CHAPTER 50

NUCLEAR PHYSICS

Deep within the atom lies its nucleus, occupying only 10^{-15} of the volume of the atom but providing most of its mass as well as the force that holds it together. The next goal in our study of physics is to understand the structure of the nucleus and the substructure of its components.

Our task is made easier by the many similarities between the study of atoms and the study of nuclei. Both systems are governed by the laws of quantum mechanics. Like atoms, nuclei have excited states that can decay to the ground state through the emission of photons (gamma rays). In certain circumstances, as we shall see, nuclei can exhibit shell effects that are very similar to those of atoms. We shall also see that there are differences between the study of atoms and the study of nuclei that keep us from achieving as complete an understanding of nuclei as we have of atoms.

In this chapter we study the structure of nuclei and their constituents. We consider some experimental techniques for studying their properties, and we conclude with a description of the theoretical basis for understanding the structure of nuclei.

50-1 DISCOVERING THE NUCLEUS

In the first years of the 20th century, not much was known about the structure of atoms beyond the fact that they contained electrons. This particle had been discovered (by J. J. Thomson) only in 1897, and its mass was unknown in those early days. Thus it was not possible even to say just how many electrons a given atom contained. Atoms are electrically neutral so they must also contain some positive charge, but at that time nobody knew what form this compensating positive charge took. How the electrons moved within the atom and how the mass of the atom was divided between the electrons and the positive charge were also open questions.

In 1911, Ernest Rutherford, interpreting some experiments carried out in his laboratory, was led to propose that the positive charge of the atom was densely concentrated at the center of the atom and that, furthermore, it was responsible for most of the mass of the atom. He had discovered the atomic nucleus!

Until this step had been taken, all attempts to understand the motions of the electrons within the atom were doomed to failure. Only 2 years after Rutherford's proposal, Niels Bohr used the concept of the nuclear atom to develop the semiclassical theory of atomic structure that we described in Chapter 47. This early work by Rutherford and Bohr marks the beginning of our understanding of the structure of atoms.

How did Rutherford come to make this proposal? It was not an idle conjecture but was based firmly on the results of an experiment suggested by him and carried out by his collaborators, Hans Geiger (of Geiger counter fame) and Ernest Marsden, a 20-year-old student who had not yet earned his bachelor's degree.

Rutherford's idea was to probe the forces acting within an atom by firing energetic alpha (α) particles through a thin target foil and measuring the extent to which they were deflected as they passed through the foil. Alpha particles, which are about 7300 times more massive than electrons, carry a charge of $+2e$ and are emitted spontaneously (with energies of a few MeV) by many radioactive materials. We now know that these useful projectiles are the nuclei of the atoms of ordinary helium. Figure 50-1 shows the experimental arrangement of Geiger and Marsden. The experiment consists of counting the number of α particles deflected through various scattering angles θ. (See Section 26-8.)

Figure 50-2 shows their results. Note especially that the vertical scale is logarithmic. We see that most of the α particles are scattered through rather small angles, but—and this was the big surprise—some of them are scattered through very large angles, approaching 180°. In Rutherford's words: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

Why was Rutherford so surprised? At the time of these experiments, many physicists believed in a model of the atom that had been proposed by J. J. Thomson. In Thomson's model, the positive charge of the atom was thought to be spread out through the entire volume of the atom. The electrons were thought to be distributed throughout this volume, somewhat like seeds in a watermelon, and to vibrate about their equilibrium positions within this sphere of charge.

The maximum deflecting force acting on the α particle as it passes through such a positive sphere of charge proves to be far too small to deflect the α particle by even as much as one degree. The electrons in the atom would also have very little effect on the massive, energetic α particle. They would, in fact, be themselves strongly deflected, much as a swarm of gnats would be brushed aside by a stone thrown through them. There is simply no mechanism in Thomson's atom model to account for the backward deflection of an α particle.

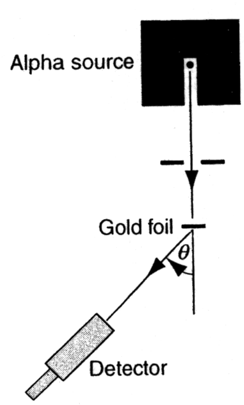

FIGURE 50-1. The experimental arrangement used in Rutherford's laboratory to study the scattering of α particles by thin metal foils. The detector can be rotated to various scattering angles θ.

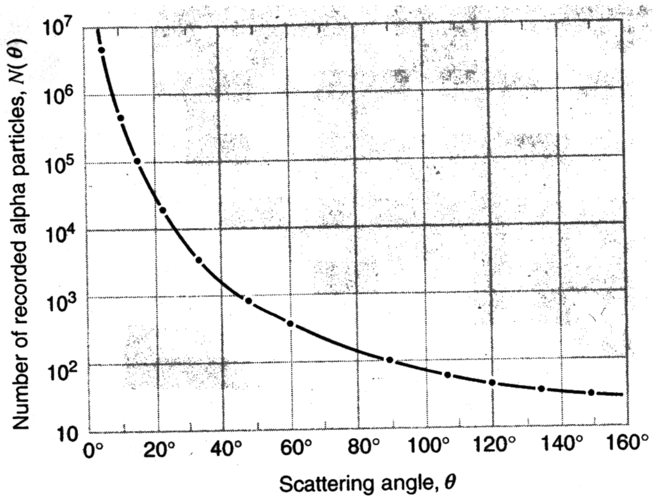

FIGURE 50-2. The dots show the α-particle scattering results from the experiments of Geiger and Marsden, and the solid curve is computed according to Rutherford's theory of the nucleus. Note that the vertical axis is marked in powers of 10.

Rutherford saw that to produce such a large deflection there must be a large force, which could be provided if the positive charge were concentrated tightly at the center of the atom, instead of being spread throughout its volume. On this model the incoming α particle can get very close to the center of the positive charge without penetrating it, resulting in a large deflecting force; see Sample Problem 50-1.

Figure 50-3 shows the paths taken by typical α particles as they pass through the atoms of the target foil. As we see, most are deflected only slightly or not at all, but a few (those whose extended incoming paths pass, by chance, close to a nucleus) are deflected through large angles. From an analysis of the data, Rutherford concluded that the dimensions of the nucleus must be smaller than the diameter

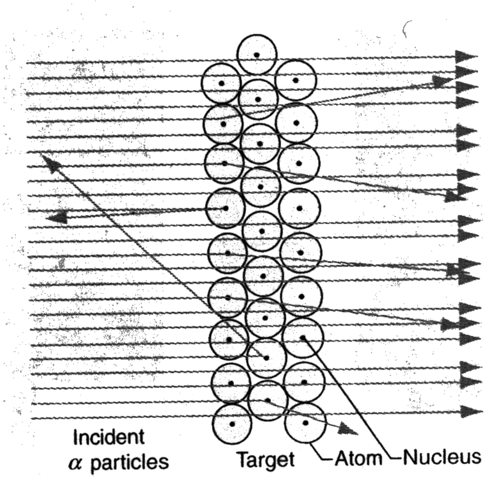

FIGURE 50-3. The angle through which an α particle is scattered depends on how close its extended incident path lies to the nucleus of an atom. Large deflections result only from very close encounters.

of an atom by a factor of about 10^4. The atom is mostly empty space! It is not often that the piercing insight of a gifted scientist, supported by a few simple calculations,* leads to results of such importance.

SAMPLE PROBLEM 50-1. A 5.30-MeV α particle happens, by chance, to be headed directly toward the nucleus of an atom of gold ($Z = 79$). How close does it get before it comes momentarily to rest and reverses its course? Neglect the recoil of the (relatively massive) gold nucleus.

Solution Initially the total mechanical energy of the two interacting particles is just equal to K_α (= 5.30 MeV), the initial kinetic energy of the α particle. The potential energy U is taken to be zero when the particles are separated by a large distance. At the moment the α particle comes to rest, the total energy is the electrostatic potential energy of the system of two particles (Eq. 28-7, $U = q_1 q_2 / 4\pi\epsilon_0 r$). Because energy must be conserved, these two quantities must be equal, or

$$K_\alpha = \frac{1}{4\pi\epsilon_0}\frac{qQ}{d}$$

in which q (= $2e$) is the charge of the α particle, Q (= $79e$) is the charge of the gold nucleus, and d is the distance between the centers of the two particles when the α particle is at rest.

Substituting for the charges and solving for d, we obtain

$$d = \frac{qQ}{4\pi\epsilon_0 K_\alpha}$$

$$= (8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})\frac{(2)(79)(1.60 \times 10^{-19}\ \mathrm{C})^2}{(5.30\ \mathrm{MeV})(1.60 \times 10^{-13}\ \mathrm{J/MeV})}$$

$$= 4.29 \times 10^{-14}\ \mathrm{m} = 42.9\ \mathrm{fm}.$$

This is a small distance by atomic standards but not by nuclear standards. As we shall see in the following section, it is considerably larger than the sum of the radii of the gold nucleus and the α particle. Thus the α particle reverses its course without ever "touching" the gold nucleus.

If the positive charge associated with the gold atom had been spread uniformly throughout the volume of the atom, the maximum retarding force acting on the α particle would have occurred at the moment the α particle began to touch the surface of the atom. This force (see Exercise 2) would have been far too weak to have had much effect on the motion of the α particle, which would have gone barreling right through such a "spongy" atom.

50-2 SOME NUCLEAR PROPERTIES

The nucleus, tiny as it may be, has a structure that is every bit as complex as that of the atom. Nuclei are made up of protons and neutrons. These particles (unlike the electron) are not true elementary particles, because they are made up of other particles, called *quarks*. However, nuclear physics—the subject of this chapter—is concerned primarily with studies of the nucleus that do not involve the internal structure of the protons and neutrons themselves. The fundamental nature of these two particles is a topic in the field of elementary particle physics, which we consider in Chapter 52.

*For an analysis of this scattering experiment, see Kenneth S. Krane, *Modern Physics*, 2nd ed. (Wiley, 1996), Chapter 6.

Nuclear Systematics

The number of protons in the nucleus (the *proton number*) is often called the atomic number and is represented by Z. The number of neutrons is called the *neutron number*, and we represent it by N. Aside from the difference in their electric charges ($q = +e$ for the proton, $q = 0$ for the neutron), the proton and the neutron are very similar particles: they have nearly equal masses and experience identical nuclear forces inside nuclei. For this reason, we classify the proton and neutron together as *nucleons*. The total number of nucleons (= $Z + N$) is called the *mass number*, and we represent it by A.

By specifying Z and A (and therefore N) we uniquely identify a particular nuclear species or *nuclide*. We use A, the total number of nucleons, as an identifying superscript in labeling nuclides. In ^{81}Br, for example, there are 81 nucleons. The symbol "Br" tells us that we are dealing with bromine, for which $Z = 35$. The remaining 46 nucleons are neutrons, so that, for this nuclide, $Z = 35$, $N = 46$, and $A = 81$. Two nuclides with the same Z but different N and A, such as ^{81}Br and ^{82}Br, are called *isotopes*.

Figure 50-4 shows a chart of the known nuclides as a plot of Z against N. The dark shading represents stable nuclides; the lighter shading represents known radioactive nuclides, or *radionuclides*. Table 50-1 shows some properties of a few selected nuclides.

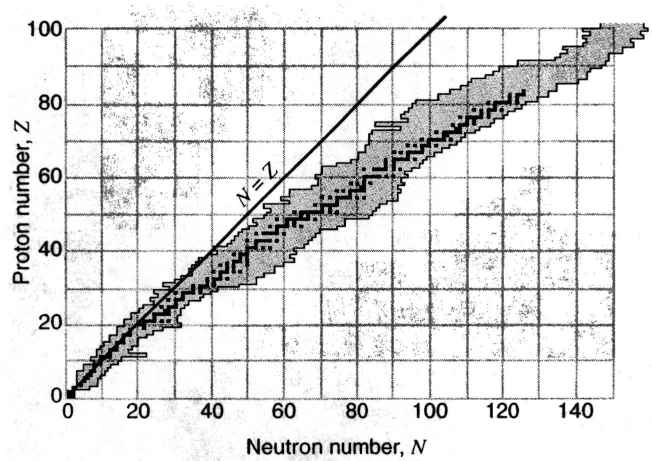

FIGURE 50-4. A plot of the known nuclides. The dark shading indicates stable nuclides and the light shading shows radioactive nuclides. Note that light stable nuclides have essentially equal numbers of protons and neutrons, whereas $N > Z$ for heavy nuclei.

