenomenon known as dispersion. Which of the wavepackets below (real part shown) will spread out the fastest?

A

B

C

D



Problem 17

The diagram above shows the allowed energy levels for an electron in some molecule, relative to the energy $E = 0eV$ it would take for the electron to escape. If the electron is in its ground state, and absorbs a photon with energy $14eV$, the final kinetic energy of the electron will be

A) 14eV

B) 12eV

C) 3eV

D) 25eV

E) The electron cannot absorb a photon with energy $14eV$.

Problem 18

For ~~a~~ HCl gas of the molecules in the previous question, how many spectral lines will be present in the emission spectrum?

A) 3

B) 4

C) 5

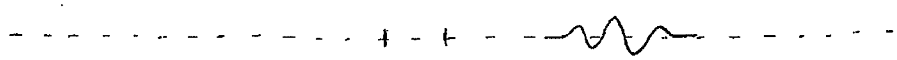
D) 6

E) 10

Problem 19

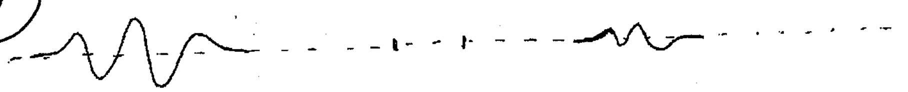
Suppose we have a short air gap between two wires. If we send an electron towards the air gap with an energy less than the work function of the metal, which of the following could be a wavefunction that results?

(A)



(B)

(C)

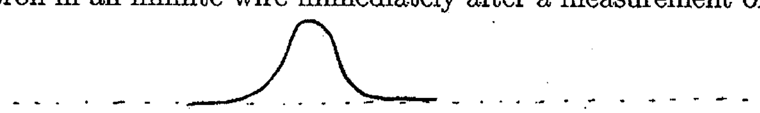


(D) Any of the above (E) None of the above

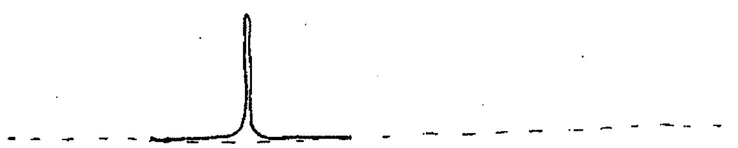
Problem 20

Which of the following is the most likely wavefunction (real part) for an electron in an infinite wire immediately after a measurement of momentum?

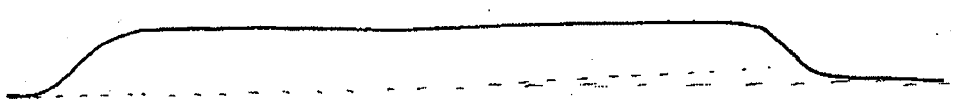
(A)



(B)



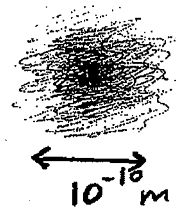
(C)



(D)

$\Delta V = \frac{10^{-34}}{2} \cdot \frac{1}{\frac{1}{2}10^{-10}} \cdot \frac{1}{10^{-30}}$

