

# PHYS 354 – Midterm 2 Exam Cheat Sheet

## Legendre Polynomials

$$\begin{aligned} P_0(\cos \theta) &= 1 & P_1(\cos \theta) &= \cos \theta \\ P_2(\cos \theta) &= \frac{3}{2} \cos^2 \theta - \frac{1}{2} & P_3(\cos \theta) &= \frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta \end{aligned}$$

## Orthogonality of Functions

$$\int_0^a \sin \frac{k\pi y}{a} \sin \frac{n\pi y}{a} dy = \begin{cases} 0, & k \neq n \\ \frac{a}{2}, & k = n \end{cases}$$

$$\int_0^\pi P_l(\cos \theta) P_n(\cos \theta) \sin \theta d\theta = \begin{cases} 0, & l \neq n \\ \frac{2}{2n+1}, & l = n \end{cases}$$

Materials    Bound Charges:  $\sigma_b = \vec{P} \cdot \hat{n}$   
 $\rho_b = -\vec{\nabla} \cdot \vec{P}$

## Multipole Expansion

$$\begin{aligned} V(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{r} d\tau' & V(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos \theta') \rho(\vec{r}') d\tau' \\ \vec{A}(\vec{r}) &= \frac{\mu_0 I}{4\pi} \oint \frac{1}{r} d\vec{l}' & \vec{A}(\vec{r}) &= \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint (r')^n P_n(\cos \theta') d\vec{l}' \end{aligned}$$

## Coulomb's Law - Scalar Potential

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{r} d\tau' \quad \vec{E} = -\vec{\nabla}V$$

## Biot-Savart Law

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}') \times \hat{r}}{r^2} d\tau'$$

## Forces

$$\vec{F}_{elec} = \int \rho \vec{E} d\tau \quad \vec{F}_{mag} = \int I d\vec{l} \times \vec{B}$$

## Vector Potential

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

## Electric and Magnetic Flux

$$\Phi_B = \int \vec{B} \cdot d\vec{a} \quad \Phi_E = \int \vec{E} \cdot d\vec{a}$$