TABLE 50-1 Some Properties of Selected Nuclides

Nuclide	Z	N	A	Stability[a]	Atomic Mass (u)	Radius (fm)	Binding Energy per Nucleon (MeV)	Spin ($h/2\pi$)	Magnetic Dipole Moment (μ_N)
^{7}Li	3	4	7	92.5%	7.016004	2.30	5.61	$\frac{3}{2}$	+3.26
^{14}N	7	7	14	99.6%	14.003074	2.89	7.48	1	+0.404
^{31}P	15	16	31	100%	30.973761	3.77	8.48	$\frac{1}{2}$	+1.13
^{88}Rb	37	51	88	18 m	87.911319	5.34	8.68	2	+0.508
^{120}Sn	50	70	120	32.4%	119.902197	5.92	8.50	0	0
^{157}Gd	64	93	157	15.7%	156.923957	6.47	8.20	$\frac{3}{2}$	−0.340
^{197}Au	79	118	197	100%	196.966552	6.98	7.92	$\frac{3}{2}$	+0.146
^{239}Pu	94	145	239	24,100 y	239.052157	7.45	7.56	$\frac{1}{2}$	+0.203

[a] For stable nuclides the isotopic abundance is given; for radionuclides, the half-life.

Note that there is a reasonably well-defined zone of stability in Fig. 50-4. Unstable radionuclides lie on either side of the stability zone.

The Nuclear Force

The force that controls the electronic structure and properties of the atom is the familiar Coulomb force. To bind the nucleus together, however, there must be a strong attractive force of a totally new kind acting between the neutrons and the protons. This force must be strong enough to overcome the repulsive Coulomb force between the (positively charged) protons and to bind both neutrons and protons into the tiny nuclear volume. Experiments suggest that this *strong force,* as it is simply called, has the same character between any pair of nuclear constituents, be they neutrons or protons.

The "strong force" has a short range, roughly equal to 10^{-15} m. This means that the attractive force between pairs of nucleons drops rapidly to zero for nucleon separations greater than a certain critical value. This in turn means that, except in the smallest nuclei, a given nucleon cannot interact through the strong force with all the other nucleons in the nucleus but only with a few of its nearest neighbors. By contrast, the Coulomb force is not a short-range force. A given proton in a nucleus exerts a Coulomb repulsion on all the other protons, no matter how large their separation; see Exercise 11.

Figure 50-4 shows that the lightest stable nuclides tend to lie on or close to the line $Z = N$. The heavier stable nuclides lie well below this line and thus typically have many more neutrons than protons. The tendency to an excess of neutrons at large mass numbers is a Coulomb repulsion effect. Because a given nucleon interacts with only a small number of its neighbors through the strong force, the amount of energy tied up in strong-force bonds between nucleons increases in proportion to A. The energy tied up in Coulomb-force bonds between protons increases more rapidly than this because each proton interacts with all other protons in the nucleus. Thus the Coulomb energy becomes increasingly important at high mass numbers.

Consider a nucleus with 238 nucleons. If it were to lie on the $Z = N$ line, it would have $Z = N = 119$. However, such a nucleus, if it could be assembled, would fly apart at once because of Coulomb repulsion. Relative stability is found only if we replace 27 of the protons by neutrons, thus greatly diluting the Coulomb repulsion effect. We then would have the nuclide ^{238}U, which has $Z = 92$ and $N = 146$, a neutron excess of 54.

Even in ^{238}U, Coulomb effects are evident in that (1) this nuclide is radioactive and emits α particles, and (2) it can easily break up (fission) into two fragments. Both of these processes reduce the Coulomb energy more than they do the energy in the strong-force bonds.

Nuclear Radii

We have used the Bohr radius a_0 ($= 5.29 \times 10^{-11}$ m $=$ 52.9 pm) as a convenient unit for measuring the dimensions of atoms. Nuclei are smaller by a factor of about 10^4, and a convenient unit for measuring distances of this scale is the *femtometer* ($= 10^{-15}$ m). This unit is often called the *fermi* and shares the same abbreviation. Thus

$$1 \text{ fermi} = 1 \text{ femtometer} = 1 \text{ fm} = 10^{-15} \text{ m}.$$

We can learn about the size and structure of nuclei by doing scattering experiments, much as suggested by Fig. 50-1, using an incident beam of high-energy electrons. The energy of the incident electrons must be high enough (> 200 MeV) so that their de Broglie wavelength will be small enough for them to act as structure-sensitive nuclear probes. In effect, these experiments measure the diffraction pattern of the scattered particles and so deduce the shape of the scattering object (the nucleus).

From a variety of scattering experiments, the nuclear density has been deduced to be of the form shown in Fig. 50-5. We see that the nucleus does not have a sharply defined surface. It does, however, have a characteristic mean

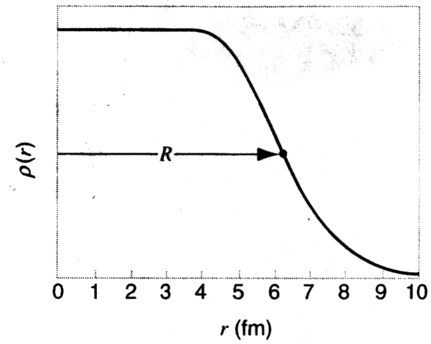

FIGURE 50-5. The variation with radial distance of the density of a nucleus of ^{197}Au.

radius R. The density $\rho(r)$ has a constant value in the nuclear interior and drops to zero through the fuzzy surface zone. From these experiments it has been found that R increases with A approximately as

$$R = R_0 A^{1/3}, \qquad (50\text{-}1)$$

in which A is the mass number and R_0 is a constant with a value of about 1.2 fm. For ^{63}Cu, for example,

$$R = (1.2 \text{ fm})(63)^{1/3} = 4.8 \text{ fm}.$$

By comparison, the mean radius of a copper ion in a lattice of solid copper is 1.8 Bohr radii, about 2×10^4 times larger.

Nuclear Masses and Binding Energies

Atomic masses can be measured with great precision using modern mass spectrometer and nuclear reaction techniques. We recall that such masses are measured in *unified atomic mass units* (abbreviation u), chosen so that the atomic mass (*not* the nuclear mass) of ^{12}C is exactly 12 u. The relation of this unit to the SI mass standard is, to six significant figures,

$$1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg}.$$

Note that the mass number (symbol A) identifying a nuclide is so named because this number is equal to the atomic mass of the nuclide, rounded to the nearest integer. Thus the mass number of the nuclide ^{137}Cs is 137. This nuclide contains 55 protons and 82 neutrons, a total of 137 particles; its atomic mass is 136.907084 u, which rounds off numerically to 137.

In nuclear physics, as contrasted with atomic physics, the energy changes per event are commonly so great that Einstein's well-known mass–energy relation $E = \Delta mc^2$ is an indispensable workaday tool. We shall often need to use the energy equivalent of 1 atomic mass unit, and we find it (using constants evaluated to six significant figures) from

$$E = \Delta m\, c^2 = \frac{(1.66054 \times 10^{-27} \text{ kg})(2.99792 \times 10^8 \text{ m/s})^2}{1.60218 \times 10^{-13} \text{ J/MeV}}$$

$$= 931.5 \text{ MeV}.$$

This means that we can write c^2 as 931.5 MeV/u and thus easily find the energy equivalent (in MeV) of any m or mass difference (in u), or conversely.

As an example, consider the deuteron, the nucleus the heavy hydrogen atom. A deuteron consists of a pro and a neutron bound together by the strong force. The ergy E_B that we must add to the deuteron to tear it ap into its two constituent nucleons is called its *binding ergy.* In effect, the binding energy is a measure of the to internal energy of the nucleus, due in part to the stro force between the nucleons, the Coulomb force betw the nucleons, and the kinetic energies of the nucleons re tive to the center of mass of the entire nucleus. From c servation of energy we can write, for this pulling-ap process,

$$m_d c^2 + E_B = m_n c^2 + m_p c^2. \qquad (50$$

If we add $m_e c^2$, the energy equivalent of one electron ma to each side of this equation, we have

$$(m_d + m_e)c^2 + E_B = m_n c^2 + (m_p + m_e)c^2$$

or

$$m(^2\text{H})c^2 + E_B = m_n c^2 + m(^1\text{H})c^2. \qquad (50$$

Here $m(^2$H) and $m(^1$H) are the masses of the neutral *he* hydrogen atom and the neutral *ordinary* hydrogen atom, spectively. They are atomic masses, not nuclear mass Solving Eq. 50-3 for E_B yields

$$E_B = [m_n + m(^1\text{H}) - m(^2\text{H})]c^2 = \Delta mc^2, \qquad (50$$

in which Δm is the mass difference. In making calculati of this kind we always use atomic, rather than nucle masses, as this is what is normally tabulated. As in this ample, the electron masses conveniently cancel.*

For the deuteron calculation the needed masses are m_n 1.008665 u, $m(^1$H) = 1.007825 u, and $m(^2$H) = 2.01410 The mass difference in Eq. 50-4 is

$$\Delta m = 1.008665 \text{ u} + 1.007825 \text{ u} - 2.014102 \text{ u}$$

$$= 0.002388 \text{ u}.$$

Substituting into Eq. 50-4 and replacing c^2 by its equi lent, 931.5 MeV/u, we find the binding energy to be

$$E_B = (0.002388 \text{ u})(931.5 \text{ MeV/u}) = 2.224 \text{ MeV}.$$

Compare this with the binding energy of the hydrogen at in its ground state, which is 13.6 eV, about five orders magnitude smaller.

If we divide the binding energy of a nucleus by its m number, we get the average binding energy per nucleon property we have listed in Table 50-1. Figure 50-6 show plot of this quantity as a function of mass number. The f that this *binding energy curve* "droops" at both high a

*See, however, Problem 11, for an exception.

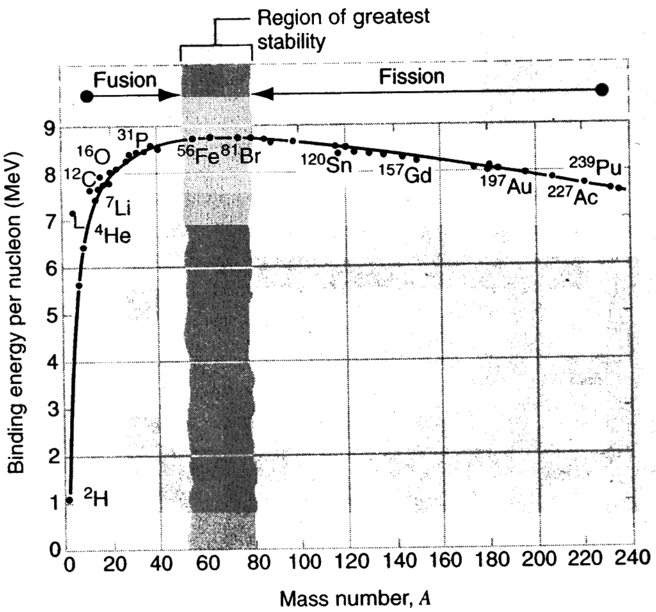

FIGURE 50-6. The binding energy per nucleon over the range of mass numbers. Some of the nuclides of Table 50-1 are identified, along with a few others. The region of greatest stability corresponds to mass numbers from about 50 to 80.

low mass numbers has practical consequences of the greatest importance.*

The drooping of the binding energy curve at high mass numbers tells us that nucleons are more tightly bound when they are assembled into two middle-mass nuclei rather than into a single high-mass nucleus. In other words, energy can be released in the *nuclear fission* of a single massive nucleus into two smaller fragments.

The drooping of the binding energy curve at low mass numbers, on the other hand, tells us that energy will be released if two nuclei of small mass number combine to form a single middle-mass nucleus. This process, the reverse of fission, is called *nuclear fusion.* It occurs inside our Sun and other stars and is the mechanism by which the Sun generates the energy it radiates to us.

Nuclear Spin and Magnetism

Nuclei, like atoms, have an intrinsic angular momentum whose maximum component along any chosen z axis is given by $J(h/2\pi)$. Here J is a quantum number, which may be integral or half-integral, called the *nuclear spin;* some values for selected nuclides are shown in Table 50-1.

Again as for atoms, a nuclear angular momentum has a nuclear magnetic moment associated with it. Recall (see Section 35-3) that in atomic magnetism, the *Bohr magneton* μ_B, defined as

$$\mu_B = \frac{eh}{4\pi m_e} = 5.79 \times 10^{-5}\ \text{eV/T},$$

is a unit of convenience. In nuclear physics the corresponding unit of convenience is the *nuclear magneton* μ_N, defined similarly to the Bohr magneton except that the electron mass m_e is replaced by the proton mass m_p. That is,

$$\mu_N = \frac{eh}{4\pi m_p} = 3.15 \times 10^{-8}\ \text{eV/T}.$$

Because the magnetic moment of the free electron is (very nearly) one Bohr magneton, it might be supposed that the magnetic moment of the free proton would be (very nearly) one nuclear magneton. It is not very close, however, the measured value being $+2.7928\ \mu_N$. To understand the magnetic moments of the proton and neutron, it is necessary to consider their internal structure. The magnetic moments of heavier nuclei can in turn be analyzed in terms of the magnetic moments of the constituent protons and neutrons.