$= 10^{6}$ m/s



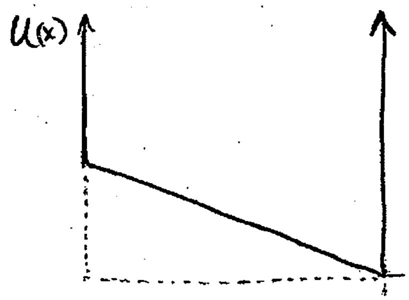
Problem 21

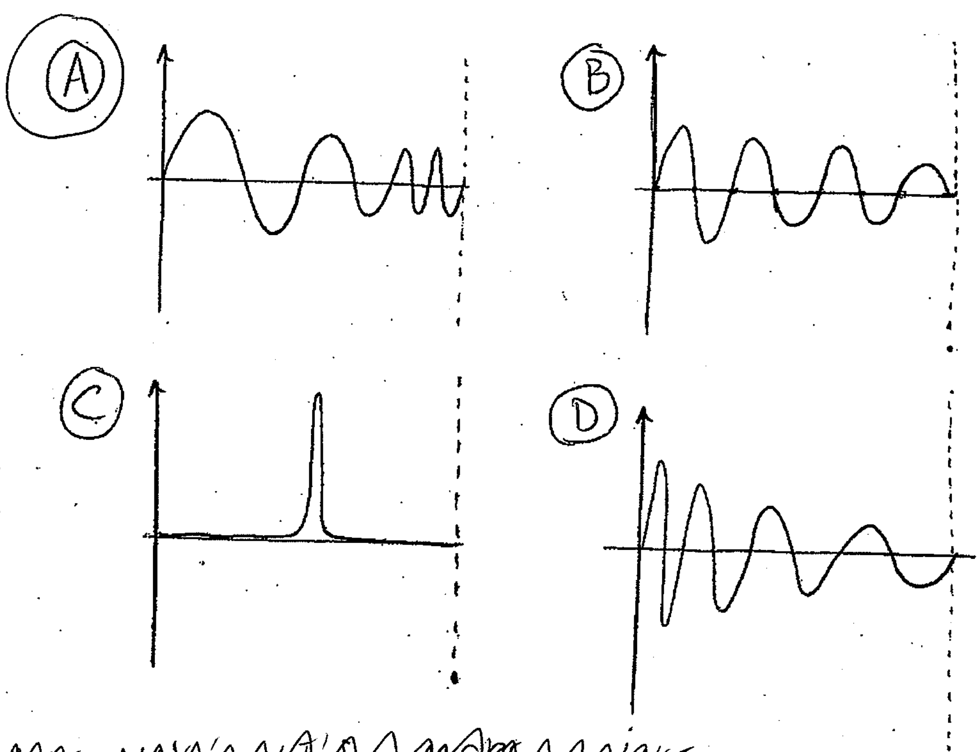
An electron in the ground state of a hydrogen atom has a wavefunction shown. If we measure the $\hat{x}$ velocity of the electron in this state, there is a range of possible values we might find. A typical value for the speed would be

A) 10m/s B) 1000m/s C) 100,000m/s D) 10,000,000m/s

Problem 22

An electron is trapped in a region where it s potential energy decreases from one side to the other, as shown. Which of the wavefunction below is most likely to correspond to an energy eigenstate in this potential?





Problem 23

Explain why the double slit experiment for electrons provides evidence that general electron states do not have definite positions and can exist in quantum superpositions. Answer as concisely as possible. (4 points)

SINCE WE SEE INTERFERENCE, THE e^-'S MUST HAVE GONE THROUGH BOTH SLITS IN SOME SENSE $\Rightarrow$ MUST BE DELOCALIZED.

INTERFERENCE ALSO REQUIRES THAT STATES CAN BE ADDED $\rightarrow$ SUPERPOSITION MUST BE APPLICABLE.

BEFORE: AFTER:

Problem 24

A particle of mass M travels at velocity v in the $+\hat{x}$ direction. At some time, it decays into two photons, one with energy $8MeV$ traveling in the $+\hat{x}$ direction, and one with energy $2MeV$ traveling in the $-\hat{x}$ direction. Determine the velocity v and the mass M of the original particle.

(give v as a fraction of c and M in units of MeV/c^2)
(5 points)

$E_{TOT} = 10 \text{ MeV}$

$p_{TOT} = 6 \text{ MeV}/c \ \hat{x}$

$M\gamma c^2 = E_{TOT} = 10 \text{ MeV}$

$Mv\gamma = p_{TOT} = 6 \text{ MeV}/c$

$v/c = \frac{cp}{E} = 0.6$ $\quad v = 0.6c$

$\gamma = 5/4$

$M = \frac{10 \text{ MeV}}{\gamma c^2} = 8 \text{ MeV}/c^2$

Problem 25

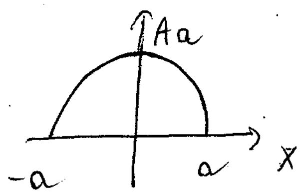
A low energy electron sitting in a ccd pixel (don't worry if you don't know what this is) has a wave function of approximately the following functional form:

$$\psi(x) = \begin{cases} A\sqrt{a^2 - x^2} & -a < x < a \\ 0 & |x| > a \end{cases}$$

where $a = 3nm$.

a) Sketch the wavefunction and determine the constant A.