SAMPLE PROBLEM 50-2. What is the approximate density of the *nuclear matter* from which all nuclei are made?

Solution We know that this density is large, because virtually all the mass of the atom resides in its tiny nucleus. The volume of the nucleus, approximated as a uniform sphere of radius R, is given by Eq. 50-1 as

$$V = \tfrac{4}{3}\pi R^3 = \tfrac{4}{3}\pi(R_0^3 A).$$

The density ρ_n of nuclear matter, expressed in nucleons per unit volume, is then

$$\rho_n = \frac{A}{V} = \frac{A}{(4\pi/3)R_0^3 A}$$

$$= \frac{1}{(4\pi/3)(1.2\ \text{fm})^3} = 0.14\ \text{nucleons/fm}^3.$$

The mass of a nucleon is 1.7×10^{-27} kg. The *mass* density ρ_m of nuclear matter is then

$$\rho_m = (0.14\ \text{nucleons/fm}^3)(1.7 \times 10^{-27}\ \text{kg/nucleon}) \times (1\ \text{fm}/10^{-15}\ \text{m})^3 = 2.4 \times 10^{17}\ \text{kg/m}^3,$$

or 2.4×10^{14} times the density of water! Unlike the orbital electrons, the nuclides have a density nearly independent of the number of their nucleons. To some extent nucleons are packed in like marbles in a bag.

SAMPLE PROBLEM 50-3. Imagine that a typical middle-mass nucleus such as ^{120}Sn is picked apart into its constituent protons and neutrons. Find (*a*) the total energy required and (*b*) the energy per nucleon. The atomic mass of ^{120}Sn is 119.902197 u; see Table 50-1.

Solution (*a*) ^{120}Sn contains 50 protons and 120 − 50 = 70 neutrons. The combined atomic mass of these free particles is

$$M = Zm_p + Nm_n = 50 \times 1.007825\ \text{u} + 70 \times 1.008665\ \text{u}$$
$$= 120.997800\ \text{u}$$

*__The Curve of Binding Energy__ has even been adopted as the title of a book

This exceeds the atomic mass of ^{120}Sn by

$$\Delta m = 120.997800 \text{ u} - 119.902197 \text{ u} = 1.095603 \text{ u}.$$

Converting this to a rest energy yields the total binding energy,

$$E_B = \Delta mc^2 = (1.0956 \text{ u})(931.5 \text{ MeV/u}) = 1020.6 \text{ MeV}.$$

(*b*) The binding energy E per nucleon is

$$E = \frac{E_B}{A} = \frac{1020.6 \text{ MeV}}{120} = 8.50 \text{ MeV/nucleon}.$$

This agrees with the value that may be read from the curve of Fig. 50-6.

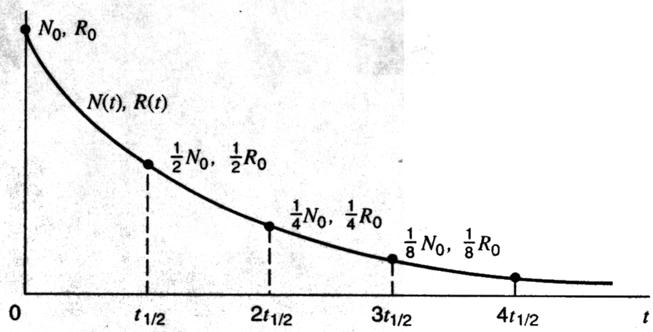

FIGURE 50-7. The exponential decrease in the number of radioactive nuclei and their decay rate.

50-3 RADIOACTIVE DECAY

As Fig. 50-4 shows, most of the nuclides that have been identified are radioactive. That is, they spontaneously emit a particle, transforming themselves in the process into a different nuclide. In this chapter we discuss the two most common situations: the emission of an α particle (alpha decay) and the emission of an electron (beta decay).

No matter what the nature of the decay, its main feature is that it is statistical. Consider, for example, a 1-mg sample of uranium metal. It contains 2.5×10^{18} atoms of the very long-lived alpha emitter ^{238}U. The nuclei of these atoms have existed without decaying since they were created (before the formation of our solar system) in the explosion of a supernova.

During any given second about 12 of the nuclei in our sample will decay, emitting an α particle in the process. We have absolutely no way of predicting, however, whether any given nucleus in the sample will be among those that do so. Every single ^{238}U nucleus has exactly the same probability as any other to decay during any 1-s observation period—namely, $12/(2.5 \times 10^{18})$, or one chance in 2×10^{17}.

In general, if a sample contains N radioactive nuclei, we can express the statistical character of the decay process by saying that the rate of change of the number of nuclei, or $-dN/dt$ (where the minus sign makes this a positive quantity), is proportional to the number of nuclei present, or

$$-\frac{dN}{dt} = \lambda N, \tag{50-5}$$

in which the proportionality constant λ, called the *disintegration constant,* has a different characteristic value for each radioactive nuclide. We can rewrite Eq. 50-5 as

$$\frac{dN}{N} = -\lambda\, dt,$$

which integrates readily to

$$N = N_0 e^{-\lambda t}. \tag{50-6}$$

Here N_0 is the number of radioactive nuclei in the sample at $t = 0$. We see that the decrease of N with time follows a simple exponential law.

We are often more interested in the *activity* or decay rate R ($= \lambda N$) of the sample than we are in N. Multiplying Eq. 50-6 by λ gives

$$R = R_0 e^{-\lambda t}, \tag{50-7}$$

in which R_0 ($= \lambda N_0$) is the decay rate at $t = 0$. The number of nuclei present and the decay rate both follow the same exponential law.

A quantity of interest is the time $t_{1/2}$, called the *half-life,* after which both N and R are reduced to one-half of their initial values. Putting $R = \frac{1}{2}R_0$ in Eq. 50-7 gives

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda t_{1/2}},$$

which leads readily to

$$t_{1/2} = \frac{\ln 2}{\lambda}, \tag{50-8}$$

a relationship between the half-life and the disintegration constant. Figure 50-7 shows how both N and R decrease exponentially with time.

The following two sample problems show how λ can be measured for decay processes with relatively short half-lives and also with relatively long half-lives.

SAMPLE PROBLEM 50-4. In short-lived decays, it is possible to measure directly the decrease in the decay rate R with time. The following table gives some data for a sample of ^{128}I, a radionuclide often used medically as a tracer to measure the iodine uptake rate of the thyroid gland. Find (*a*) the disintegration constant λ and (*b*) the half-life $t_{1/2}$ from these data.

Time (min)	*R* (counts/s)	*Time* (min)	*R* (counts/s)
4	392.2	132	10.9
36	161.4	164	4.56
68	65.5	196	1.86
100	26.8	218	1.00

Solution (*a*) If we take the natural logarithm of each side of Eq. 50-7, we find that

$$\ln R = \ln R_0 - \lambda t. \tag{50-9}$$

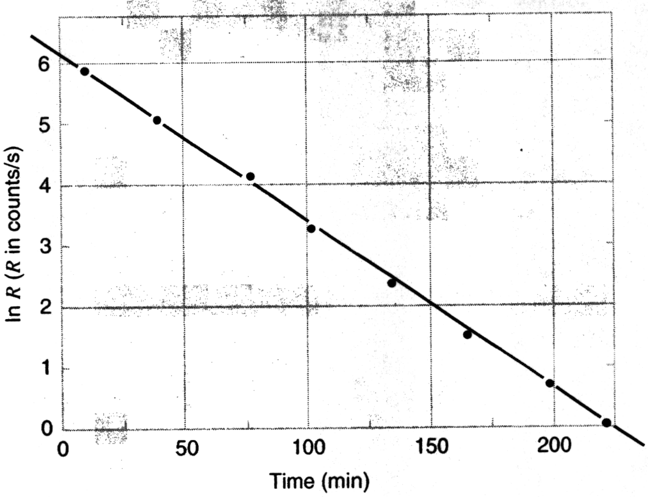

FIGURE 50-8. Sample Problem 50-4. A logarithmic plot of the decay data is fitted by a straight line, showing the exponential nature of the decay. The disintegration constant λ can be found from the slope of the line.

Thus if we plot the natural logarithm of R against t, we should obtain a straight line whose slope is $-\lambda$. Figure 50-8 shows such a plot, using the data given for ^{128}I. Equating the slope of the line to $-\lambda$ yields

$$-\lambda = -\frac{(6.1 - 0)}{(225 \text{ min} - 0)}$$

or

$$\lambda = 0.0271 \text{ min}^{-1}.$$

(*b*) Equation 50-8 yields for $t_{1/2}$:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{0.0271 \text{ min}^{-1}} = 25.6 \text{ min}.$$

SAMPLE PROBLEM 50-5. A 1.00-g sample of pure KCl from the chemistry stockroom is found to be radioactive and to decay at an absolute rate R of 1600 counts/s. The decays are traced to the element potassium and in particular to the isotope ^{40}K, which constitutes 1.18% of normal potassium. What is the half-life for this decay?

Solution In the case of long-lived decays, it is not possible to wait long enough to observe a measurable decrease in the decay rate R with time. We must find λ by measuring both N and $-dN/dt$ in Eq. 50-5. The molar mass of KCl is 74.9 g/mol, so the number of potassium atoms in the sample is

$$N_K = \frac{(6.02 \times 10^{23} \text{ mol}^{-1})(1.00 \text{ g})}{74.9 \text{ g/mol}} = 8.04 \times 10^{21}.$$

The number of ^{40}K atoms is 1.18% of N_K, or

$$N_{40} = (0.0118)(8.04 \times 10^{21}) = 9.49 \times 10^{19}.$$

From Eq. 50-5 we have

$$\lambda = \frac{-dN/dt}{N} = \frac{R}{N_{40}} = \frac{1600 \text{ s}^{-1}}{9.49 \times 10^{19}} = 1.69 \times 10^{-17} \text{ s}^{-1},$$

and the half-life, from Eq. 50-8, is

$$t_{1/2} = \frac{\ln 2}{\lambda} = \left(\frac{0.693}{1.69 \times 10^{-17} \text{ s}^{-1}}\right)\left(\frac{1 \text{ y}}{3.16 \times 10^7 \text{ s}}\right)$$
$$= 1.30 \times 10^9 \text{ y}.$$

This is of the order of magnitude of the age of the universe. No wonder we cannot measure the half-life of this nuclide by waiting around for its decay rate to decrease! (Interestingly, the potassium in our own bodies has its normal share of the ^{40}K isotope. We are all slightly radioactive.)

50-4 ALPHA DECAY

In a radioactive decay process, the nucleus of an atom with mass m_i undergoes a transformation usually leading to a different nuclide, with the accompanying emission of one or more additional particles. The total mass of all the final particles is m_f. The decay will be possible only if $m_i > m_f$; put another way, according to conservation of energy the total energy before the decay (the rest energy m_ic^2 of the initial atom, which we assume to be at rest in our reference frame) must equal the total energy after the decay (the total rest energy m_fc^2 of all the final products plus their total kinetic energy K_f):

$$m_ic^2 = m_fc^2 + K_f. \tag{50-10}$$

It is convenient to define the *Q-value for the decay* to be the difference between the initial and final rest energies:

$$Q = m_ic^2 - m_fc^2. \tag{50-11}$$

In effect, Q is the energy released in the decay. Comparison with Eq. 50-10 shows that, if the initial decaying nucleus is at rest, the Q-value gives the total kinetic energy of all the decay products:

$$Q = K_f. \tag{50-12}$$

The energy Q is shared among the decay products consistent with conservation of momentum.

From Eq. 50-12 we can clearly see that the decay is possible only for $Q > 0$, and thus according to Eq. 50-11 m_i must be greater than m_f, as we have already concluded.

Let us apply these concepts to a specific decay process—namely, to alpha decay in which an alpha (α) particle (^{4}He nucleus) is emitted by a nucleus. We consider as an example the decay of ^{238}U. In emitting an α particle, the ^{238}U nucleus loses two protons (from 92 to 90) and two neutrons (from 146 to 144). According to the periodic table, the element with atomic number 90 is Th, and so the complete decay process is

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^4\text{He}. \tag{50-13}$$

In Sample Problem 50-6 we show that the decay energy (Q value) for this process is 4.27 MeV, which is shared between the α particle and the recoiling ^{234}Th nucleus. The measured half-life for this decay is 4.47×10^9 y.

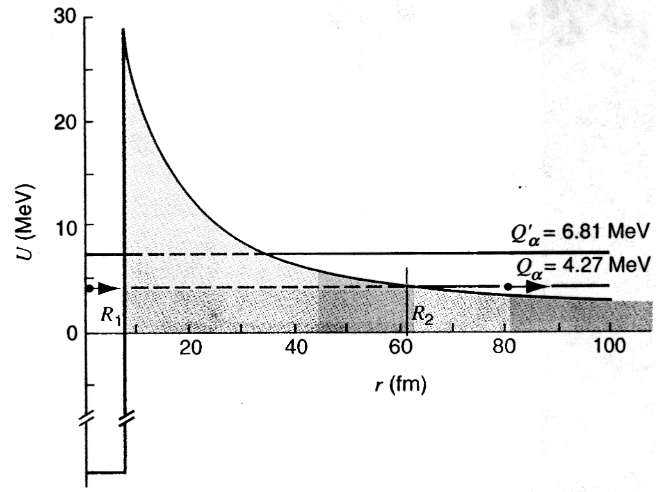

FIGURE 50-9. A potential energy function representing the emission of α particles by ^{238}U. The shaded area represents the potential barrier that inhibits the decay process. The horizontal lines represent the decay energies of ^{238}U (4.27 MeV) and ^{228}U (6.81 MeV).

We now ask ourselves: "If energy is released in every such decay event, why did the ^{238}U nuclei not decay shortly after they were created?" The creation process is believed to have occurred in the violent explosions of ancestral stars (supernovas), predating the formation of our solar system. Why did these nuclei wait so very long before getting rid of their excess energy by emitting an α particle? To answer this question, we must study the detailed mechanism of alpha decay.

We choose a model in which the α particle is imagined to exist preformed inside the nucleus before it escapes. Figure 50-9 shows the approximate potential energy function $U(r)$ for the α particle and the residual ^{234}Th nucleus as a function of their separation. It is a combination of a potential well associated with the (attractive) strong nuclear force that acts in the nuclear interior ($r < R_1$) and a Coulomb potential associated with the (repulsive) electrostatic force that acts between the two particles after the decay has occurred ($r > R_1$).

The horizontal line marked $Q_\alpha = 4.27$ MeV shows the disintegration energy for the process, as calculated in Sample Problem 50-6. Note that this line intersects the potential energy curve at two points, R_1 and R_2. We now see why the α particle is not immediately emitted from the ^{238}U nucleus! That nucleus is surrounded by an impressive potential barrier, shown by the shaded area in Fig. 50-9. Visualize this barrier as a spherical shell whose inner radius is R_1 and whose outer radius is R_2, its volume being forbidden to the α particle under the laws of classical physics. If the α particle found itself in that region, its potential energy U would exceed its total energy E, which would mean, classically, that its kinetic energy K ($= E - U$) would be negative, an impossible situation.

TABLE 50-2 The Alpha Decay of ^{238}U and ^{228}U

Nuclide	Q_α	*Half-life*
^{238}U	4.27 MeV	4.5×10^9 y
^{228}U	6.81 MeV	550 s

Indeed, we now change our question and ask: "How can the ^{238}U nucleus *ever* emit an α particle?" The α particle seems permanently trapped inside the nucleus by the barrier.

The answer is that, as we learned in Section 46-7, in wave mechanics there is always a chance (described by Eq. 46-24) that a particle can tunnel through a barrier that is classically insurmountable. In fact, the explanation of alpha decay by wave mechanical barrier tunneling was one of the very first applications of the new quantum physics.

For the long-lived decay of ^{238}U the barrier is actually not very "leaky." We can show that the α particle, presumed to be rattling back and forth within the nucleus, must present itself at the inner surface of the barrier about 10^{38} times before it succeeds in tunneling through. This is about 10^{20} times per second for about 10^9 years! We, of course, are waiting on the outside, taking note of only those α particles that *do* manage to escape.

We can test this barrier-tunneling explanation of alpha decay by looking at other alpha emitters, for which the barrier would be different. For an extreme contrast, consider the alpha decay of another uranium nuclide, ^{228}U, which has a disintegration energy Q'_α of 6.81 MeV, as shown in Fig. 50-9. The barrier in this case is both thinner (compare the lengths of the dashed lines in Fig. 50-9) and lower (compare the heights of the barrier above the dashed lines); if our barrier tunneling notions are correct, we would expect alpha decay to occur more readily for ^{228}U than for ^{238}U. Indeed it does. As Table 50-2 shows, the half-life of ^{228}U is only 550 s! We recall from Section 46-7 that the transmission coefficient of a barrier—because of the exponential nature of Eq. 46-24—is very sensitive to small changes in the dimensions of the barrier. We see that an increase in Q_α by a factor of only 1.6 produces a decrease in half-life (that is, in the effectiveness of barrier tunneling) by a factor of 3×10^{14}.

SAMPLE PROBLEM 50-6. (*a*) Find the energy released during the alpha decay of ^{238}U. (*b*) Show that this nuclide cannot spontaneously emit a proton. The needed atomic masses are

^{238}U	238.050783 u	^{4}He	4.002603 u
^{234}Th	234.043596 u	^{1}H	1.007825 u
^{237}Pa	237.051144 u.		

Solution (*a*) In the alpha decay process of Eq. 50-13 the total atomic mass m_f of the decay products ^{234}Th + ^{4}He is 238.046199 u. According to Eq. 50-11, the Q-value is

$$Q = (m_i - m_f)c^2$$
$$= (238.050783\text{ u} - 238.046199\text{ u})(931.5\text{ MeV/u})$$
$$= 4.27\text{ MeV}.$$

This energy is available to share as kinetic energy between the α particle and the recoiling ^{234}Th atom.
(*b*) If ^{238}U were to emit a proton, the decay process would be

$$^{238}\text{U} \rightarrow {}^{237}\text{Pa} + {}^{1}\text{H}.$$

In this case the Q-value is

$$Q = (m_i - m_f)c^2$$
$$= (238.050783\text{ u} - 238.058969\text{ u})(931.5\text{ MeV/u})$$
$$= -7.62\text{ MeV}.$$

The negative Q-value shows that ^{238}U is stable against spontaneous proton emission.

50-5 BETA DECAY

A nucleus that decays spontaneously by emitting an electron (either positive or negative) is said to undergo *beta decay.** Here are two examples:

$$^{32}\text{P} \rightarrow {}^{32}\text{S} + e^- + \bar{\nu} \qquad (t_{1/2} = 14.3\text{ d}) \qquad (50\text{-}14)$$

and

$$^{64}\text{Cu} \rightarrow {}^{64}\text{Ni} + e^+ + \nu \qquad (t_{1/2} = 12.7\text{ h}). \qquad (50\text{-}15)$$

The symbols ν and $\bar{\nu}$ represent the *neutrino* and its antiparticle, the *antineutrino,* neutral particles that are emitted from the nucleus along with the electron or positron (positive electron) during the decay process. Neutrinos interact only very weakly with matter and—for that reason—are so extremely difficult to detect that, for many years, their presence went unnoticed. We consider the fundamental nature and importance of these elusive particles in Chapter 52.

It may seem surprising that nuclei can emit electrons (and neutrinos) in view of the fact that we have said that nuclei are made up of neutrons and protons only. However, we saw earlier that atoms emit photons, and we certainly do not say that atoms "contain" photons. We say that the photons are created during the emission process.

So it is with the electrons and the neutrinos emitted from nuclei during beta decay. They are both created during the emission process, a neutron transforming itself into a proton within the nucleus (or conversely) according to

$$\text{n} \rightarrow \text{p} + e^- + \bar{\nu} \qquad (\beta^- \text{ decay}) \qquad (50\text{-}16)$$

or

$$\text{p} \rightarrow \text{n} + e^+ + \nu \qquad (\beta^+ \text{ decay}). \qquad (50\text{-}17)$$

These are the basic beta-decay processes.

*Beta decay also includes electron capture, in which a nucleus decays by absorbing one of its orbital electrons. We do not consider that process here.

In any decay process, the amount of energy released is uniquely determined by the difference in rest energy between the initial nucleus and the final nucleus plus decay products (see Eq. 50-11). In a particular alpha-decay process, such as that of ^{238}U, every emitted α particle carries the same kinetic energy. In beta decay, however, the kinetic energy of the emitted electrons is not uniquely determined. Instead, the emitted electrons have a continuous spectrum of energies, from zero up to a maximum K_{max}, as Fig. 50-10 illustrates for the beta decay of ^{64}Cu (Eq. 50-15).

For many years, before the neutrino was identified, curves such as that of Fig. 50-10 were a challenging puzzle. They suggested that some energy was "missing" in the decay process and led many reputable physicists, including Niels Bohr, to speculate that perhaps the law of conservation of energy might be valid only statistically in such decays.

The answer to this puzzle lies in the emission of the neutrino or antineutrino, which carries a share of the decay energy. If we were to measure the energies of both particles (electron and antineutrino or positron and neutrino) in a particular decay process and add them up, we would come out every time with the same fixed value, equal to the disintegration energy. Energy is indeed conserved in each individual decay process.

The existence of an undetected particle as a solution to the missing energy problem was proposed by Pauli in 1931, and the neutrino was made a part of a formal theory of beta decay by Fermi in 1934. Nevertheless, it took another 20 years before neutrinos were detected in the laboratory. The difficulty in their measurement results from their exceedingly weak interactions with matter—their mean free path through solid matter is of the order of several thousand light years. Today neutrino physics is an important subfield of nuclear and particle physics, and its practitioners study neutrinos emitted from radioactive sources or produced in nuclear reactions on Earth, in the Sun, or in supernovas.

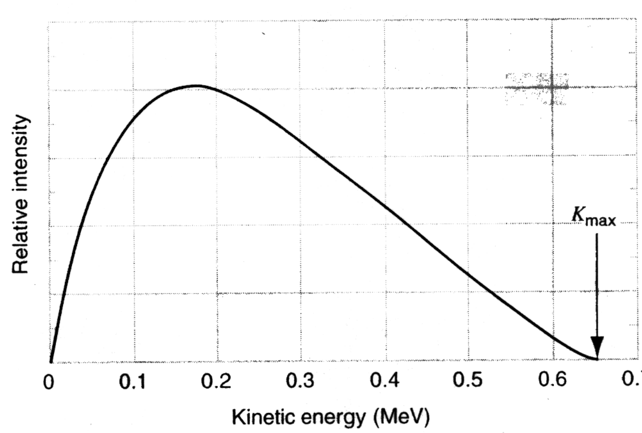

FIGURE 50-10. The kinetic energy distribution of the positrons emitted in the beta decay of ^{64}Cu. The maximum kinetic energy is 0.653 MeV.

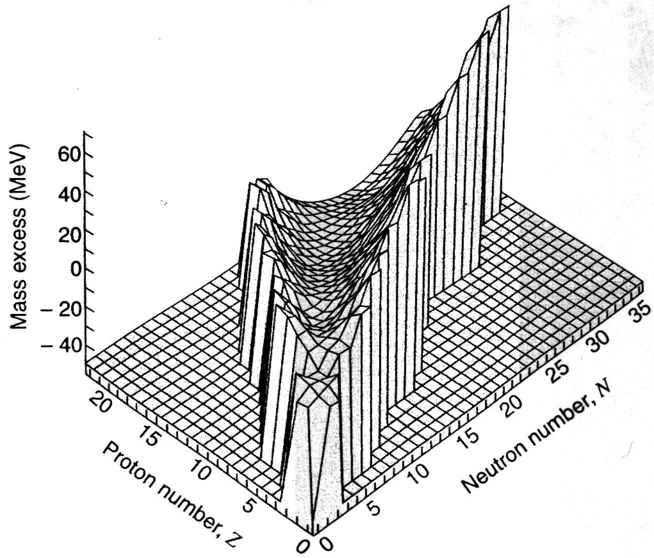

FIGURE 50-11. A portion of the valley of the nuclides, showing only the lightest nuclides. The quantity plotted on the vertical axis is the mass excess, defined as $(m - A)c^2$, where m is the atomic mass in u.

Our study of alpha and beta decay permits us to look at the nuclidic chart of Fig. 50-4 in a new way. Let us construct a three-dimensional surface by plotting the mass of each nuclide in a direction at right angles to the NZ plane of that figure. The surface so formed gives a graphic representation of nuclear stability. As Fig. 50-11 shows (for the light nuclides), it describes a "valley of the nuclides," the stability zone of Fig. 50-4 running along its bottom. Nuclides on the headwall of the valley (a region not displayed in Fig. 50-11) decay into it largely by chains of alpha decay and by spontaneous fission. Nuclides on the proton-rich side of the valley decay into it by emitting positive electrons and those on the neutron-rich side do so by emitting negative electrons.

SAMPLE PROBLEM 50-7. Calculate the disintegration energy (Q-value) in the beta decay of ^{32}P, as described by Eq. 50-14. The needed atomic masses are 31.973907 u for ^{32}P and 31.972071 u for ^{32}S.