(3 points)



$$1 = \int_{-\infty}^{\infty} |\psi|^2 \, dx = \int_{-a}^{a} |A|^2 (a^2 - x^2) \, dx$$

$$= |A|^2 \left(a^2 x - \frac{1}{3} x^3 \right) \Big|_{-a}^{a} =$$

$$= |A|^2 \left(2a^3 - \frac{2}{3} a^3 \right) = |A|^2 \frac{4}{3} a^3$$

$$|A|^2 = \frac{3}{4} \frac{1}{a^3}$$

$$A = \left(\frac{\sqrt{3}}{2} a^{-3/2} \right) (\text{ARBITRARY PHASE})$$

$e^{i\theta}$ $\theta \in \mathbb{R}$

b) If we perform position measurements on 1000 electrons in this same initial state, how many of the electrons would we expect to find in the region $x > 2nm$ (3 points)

$$\int_{2nm}^{3nm} |\psi(x)|^2\, dx = |A|^2 \left(a^2 x - \frac{1}{3} x^3 \right)\Big|_{2nm}^{3nm}$$

$$= \frac{3}{4} \frac{1}{(3nm)^3} \left((3nm)^3 \left[\frac{2}{3}\right] - (3m)^2 (2nm) + \frac{1}{3} (2nm)^3 \right)$$

$$= \frac{3}{4} \left(\frac{2}{3} - \frac{2}{3} + \frac{8}{81} \right) = \frac{2}{27}$$

$$\therefore \quad 1000 \frac{2}{27} \simeq 74$$

ABOUT 74 ELECTRONS

Problem 26

Abby and Max are at opposite ends of the country, 6000km apart. Abby's cell phone rings, and then Max's cell phone rings 0.01s later. Is there a frame of reference where both phones ring simultaneously? If so, how fast and in what direction (i.e. in the direction from Max to Abby or in the direction from Abby to Max) would an observer in this frame be traveling.

(3 points)

$$c\Delta t = 3\cdot 10^{8}\,\text{m/s}\;\; 0.01\,\text{s} = 3\cdot 10^{6}\,\text{m} = 3000\ \text{km}$$

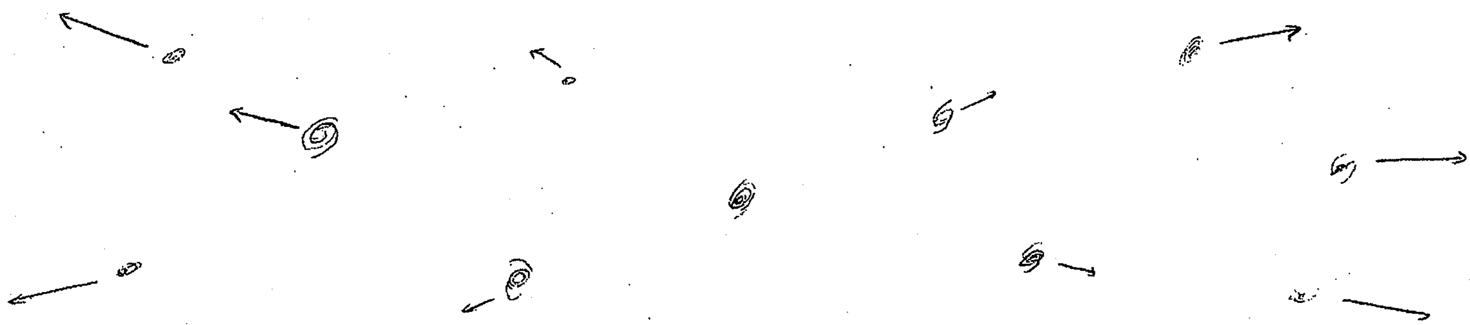
$\Delta x = 6000\ \text{km} > c\Delta t$ $\Rightarrow$ SPACELIKE

YES

$\Delta \tilde{t} = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = 0$ ← SIMULTANEOUS

$$v = \frac{\Delta t\, c}{\Delta x}\, c = \frac{1}{2} c$$ +IVE!