Solution Because of the presence of the emitted electron, we must be especially careful to distinguish between nuclear and atomic masses. Let m' represent the nuclear masses of ^{32}P and ^{32}S, and let m represent their atomic masses. We take the disintegration energy Q to be $\Delta m\, c^2$, where

$$\Delta m = m'(^{32}\text{P}) - [m'(^{32}\text{S}) + m_e],$$

with m_e the mass of the electron and the neutrino assumed to be massless. If we add and subtract $15m_e$ on the right-hand side, we have

$$\Delta m = [m'(^{32}\text{P}) + 15m_e] - [m'(^{32}\text{S}) + 16m_e].$$

The quantities in brackets are the atomic masses. Thus we have

$$\Delta m = m(^{32}\text{P}) - m(^{32}\text{S}).$$

If we subtract the atomic masses in this way, the mass of the emitted electron is automatically taken into account.*

The disintegration energy for the ^{32}P decay is then

$$Q = \Delta m\, c^2 = (31.973907 \text{ u} - 31.972071 \text{ u})(931.5 \text{ MeV/u}) = 1.71 \text{ MeV}.$$

This is just equal to the measured value of K_{max}, the maximum energy of the emitted electrons. Thus although 1.71 MeV is released every time a ^{32}P nucleus decays, in essentially every case the electron carries away less energy than this. The neutrino gets the rest, carrying it away from the laboratory undetected. (A negligible share, of the order of eV, also goes to the ^{32}S nucleus in order to conserve momentum in the decay.)

50-6 MEASURING IONIZING RADIATION†

When radiations such as x rays, gamma rays, beta particles, or alpha particles encounter an atom, they can cause the atom to eject electrons and to become ionized. Because ionization can damage individual cells of living tissue, the effects of ionizing radiations have become a matter of general public interest. Such radiations arise in nature from the cosmic rays and also from radioactive elements in the Earth's crust. Artificially produced radiations also contribute, including diagnostic and therapeutic x rays and radiations from radionuclides used in medicine and in industry. The disposal of radioactive waste and the evaluation of the probabilities of nuclear accidents continue to be dealt with at the level of national policy.

It is not our task here to explore the various sources of ionizing radiations but simply to describe the units in which the properties and effects of these radiations are expressed. There are four such units, and they are often used loosely or incorrectly in popular reporting.

1. *The curie (abbreviation Ci).* This is a measure of the *activity* or rate of decay of a radioactive source. It was originally defined as the activity of 1 g of radium in equilibrium with its by-products, but it is now defined simply as

$$1 \text{ curie} = 3.7 \times 10^{10} \text{ disintegrations per second.}$$

This definition says nothing about the nature of the decays or the amount of energy released in the decay. Note also that this unit is not appropriate to describe the ionizing effects of x rays from, say, a medical x-ray machine. The radiations must be emitted from a radionuclide.

*This is not the case for positron decay or for electron capture; see Problems 11 and 12. Note also that in this sample problem we neglect the (small) difference in the binding energies of the atomic electrons before and after the beta decay.

†See "Radiation Exposure in Our Daily Lives," by Stewart C. Bushong, *The Physics Teacher,* March 1977, p. 135.

An example of the proper use of the curie is the statement: "One milligram of ^{239}Pu has an activity of 62 μCi." The fact that ^{239}Pu is an alpha emitter does not enter.

2. *The roentgen (abbreviation R).* This is a measure of *exposure*—that is, of the ability of a beam of x rays or gamma rays to produce ions in a particular substance. Specifically, one roentgen is defined as that exposure that would produce 1.6×10^{12} ion pairs per gram of air, the air being dry and at standard temperature and pressure. We might say, for example: "In 0.1 s, this dental x-ray beam provides an exposure of 30 mR." This says nothing about whether ions are actually produced or whether or not there is a patient in the chair.

3. *The rad.* This is an acronym for *r*adiation *a*bsorbed *d*ose and is a measure, as its name suggests, of the dose actually delivered to a specific object, in terms of the energy transferred to it. An object, which might be a person (whole body) or a specific part of the body (the hands, say) is said to have received an absorbed dose of 1 rad when 10^{-2} J/kg have been delivered to it by ionizing radiations. A typical statement to show the usage is: "A whole-body gamma-ray dose of 300 rad will cause death in 50% of the population exposed to it." By way of comfort we note that the present average exposure to radiation from both natural and artificial sources is about 0.2 rad (= 200 mrad) per year. The SI unit for absorbed dose is the *gray* (Gy), which is equal to 100 rad.

4. *The rem.* This is an acronym for *r*oentgen *e*quivalent in *m*an and is a measure of *dose equivalent.* It takes account of the fact that, although different types of radiation (gamma rays and neutrons, say) may deliver the same energy per unit mass to the body, they do not have the same biological effect. The dose equivalent (in rems) is found by multiplying the absorbed dose (in rads) by a *quality factor* QF, which may be found tabulated in various reference sources. For x rays and electrons, QF = 1. For slow neutrons, QF = 5, and so on. Personnel monitoring devices such as film badges or dosimeters are designed to register the dose equivalent in rems.

An example of correct usage of the rem is: "The recommendation of the National Council on Radiation Protection is that no individual who is (nonoccupationally) exposed to radiations should receive a dose equivalent greater than 500 mrem (= 0.5 rem) in any one year." This includes radiations of all kinds, using the appropriate quality factors. The SI unit for dose equivalent is the *sievert* (Sv), where 1 Sv = 100 rem.

SAMPLE PROBLEM 50-8. A dose of 300 rad is lethal to 50% of the population that receives it. If the equivalent amount of energy were absorbed directly as heat, what temperature increase would result? Assume that c, the specific heat capacity of the human body, is the same as that of water—namely, 4180 J/kg · K.

Solution An absorbed dose of 300 rad corresponds to an absorbed energy per unit mass of

$$(300 \text{ rad})\left(\frac{10^{-2} \text{ J/kg}}{1 \text{ rad}}\right) = 3 \text{ J/kg}.$$

The temperature increase that would result from such an influx of heat is found from

$$\Delta T = \frac{Q/m}{c} = \frac{3 \text{ J/kg}}{4180 \text{ J/kg}\cdot\text{K}} = 7.2 \times 10^{-4} \text{ K}.$$

We see from this tiny temperature increase that the damage done by ionizing radiation has very little to do with thermal heating. The harmful effects arise because the ionizing radiation succeeds in breaking molecular bonds and thus interferes with the normal functioning of the tissue in which it has been absorbed.

50-7 NATURAL RADIOACTIVITY

All the elements beyond hydrogen and helium were made in nuclear reactions in the interiors of stars or in explosive supernovas. Both radioactive and stable nuclides are created in these processes. The solar system is composed of nuclides that were formed about 4.5×10^9 years ago. (How this is determined is discussed later in this section.) Most of the radioactive nuclides that were formed at that time have half-lives that are far shorter than a billion years, and so they have long since decayed to stable nuclides through alpha or beta emission. A few of the original radioactive nuclides, however, have half-lives that are not short in comparison with the age of the solar system. The decay of these nuclides can still be observed, and these decays form part of the background of natural radioactivity in our environment.

Some of these radioactive species are part of decay chains that start with heavy nuclides, such as ^{232}Th ($t_{1/2} = 1.4 \times 10^{10}$ y) or ^{238}U ($t_{1/2} = 4.5 \times 10^9$ y). These nuclides decay through a sequence of alpha and beta decays, eventually reaching stable end products (respectively, ^{208}Pb and ^{206}Pb). The intermediate nuclei in these decay chains have much shorter half-lives; the rate at which the original nuclide disappears and is replaced with the stable end product is determined by the longest-lived member of the chain. These decay processes have presumably been going on since the solar system was formed, and so (as we discuss later) the relative amounts of the initial nuclide and stable decay products present in a material can give a measure of the age of the material. These decays are also thought to contribute to the internal heating of the planets.

Normally, the products of these decays remain in place in the rocks or minerals containing the parent nuclide. However, one of the intermediate substances produced in these decay chains, radon, is a gas. Natural decays that occur near the surface of the Earth (and in building materials, such as concrete) release radioactive radon gas into the atmosphere. The hazards of breathing this radon gas are currently the subject of active research. Radon gas can also be released from the fracture of rocks beneath the surface; therefore the detection of radon gas has been explored as a way of predicting earthquakes.

TABLE 50-3 Some Natural Radioactive Isotopes

Isotope	$t_{1/2}$ (y)
^{40}K	1.28×10^9
^{87}Rb	4.8×10^{10}
^{113}Cd	9×10^{15}
^{115}In	4.4×10^{14}
^{138}La	1.3×10^{11}
^{176}Lu	3.6×10^{10}
^{187}Re	5×10^{10}

In addition to the heavy elements, other long-lived radioactive nuclides are present in natural substances. Some of these are listed in Table 50-3.

Other radioactive nuclides are continually produced by natural processes, generally in the Earth's atmosphere by reactions of molecules of the air with cosmic rays (high-energy protons from space). Notable among these is ^{14}C ($t_{1/2} = 5730$ y), which has important applications in radioactive dating of organic materials.

Radioactive Dating

Suppose we have an initial radionuclide I that decays to a final product F with a known half-life $t_{1/2}$. At a particular time $t = 0$, we start with N_0 initial nuclei and none of the final product nuclei. At a later time t, we find N_I of the original nuclei remain, while N_F ($= N_0 - N_I$) of the product nuclei have appeared. The initial nuclei decay according to

$$N_I = N_0 e^{-\lambda t},$$

and thus

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N_I} = \frac{t_{1/2}}{\ln 2} \ln \frac{N_0}{N_I}$$

or, substituting $N_I + N_F$ for N_0,

$$t = \frac{t_{1/2}}{\ln 2} \ln \left(1 + \frac{N_F}{N_I}\right). \qquad (50\text{-}18)$$

That is, a measurement of the present ratio of product and original nuclei can determine the age of the sample.

This calculation has been based on the assumption that none of the product nuclei were present at $t = 0$. This assumption may not always be valid, but there are techniques for radioactive dating that can correct for the presence of these original product nuclei.

This method can be used to determine the time since the formation of the solar system; examples include the ratios of ^{238}U to ^{206}Pb, ^{87}Rb to ^{87}Sr, and ^{40}K to ^{40}Ar. Terrestrial rocks, Moon rocks, and meteorites analyzed by these methods all seem to have common ages of around 4.5×10^9 y, which we take to be the age of the solar system.

The radioactive isotope ^{14}C is present in the atmosphere; about 1 carbon atom in 10^{12} is radioactive ^{14}C. Each gram of carbon has an activity of about 12 decays per minute due to ^{14}C. Living organisms can absorb this activity by aspiration of CO_2 or by eating plants that have done so. When the organism dies, it stops absorbing ^{14}C, and the ^{14}C present at its death begins to decay. By measuring the decay rate of ^{14}C, we can determine the age of the sample. For example, if we examine a sample and it shows 6 decays per minute per gram of carbon, we know that the original activity has been reduced by half, and the sample must be one half-life (5730 y) old.

This method of *radiocarbon dating* (which was developed in 1947 by Willard Libby, who was awarded the 1960 Nobel Prize in chemistry for this work) is useful for samples of organic matter that are less than about 10 half-lives in age. In 10 half-lives, the activity of a sample drops by a factor of 2^{-10}, or about 10^{-3}, and the decay rate becomes too small to be determined with precision. The practical upper limit on the age of samples that can be dated by this method is about 50,000 y. In recent years, a new technique has been developed in which an accelerator is used as a mass spectrometer to determine the ^{14}C/^{12}C ratio to high precision. In this way the usefulness of radiocarbon dating has been extended to samples as old as 100,000 y.

SAMPLE PROBLEM 50-9. In a sample of rock, the ratio of ^{206}Pb to ^{238}U nuclei is found to be 0.65. What is the age of the rock?

Solution From Eq. 50-18, using 4.5×10^9 y for the half-life of ^{238}U, we have

$$t = \frac{4.5 \times 10^9 \text{ y}}{0.693} \ln (1 + 0.65) = 3.3 \times 10^9 \text{ y}.$$

This rock is somewhat younger than the maximum age of 4.5×10^9 y that we determine for rocks in the solar system, which may suggest that the rock did not solidify until 3.3×10^9 y ago. The ^{206}Pb that was formed prior to that time was "boiled off" from the molten rock. Only after the rock solidified could the ^{206}Pb begin to accumulate.

50-8 NUCLEAR REACTIONS

We can represent a nuclear reaction by

$$X + a \rightarrow Y + b \qquad (50\text{-}19)$$

or, in more compact notation,

$$X(a,b)Y. \qquad (50\text{-}20)$$

Usually, particle a is the *projectile nucleus* and particle X is the *target nucleus,* which is often at rest in the laboratory. If the projectile is a charged particle, it may be raised to its desired energy in a Van de Graaff accelerator (see Section 28-10) or a cyclotron (see Section 32-3). The projectile may also be a neutron from a nuclear reactor. It is customary to designate product particle Y as the heavier *residual nucleus* and b as the lighter *emerging nucleus.*

The reaction energy Q is defined in analogy with Eq. 50-11 as $m_i c^2 - m_f c^2$, or

$$Q = (m_X + m_a)c^2 - (m_Y + m_b)c^2. \qquad (50\text{-}21)$$

Using energy conservation, we can write Eq. 50-21 as

$$Q = (K_Y + K_b) - (K_X + K_a), \qquad (50\text{-}22)$$

in which K represents the kinetic energy.

Equations 50-21 and 50-22 are valid only when Y and b are in their ground states. As we discuss later in this section, if either nuclide is produced in an excited state, the reaction energy is reduced by the excitation energy.

A typical reaction is

$$^{19}\mathrm{F}(\mathrm{p},\alpha)^{16}\mathrm{O},$$

for which $Q = 8.13$ MeV. Equations 50-21 and 50-22 tell us that, in this reaction, the system loses rest energy and gains kinetic energy, in amount 8.13 MeV per event. Reactions, like this one, for which $Q > 0$ are called *exothermic.* Reactions for which $Q < 0$ are called *endothermic;* such reactions will not "go" unless a certain minimum kinetic energy (the *threshold energy*) is carried into the system by the projectile.

If a and b are identical particles, which requires that X and Y also be identical, we describe the reaction as *scattering.* If the kinetic energy of the system is the same both before and after the event (which means that $Q = 0$ and all nuclides remain in their ground states), we have *elastic scattering.* If these energies are different ($Q \neq 0$), we have *inelastic scattering,* in which case Y or b may be left in an excited state.

We can easily keep track of nuclear reactions by plotting them on a nuclidic chart like that of Fig. 50-4. Figure 50-12 shows an enlarged portion of such a chart, centered

Proton number, Z							
82	^{197}Pb 8 min	^{198}Pb 2.4 h	^{199}Pb 1.5 h	^{200}Pb 21.5 h	^{201}Pb 9.42 h	^{202}Pb 5250 y	^{203}Pb 52.0 h
81	^{196}Tl 1.84 h	^{197}Tl 2.83 h	^{198}Tl 5.3 h	^{199}Tl 7.4 h	^{200}Tl 26.1 h	^{201}Tl 73.6 h	^{202}Tl 12.2 d
80	^{195}Hg 9.5 h	^{196}Hg 0.15 %	^{197}Hg 64.1 h	^{198}Hg 10.0 %	^{199}Hg 16.9 %	^{200}Hg 23.1 %	^{201}Hg 16.2 %
79	^{194}Au 39.5 h	^{195}Au 183 d	^{196}Au 6.18 d	^{197}Au 100 %	^{198}Au 2.70 d	^{199}Au 3.14 d	^{200}Au 48.4 min
78	^{193}Pt 50 y	^{194}Pt 32.9 %	^{195}Pt 33.8 %	^{196}Pt 25.3 %	^{197}Pt 18.3 h	^{198}Pt 7.2 %	^{199}Pt 30.8 min
77	^{192}Ir 74.2 d	^{193}Ir 62.7 %	^{194}Ir 19.2 h	^{195}Ir 2.5 h	^{196}Ir 52 s	^{197}Ir 5.8 min	^{198}Ir 8 s
76	^{191}Os 15.4 d	^{192}Os 41.0 %	^{193}Os 30.5 h	^{194}Os 6.0 y	^{195}Os 6.5 min	^{196}Os 35 min	
	115	116	117	118	119	120	121

Neutron number, N

FIGURE 50-12. An expanded portion of the chart of the nuclides (Fig. 50-4).

			α,n	α,γ
	p,n	p,γ d,n	d,γ α,d	α,p
	γ,n p,d	(shaded)	n,γ d,p	
p,α	γ,d d,α	n,d γ,p	n,p	
γ,α	n,α			

FIGURE 50-13. Placing this as an overlay on Fig. 50-12, with the shaded central square over a particular target nuclide, shows the residual nuclides that result from the indicated reactions.

arbitrarily on the nuclide ^{197}Au. Stable nuclides are shaded, and their isotopic abundances are shown. The unshaded squares represent radionuclides, with their half-lives shown.

Figure 50-13 suggests a transparent overlay that we can place over a nuclidic chart such as that of Fig. 50-12. If the shaded central square of Fig. 50-13 overlays a particular target on the chart of Fig. 50-12, the residual nuclides resulting from the various reactions printed on the overlay are identified.

Thus if we chose ^{197}Au as a target, a (p,α) reaction will produce (stable) ^{194}Pt, and either an (n,γ) or a (d,p) reaction will produce the radionuclide ^{198}Au, whose half-life is 2.70 d.

Nuclei, like atoms, have stationary states of definite energy, and reaction studies can be used to identify them. Consider, for example, the reaction

$$^{27}\mathrm{Al}(\mathrm{d},\mathrm{p})^{28}\mathrm{Al}, \qquad Q = 5.49 \text{ MeV},$$

in which a thin aluminum target foil is bombarded with 2.10-MeV deuterons. In the laboratory the emerging protons are seen to come off with a number of well-defined discrete energies and are accompanied by gamma rays. Fig-

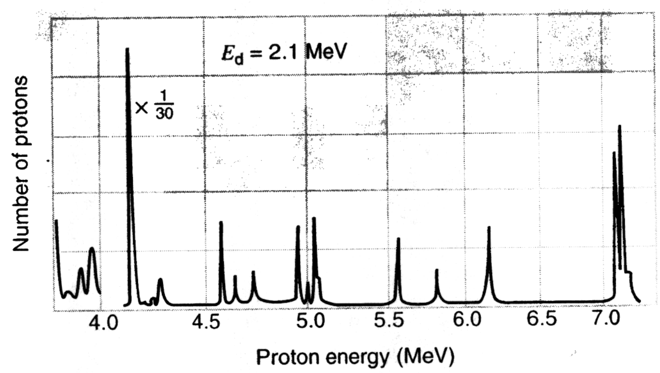

FIGURE 50-14. The energy distribution of protons resulting from the reaction ^{27}Al(d,p)^{28}Al. The incident deuteron has an energy of 2.10 MeV. The protons are detected as they emerge from the target at right angles to the incident beam.

ure 50-14 shows the energy distribution of the emerging protons.

In every reaction event we know that an energy equal to the kinetic energy of the incident deuteron (=2.10 MeV) plus the reaction energy Q (=5.49 MeV) is available to be shared between the two reaction products—that is, between the residual nucleus ^{28}Al and the emerging proton p. How is this total energy (2.10 MeV + 5.49 MeV = 7.59 MeV) to be shared between these two particles?

It all depends on whether the residual nucleus ^{28}Al is produced in its ground state or in one of its excited stationary states. In the former case, the emerging proton will have the maximum possible energy, corresponding to the peak on the extreme right of the proton spectrum in Fig. 50-14. If, however, the residual nucleus is formed in an excited state, that nucleus will retain more of the available energy and there will be less energy left for the emerging proton. The residual nucleus will not remain in its excited state very long but will get rid of its excess energy, such as by emitting a gamma ray.

Every proton peak in the spectrum of Fig. 50-14 corresponds to a stationary state of the residual nucleus ^{28}Al. Figure 50-15 shows the energy levels that may be deduced by analyzing this spectrum. You can see the correspondence between the peaks of Fig. 50-14 and the energy levels of Fig. 50-15. We have seen that our understanding of the way atoms are put together rests on the measured energies of the hydrogen atom states as its firm foundation. In the same way, we can learn how nuclei are put together by studying the energies and other properties of their stationary states.

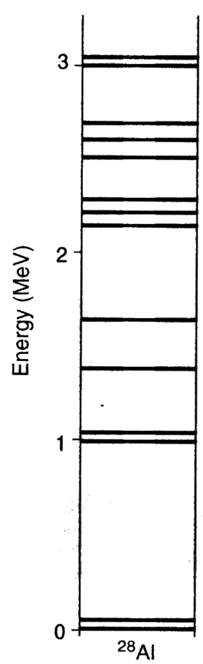

FIGURE 50-15. Energy levels of ^{28}Al, deduced from data such as those of Fig. 50-14.

SAMPLE PROBLEM 50-10. In the reaction

$$^1H + {}^3H \rightarrow {}^2H + {}^2H,$$

protons (^{1}H) with kinetic energy 5.70 MeV are incident on ^{3}H at rest. (*a*) What is the Q value for this reaction? (*b*) Find the kinetic energies of the deuterons emitted along the direction of the incident proton.

Solution (*a*) From Eq. 50-21 we have

$$\begin{aligned} Q &= [m(^1H) + m(^3H) - m(^2H) - m(^2H)]c^2 \\ &= (1.007825\ u + 3.016049\ u - 2.014102\ u \\ &\quad - 2.014102\ u)(931.5\ MeV/u) \\ &= -4.03\ MeV. \end{aligned}$$

This reaction is endothermic; the final products have the greater mass and correspondingly the smaller kinetic energy by Eq. 50-22.

(*b*) Using Eq. 50-22, with $K = 0$ for the initial ^{3}H, we have

$$\begin{aligned} K_1 + K_2 &= Q + K_p = -4.03\ MeV + 5.70\ MeV \\ &= 1.67\ MeV. \end{aligned} \quad (50\text{-}23)$$

Here the subscripts 1 and 2 refer to the two ^{2}H product nuclei. Conservation of momentum along the direction of the incident protons gives

$$\begin{aligned} p_1 + p_2 = p_p &= \sqrt{2m(^1H)K_p} = \sqrt{2(938\ MeV/c^2)(5.70\ MeV)} \\ &= 103.4\ MeV/c. \end{aligned} \quad (50\text{-}24)$$

Equations 50-23 and 50-24 can be solved as two equations in two unknowns (either p_1 and p_2 or K_1 and K_2). The results are

$$K_1 = 0.24\ MeV \quad \text{and} \quad K_2 = 1.43\ MeV.$$

Note that we have used nonrelativistic dynamics in solving this problem. Is this a good approximation?

50-9 NUCLEAR MODELS (Optional)

The structure of atoms is now well understood. The Coulomb force is exerted by the massive center (the nucleus) on the electrons, and (given enough computer time) we can use the methods of quantum mechanics to calculate properties of the atom.

Things are not quite so well understood in the case of nuclei. The force law is complicated and cannot be written down explicitly in full detail. Nor is there a natural force center to simplify the calculations. To understand nuclear structure, we face a many-body problem of great complexity.

In the absence of a comprehensive theory of nuclear structure, we try instead to construct *nuclear models.* Physicists use models as simplified ways of looking at a complex system to give physical insight into its properties. The usefulness of a model is tested by its ability to make predictions that can be verified experimentally in the laboratory.

Two models of the nucleus have proved useful. One model (the *collective model*) describes situations in which we can consider all the protons and neutrons to behave cooperatively. The other model (the *independent particle* model) neglects all but one proton or neutron in determining the properties of the nucleus. These two models represent quite opposing views of nuclear structure, but they can be combined to create a single unified model of the nucleus.

The Collective Model

In the collective model, we ignore the motions of individual nucleons and treat the nucleus as a single entity. This model, originally called the "liquid drop model," was developed by Niels Bohr to explain nuclear fission. We imagine the nucleus as a body analogous to a liquid drop, in which the nucleons interact with each other like molecules in the liquid.

The equilibrium shape of the liquid drop is determined by the interactions of its molecules, and similarly the equilibrium shape of a nucleus is determined by the interactions of its nucleons. Many nuclei have spherical equilibrium shapes, whereas others may be ellipsoidal.

Like a liquid drop, a nucleus can absorb energy by the entire nucleus rotating about an axis or vibrating about its equilibrium shape. Through radioactive decay or nuclear reaction experiments, it is possible to study these excited states. Figure 50-16 shows examples of the two kinds of situations. The rotational energy $\frac{1}{2}I\omega^2$ can be written in terms of the angular momentum L $(=I\omega)$ as $L^2/2I$. Writing the quantized angular momentum according to Eq. 47-28 as $L = \sqrt{J(J+1)}(h/2\pi)$, where J is the rotational angular momentum quantum number of the entire nucleus, we obtain

$$E_J = \frac{h^2}{8\pi^2 I} J(J+1). \qquad (50\text{-}25)$$

Note that the spacing between the states grows as the angular momentum increases.