$\Rightarrow$ OBSERVER TRAVELING AT 0.5 c IN THE DIRECTION FROM ABBY TO MAX.



Problem 27

Hubble's Law in astronomy states that due to the expansion of the universe, objects at a distance D from our galaxy appear on average to be moving away from us at a velocity

$$v = H_0 D$$

where $H_0 = c/(1.38 \times 10^{10} lyr)$ is Hubble's constant. Astronomers spot a distant cluster of galaxies of a type that emits light with a peak wavelength of 500nm. For these galaxies, the peak wavelength that they observe is actually 1000nm.

a) How far away is this cluster of galaxies (assuming they are moving directly away from us)? (2 points)

$$\underbrace{\lambda'}_{1000\,nm} = \gamma\left(1 + v/c\right)\lambda = \sqrt{\frac{1+v/c}{1-v/c}}\ \underbrace{\lambda}_{500\,nm} \qquad \sqrt{\frac{1+v/c}{1-v/c}} = 2$$

$$1 + v/c = 4(1 - v/c) \qquad V = \frac{3}{5} c = 1.8\ 10^8\ m/s$$

$$D = V/H_0 = \frac{3/5\, c}{c/1.38 \cdot 10^{10}\, lyr} = 8.29 \cdot 10^9\ lyr$$

b) Upon further observations, the astronomers notice that one of the galaxies in the cluster actually has a peak wavelength of 1500nm. How fast is this galaxy moving relative to the rest of the cluster (assuming that it is also moving directly away from us)? (2 points)

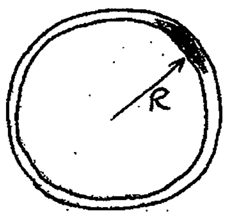
$$\sqrt{\frac{1+v/c}{1-v/c}} = \frac{1500\,nm}{500\,nm} = 3 \Rightarrow v/c = 8/10 = 4/5$$

SPEED OF FAST GALAXY WRT EARTH

SUBTRACT VELOCITIES, $u = 3/5\, c$

$$\frac{v-u}{1-uv} = \frac{4/5 - 3/5}{1 - 4/5 \cdot 3/5} c = \frac{1/5}{\frac{25-12}{25}} c = \frac{5}{13} c$$

Problem 28: An electron is trapped inside a wire which is shaped like a ring of radius R.



Determine the possible values of ANGULAR MOMENTUM for the electron. (Hint: if $0 \leq x \leq 2\pi R$ is the coordinate for the electron along the wire, the Schrödinger equation for $\phi(x)$ is the same as if the wire were straight.)

(2 points)

WAVEFUNCTION $e^{ipx/\hbar}$ IS A SOL'N TO THE SCHRÖDINGER EQUATION WITH $V = 0$.

BOUNDARY CONDITION: MUST BE SINGLE-VALUED GOING AROUND CIRCLE ONCE, $x \to x + 2\pi R$

$$e^{ip(x+2\pi R)/\hbar} = e^{ipx/\hbar}$$

$$e^{ip\,2\pi R/\hbar} = 1 \implies$$ $p\,2\pi R/\hbar$ IS A MULTIPLE OF 2π, OR

$pR/\hbar$ IS AN INTEGER, n

$$pR/\hbar = n \implies p = \frac{\hbar}{R} n$$

ANGULAR MOMENTUM $L = Rp = \hbar n$

$\therefore$ POSSIBLE VALUES ARE INTEGER MULTIPLES OF $\hbar$.

P.S. THAT WAS A HARD QUESTION!