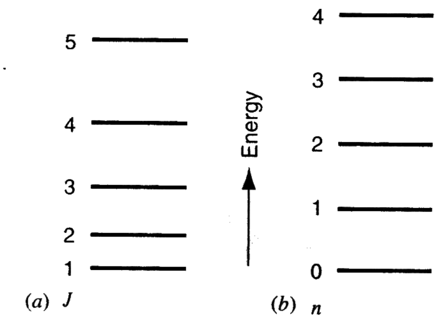

FIGURE 50-16. (*a*) Rotational excited states, labeled with the angular momentum quantum number J. (*b*) Vibrational states, labeled with the vibrational quantum number n.

The vibrational states have energies given by

$$E_n = nhf \qquad n = 1, 2, 3, \ldots, \qquad (50\text{-}26)$$

where f is the vibrational frequency. Typically the vibrational energy quantum hf has an energy of about 0.5 MeV, corresponding to a frequency of the order of 10^{20} Hz. Note that the vibrational states shown in Fig. 50-16*b* are equally spaced, in contrast to the rotational states.

Experimental evidence for collective structure in nuclei includes not only the observation of rotational and vibrational excited states similar to those of Fig. 50-16, but also nuclear reaction processes in which the energy is shared by all of the nucleons in the nucleus, gamma-ray emission probabilities, and nonspherical (often ellipsoidal) equilibrium shapes of many nuclei.

The Independent-Particle Model

At the other extreme, the independent-particle model assumes that each nucleon can be assigned well-defined states similar to the states of electrons in atoms. Because neutrons and protons, like electrons, must obey the Pauli exclusion principle, the independent-particle structure of a nucleus is very similar to the structure for atoms we discussed in Chapter 48. Like electrons, nucleons arrange themselves into shells with well-defined quantum numbers. When a shell is filled, the result is a nucleus of unusual stability, in analogy with the inert gases that correspond to filled atomic shells.

The closed shells for nucleons occur at so-called "magic" neutron or proton numbers of 2, 8, 20, 28, 50, 82, and 126; these differ slightly from the closed atomic shells (2, 10, 18, 36, 54, and 86) due in part to the differences between the nuclear force and the Coulomb force that acts on the electrons. Some nuclei, called "doubly magic," have closed shells of both protons and neutrons and are therefore especially stable—for example, ^{40}Ca with 20 protons and 20 neutrons. To remove a proton from ^{40}Ca takes 8.3 MeV; however, to remove a proton from a nucleus with one additional proton (^{41}Sc, with a nonmagic 21 protons), takes only 1.1 MeV. Similarly, to remove a neutron from ^{40}Ca takes 15.6 MeV, but only 8.4 MeV is required to remove a neutron from ^{41}Ca, with nonmagic 21 neutrons. This sudden increase in the energy to remove a particle from a closed shell is also observed in atoms (see Fig. 48-6 for the ionization energies of atoms). A similar effect occurs in the ionic radii of atoms, which show a sudden increase as one electron shell is filled and the next electron must start a new shell. Nuclear radii show the same shell effect, as illustrated in Fig. 50-17. Note the sudden jumps in the radius at $N = 20$, 28, 50, 82, and 126, corresponding to filled neutron shells.

As in the case of the nuclear radius or the energy needed to remove one proton or neutron, the independent-particle model at its extreme limit assumes that the properties of the nucleus are determined by a single "valence"

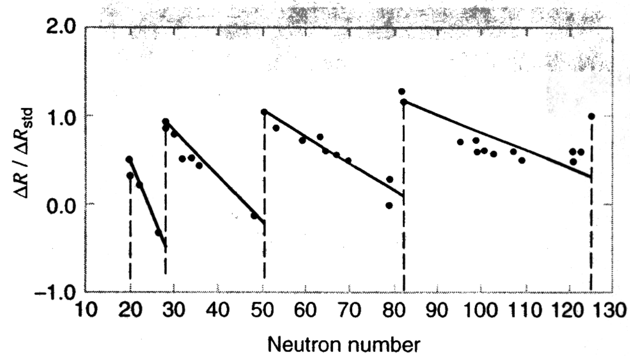

FIGURE 50-17. The variation in nuclear radius as a function of neutron number. The variation is expressed relative to the "standard" variation expected from the "collective" structure of $R = R_0 A^{1/3}$. The sudden jumps indicate shell structure.

proton or neutron. Other nuclear properties that can be successfully interpreted in the independent-particle model include alpha-decay half-lives, magnetic dipole moments, reaction probabilities for capturing a single particle such as a neutron, and energies of excited states.

SAMPLE PROBLEM 50-11. The nuclide ^{120}Sn ($Z = 50$) has a filled proton shell, 50 being one of the magic nucleon numbers. The nuclide ^{121}Sb ($Z = 51$) has an "extra" proton outside this shell. According to the shell concept, this extra proton should be easier to remove than a proton from the filled shell. Verify this by calculating the required energy in each case. Use the following mass data:

Nuclide	Z	N	*Atomic Mass* (u)
^{121}Sb	50 + 1	70	120.903818
^{120}Sn	50	70	119.902197
^{119}In	50 − 1	70	118.905846

The proton atomic mass is 1.007825 u.

Solution Removing the "extra" proton corresponds to the process

$$^{121}Sb \rightarrow {}^{120}Sn + p.$$

The required energy E follows from

$$\begin{aligned} E &= [m(^{120}Sn) + m(^{1}H) - m(^{121}Sb)]c^2 \\ &= (119.902197\text{ u} + 1.007825\text{ u} - 120.903818\text{ u}) \\ &\quad \times (931.5\text{ MeV/u}) \\ &= 5.8\text{ MeV}. \end{aligned}$$

Removing the proton from the filled shell corresponds to

$$^{120}Sn \rightarrow {}^{119}In + p.$$

The required energy follows from

$$\begin{aligned} E &= [m(^{119}In) + m(^{1}H) - m(^{120}Sn)]c^2 \\ &= (118.905846\text{ u} + 1.007825\text{ u} - 119.902197\text{ u}) \\ &\quad \times (931.5\text{ MeV/u}) \\ &= 10.7\text{ MeV}. \end{aligned}$$

This is considerably greater than the energy required to remove an "extra" proton ($=5.8$ MeV), just as the shell model predicts. In much the same way, the energy needed to remove an *electron* from a filled *electron shell* ($=22$ eV for the filled shell of neon) is much greater than that needed to remove an "extra" electron from outside such a filled shell ($=5$ eV for the "extra" electron from sodium).

MULTIPLE CHOICE

50-1 Discovering the Nucleus

1. Rutherford was able to ignore the effect of the electrons in his analysis of alpha-particle scattering experiments because
   (A) electrons do not exert a force on alpha particles.
   (B) the mass of an electron is much less than the mass of an alpha particle.
   (C) the electrons are uniformly distributed throughout the atom.
   (D) the electrons are in such rapid motion that an alpha particle cannot collide with them.

50-2 Some Nuclear Properties

2. How many neutrons are in the nuclide ^{66}Zn?
   (A) 26 (B) 30 (C) 36 (D) 66
3. The binding energy for nucleus A is 7.7 MeV, and that for nucleus B is 7.8 MeV. Which nucleus has the larger mass?
   (A) Nucleus A (B) Nucleus B
   (C) More information is needed to answer.
4. If a nucleus were as big as a grape, an atom would be as big as
   (A) a house. (B) a football field.
   (C) a city. (D) the Moon.

50-3 Radioactive Decay

5. The half-life of a certain radioactive sample is 30 minutes. At 2:00 P.M. the decay rate is measured to be 1200/s. What will be the outcome of a measurement of the decay rate at 3:00 P.M. on that same day?
   (A) 4800/s (B) 1200/s (C) 600/s (D) 300/s

50-4 Alpha Decay

6. A certain heavy nucleus alpha decays with a disintegration energy of 1.50 MeV. From this we can conclude that the kinetic energies observed for the alpha particles
   (A) are all equal to 1.50 MeV.
   (B) are all a little less than 1.50 MeV.
   (C) could occasionally be greater than 1.50 MeV.
   (D) vary continuously from zero to 1.50 MeV.

50-5 Beta Decay

7. A certain stable nuclide, after absorbing a neutron, emits a negative electron and then splits spontaneously into two alpha particles. The original nuclide is
   (A) ^{8}Be. (B) ^{7}Be. (C) ^{7}B. (D) ^{7}Li.

8. In a beta-decay experiment, an electron is observed with a kinetic energy of 1.0 MeV. From this observation, what can be concluded about the disintegration energy Q of the decay?
   (A) $Q = 1.0$ MeV
   (B) $Q \leq 1.0$ MeV
   (C) $Q \geq 1.0$ MeV
   (D) Nothing can be concluded about Q.

50-6 Measuring Ionizing Radiation

9. The decay rate of a radioactive source is measured in units of ________, and the biological effect of that radiation on a human being is measured in units of ________.
   (A) curies (B) roentgens (C) rads (D) rems

50-7 Natural Radioactivity

10. The decay chain that leads from $^{238}_{92}U$ to $^{206}_{82}Pb$ consists of a series of alpha decays and beta decays. How many alpha particles are emitted?
   (A) 4 (B) 5 (C) 6 (D) 8

50-8 Nuclear Reactions

11. In a nuclear reaction, a beam of alpha particles (^4He) strikes a target of ^{60}Ni. The products of this reaction might be
   (A) $^{63}Zn + n$. (B) $^{63}Cu + p$. (C) $^{61}Ni + {}^3He$.
   (D) All of the above

50-9 Nuclear Models

12. How many elements are found in nature that have filled electron shells in the atom as well as filled proton and neutron shells in the nucleus?
   (A) 1 (B) 2 (C) 3 (D) 4

QUESTIONS

1. When a thin foil is bombarded with α particles, a few of them are scattered back toward the source. Rutherford concluded from this that the positive charge of the atom—and also most of its mass—must be concentrated in a very small "nucleus" within the atom. What was his line of reasoning?
2. In what ways do the so-called strong force and the electrostatic or Coulomb force differ?
3. Why does the *relative* importance of the Coulomb force compared to the strong nuclear force increase at large mass numbers?
4. In your body, are there more neutrons than protons? More protons than electrons? Discuss.
5. Why do nuclei tend to have more neutrons than protons at high mass numbers?
6. Why do we use atomic rather than nuclear masses in analyzing most nuclear decay and reaction processes?
7. How might the equality $1\ u = 1.6605 \times 10^{-27}$ kg be arrived at in the laboratory?
8. The atoms of a given element may differ in mass, have different physical characteristics, and yet not vary chemically. Why is this?
9. The deviation of isotopic masses from integer values is due to many factors. Name some. Which is most responsible?
10. How is the mass of the neutron determined?
11. The most stable nuclides have a mass number A near 60 (see Fig. 50-6). Why don't *all* nuclides have mass numbers near 60?
12. If we neglect the very lightest nuclides, the binding energy per nucleon in Fig. 50-6 is roughly constant at 7 to 8 MeV/nucleon. Do you expect the mean electronic binding energy per electron in atoms also to be roughly constant throughout the periodic table?
13. Why is the binding energy per nucleon (Fig. 50-6) low at low mass numbers? At high mass numbers?
14. In the binding energy curve of Fig. 50-6, what is special or notable about the nuclides ^2H, ^4He, ^{62}Ni, and ^{239}Pu?
15. The magnetic moment of the neutron is $-1.9130\ \mu_N$. What is a nuclear magneton and how does it differ from a Bohr magneton? What does the minus sign mean? How can the neutron, which carries no net charge, have a magnetic moment in the first place?
16. A particular ^{238}U nucleus was created in a massive stellar explosion, perhaps 10^{10} y ago. It suddenly decays by α emission while we are observing it. After all those years, why does it decide to decay at this particular moment?
17. Can you justify this statement: "In measuring half-lives by the method of Sample Problem 50-4, it is not necessary to measure the absolute decay rate R; any quantity proportional to it will suffice. However, in the method of Sample Problem 50-5 an absolute rate *is* needed."
18. Does the temperature affect the rate of decay of radioactive nuclides? If so, how?
19. You are running longevity tests on lightbulbs. Do you expect their "decay" to be exponential? What is the essential difference between the decay of lightbulbs and of radionuclides?
20. Generally clocks exhibit complete regularity of some periodic process. Considering that radioactive decay is completely random, how can it nevertheless be used for the measurement of time?
21. Can you give a justification, even a partial one, for the barrier-tunneling phenomenon in terms of basic ideas about the wave nature of matter?
22. Explain why, in alpha decay, short half-lives correspond to large disintegration energies, and conversely.
23. A radioactive nucleus can emit a positron, e^+. This corresponds to a proton in the nucleus being converted to a neutron. The mass of a neutron, however, is greater than that of a proton. How then can positron emission occur?
24. In beta decay the emitted electrons form a continuous spectrum, but in alpha decay the alpha particles form a discrete spectrum. What difficulties did this cause in the explanation of beta decay, and how were these difficulties finally overcome?
25. How do neutrinos differ from photons? Each has zero charge and (presumably) zero rest mass and travels at the speed of light.

26. The decay of radioactive elements produces helium, which eventually passes into the Earth's atmosphere. The amount of helium actually present in the atmosphere, however, is very much less than the amount released in this way. Explain.
27. The half-life of ^{238}U is 4.5×10^9 y, about the age of the solar system. How can such a long half-life be measured?
28. In radioactive dating with ^{238}U, how do you get around the fact that you do not know how much ^{238}U was present in the rocks to begin with? (*Hint:* What is the ultimate decay product of ^{238}U?)
29. Make a list of the various sources of ionizing radiation encountered in our environment, whether natural or artificial.
30. Which of these conservation laws apply to all nuclear reactions: conservation of (*a*) charge, (*b*) mass, (*c*) total energy, (*d*) rest energy, (*e*) kinetic energy, (*f*) linear momentum, (*g*) angular momentum, and (*h*) total number of nucleons?
31. Small temperature changes have a large effect on the rate of chemical reactions but generally have a negligible effect on the rate of nuclear reactions. Explain.
32. In the development of our understanding of the atom, did we use atomic models as we now use nuclear models? Is Bohr's theory such an atomic model? Are models now used in atomic physics? What is the difference between a model and a theory?
33. What are the basic assumptions of the collective and the independent particle models of nuclear structure? How do they differ? Are there similarities between them?
34. Does the collective model of the nucleus give us a picture of the following phenomena: (*a*) acceptance by the nucleus of a colliding particle, (*b*) loss of a particle by spontaneous emission, (*c*) fission, (*d*) dependence of stability on energy content?
35. What is so special ("magic") about the magic nucleon numbers?
36. Why are the magic nucleon numbers and the magic electron numbers not the same? What accounts for each?
37. The average number of stable (or very long-lived) isotopes of the inert gases is 3.7. The average number of stable nuclides for the four magic neutron numbers, however, is 5.8, considerably greater. If the inert gases are so stable, why were not more stable isotopes of them created when the elements were formed?

EXERCISES

50-1 Discovering the Nucleus

1. Calculate the distance of closest approach for a head-on collision between a 5.30-MeV α particle and the nucleus of a copper atom.
2. (*a*) Calculate the electric force on an α particle at the surface of a gold atom, presuming that the positive charge is spread uniformly throughout the volume of the atom. Ignore the atomic electrons. A gold atom has a radius of 0.16 nm; treat the α particle as a point particle. (*b*) Through what distance would this force, presumed constant, have to act to bring a 5.30-MeV α particle to rest? Express your answer in terms of the diameter of a gold atom.
3. Assume that a gold nucleus has a radius of 6.98 fm (see Table 50-1), and an α particle has a radius of 1.8 fm. What energy must an incident α particle have to just touch the gold nucleus?

50-2 Some Nuclear Properties

4. Locate the nuclides displayed in Table 50-1 on the nuclidic chart of Fig. 50-4. Which of these nuclides are within the stability zone?
5. The radius of a nucleus is measured, by electron-scattering methods, to be 3.6 fm. What is the likely mass number of the nucleus?
6. Arrange the 25 nuclides given here in squares as a section of the nuclidic chart similar to Fig. 50-4. Draw in and label (*a*) all isobaric (constant *A*) lines and (*b*) all lines of constant neutron excess, defined as $N - Z$. Consider nuclides $^{118-122}Te$, $^{117-121}Sb$, $^{116-120}Sn$, $^{115-119}In$, and $^{114-118}Cd$.
7. A neutron star is a stellar object whose density is about that of nuclear matter, as calculated in Sample Problem 50-2. Suppose that the Sun were to collapse into such a star without losing any of its present mass. What would be its expected radius?
8. Verify that the binding energy per nucleon given in Table 50-1 for ^{239}Pu is indeed 7.56 MeV/nucleon. The needed atomic masses are 239.052156 u (^{239}Pu), 1.007825 u (^1H), and 1.008665 u (neutron).
9. Calculate the average binding energy per nucleon of ^{62}Ni, which has an atomic mass of 61.928349 u. This nucleus has the greatest binding energy per nucleon of all the known stable nuclei.
10. The atomic masses of ^1H, ^{12}C, and ^{238}U are 1.007825 u, 12.000000 u (by definition), and 238.050783 u, respectively. (*a*) What would these masses be if the mass unit were defined so that the mass of ^1H was (exactly) 1.000000 u? (*b*) Use your result to suggest why this perhaps obvious choice was not made.
11. (*a*) Convince yourself that the energy tied up in nuclear, or strong-force, bonds is proportional to *A*, the mass number of the nucleus in question. (*b*) Convince yourself that the energy tied up in Coulomb-force bonds between the protons is proportional to $Z(Z-1)$. (*c*) Show that, as we move to larger and larger nuclei (see Fig. 50-4), the importance of (*b*) increases more rapidly than does that of (*a*).
12. In the periodic table, the entry for magnesium is

12 Mg 24.305

There are three isotopes:

^{24}Mg, atomic mass = 23.985042 u;

^{25}Mg, atomic mass = 24.985837 u;

^{26}Mg, atomic mass = 25.982593 u.

The abundance of ^{24}Mg is 78.99% by mass. Calculate the abundances of the other two isotopes.

13. Because a nucleon is confined to a nucleus, we can take its uncertainty in position to be approximately the nuclear radius R. What does the uncertainty principle yield for the kinetic energy of a nucleon in a nucleus with, say, $A = 100$? (*Hint:* Take the uncertainty in momentum Δp to be the actual momentum p.)

14. You are asked to pick apart an α particle (^{4}He) by removing, in sequence, a proton, a neutron, and a proton. Calculate (*a*) the work required for each step, (*b*) the total binding energy of the α particle, and (*c*) the binding energy per nucleon. Needed atomic masses are

^{4}He	4.002603 u	^{2}H	2.014102 u
^{3}H	3.016049 u	^{1}H	1.007825 u
n	1.008665 u.		

15. To simplify calculations, atomic masses are sometimes tabulated, not as the actual atomic mass m but as $(m - A)c^2$, where A is the mass number expressed in mass units. This quantity, usually reported in MeV, is called the *mass excess,* symbol Δ. Using data from Sample Problem 50-3, find the mass excesses for (*a*) ^{1}H, (*b*) the neutron, and (*c*) ^{120}Sn.

16. (*a*) Show that the total binding energy of a nuclide can be written as

$$E_B = Z\Delta_H + N\Delta_n - \Delta,$$

where Δ_H, Δ_n, and Δ are the appropriate mass excesses; see Exercise 15. (*b*) Using this method calculate the binding energy per nucleon for ^{197}Au. Compare your result with the value listed in Table 50-1. The needed mass excesses are $\Delta_H = +7.289$ MeV, $\Delta_n = +8.071$ MeV, and $\Delta_{197} = -31.157$ MeV. Δ_H is the mass excess of ^{1}H. Note the economy of calculation that results when mass excesses are used in place of the actual masses.

17. A penny has a mass of 3.00 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons in this coin. Ignore the binding energy of the electrons. For simplicity assume that the penny is made entirely of ^{63}Cu atoms (mass = 62.929601 u). The atomic masses of the proton and the neutron are 1.007825 u and 1.008665 u, respectively.

18. Nuclear radii may be measured by scattering high-energy electrons from nuclei. (*a*) What is the de Broglie wavelength for 480-MeV electrons? (*b*) Are they suitable probes for this purpose? Relativity must be taken into account.

50-3 Radioactive Decay

19. The half-life of a radioactive isotope is 140 d. How many days would it take for the activity of a sample of this isotope to fall to one-fourth of its initial decay rate?

20. The half-life of a particular radioactive isotope is 6.5 h. If there are initially 48×10^{19} atoms of this isotope in a particular sample, how many atoms of this isotope remain after 26 h?

21. A radioactive isotope of mercury, ^{197}Hg, decays into gold, ^{197}Au, with a decay constant of 0.0108 h^{-1}. (*a*) Calculate its half-life. (*b*) What fraction of the original amount will remain after three half-lives? (*c*) After 10 days?

22. From data presented in the first few paragraphs of Section 50-3, deduce (*a*) the disintegration constant λ and (*b*) the half-life of ^{238}U.

23. ^{67}Ga, atomic mass = 66.93 u, has a half-life of 78.25 h. Consider an initially pure 3.42-g sample of this isotope. (*a*) Find its activity (decay rate). (*b*) Find its activity 48.0 h later.

24. Show that the law of radioactive decay (Eq. 50-6) can be written in the form

$$N = N_0(\tfrac{1}{2})^{t/t_{1/2}}.$$

25. ^{223}Ra decays by alpha decay with a half-life of 11.43 d. How many helium atoms are created in 28 d from an initially pure sample of ^{223}Ra containing 4.70×10^{21} atoms?

26. The radionuclide ^{64}Cu has a half-life of 12.7 h. How much of an initially pure 5.50-g sample of ^{64}Cu will decay during the 2-h period beginning 14.0 h later?

27. The radionuclide ^{32}P (half-life = 14.28 d) is often used as a tracer to follow the course of biochemical reactions involving phosphorus. (*a*) If the counting rate in a particular experimental setup is 3050 counts/s, after what time will it fall to 170 counts/s? (*b*) A solution containing ^{32}P is fed to the root system of an experimental tomato plant and the ^{32}P activity in a leaf is measured 3.48 d later. By what factor must this reading be multiplied to correct for the decay that has occurred since the experiment began?

28. A 1.00-g sample of samarium emits α particles at a rate of 120 particles/s. ^{147}Sm, whose natural abundance in bulk samarium is 15.0%, is the responsible isotope. Calculate the half-life of this isotope.

29. ^{239}Pu, atomic mass = 239 u, decays by alpha decay with a half-life of 24,100 y. How many grams of helium are produced by an initially pure 12.0-g sample of ^{239}Pu after 20,000 y? (Recall that an α particle is a helium nucleus, with an atomic mass of 4.00 u.)

30. A source contains two phosphorus radionuclides, ^{32}P ($t_{1/2}$ = 14.3 d) and ^{33}P ($t_{1/2}$ = 25.3 d). Initially 10.0% of the decays come from ^{33}P. How long must one wait until 90.0% do so?

50-4 Alpha Decay

31. Consider a ^{238}U nucleus to be made up of an α particle (^{4}He) and a residual nucleus (^{234}Th). Plot the electrostatic potential energy $U(r)$, where r is the distance between these particles. Cover the range 10 fm $< r <$ 100 fm and compare your plot with that of Fig. 50-9.

32. Generally, heavier nuclides tend to be more unstable to alpha decay. For example, the most stable isotope of uranium, ^{238}U, has an alpha decay half-life of 4.5×10^9 y. The most stable isotope of plutonium is ^{244}Pu with a 8.2×10^7 y half-life, and for curium we have ^{248}Cm and 3.4×10^5 y. When half of an original sample of ^{238}U has decayed, what fractions of the original isotopes of (*a*) plutonium and (*b*) curium are left?

33. Heavy radionuclides emit an α particle rather than other combinations of nucleons because the α particle is such a stable, tightly bound structure. To confirm this, calculate the disintegration energies for these hypothetical decay processes and discuss the meaning of your findings:

$$^{235}\text{U} \rightarrow {}^{232}\text{Th} + {}^3\text{He}, \quad Q_3;$$

$$^{235}\text{U} \rightarrow {}^{231}\text{Th} + {}^4\text{He}, \quad Q_4;$$

$$^{235}\text{U} \rightarrow {}^{230}\text{Th} + {}^5\text{He}, \quad Q_5.$$

The needed atomic masses are

^{232}Th	232.038050 u	^{3}He	3.016029 u
^{231}Th	231.036297 u	^{4}He	4.002603 u
^{230}Th	230.033127 u	^{5}He	5.012228 u.
^{235}U	235.043923 u		

34. Under certain circumstances, a nucleus can decay by emitting a particle heavier than an α particle. Such decays are very rare. Consider the decays

$$^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}$$

and

$$^{223}\text{Ra} \rightarrow {}^{219}\text{Rn} + {}^{4}\text{He}.$$

(*a*) Calculate the Q-values for these decays and determine that both are energetically possible. (*b*) The Coulomb barrier height for α particles in this decay is 30 MeV. What is the barrier height for ^{14}C decay? Atomic masses are

^{223}Ra	223.018497 u	^{14}C	14.003242 u
^{209}Pb	208.981075 u	^{4}He	4.002603 u.
^{219}Rn	219.009475 u		

50-5 Beta Decay

35. ^{137}Cs is present in the fallout from above-ground detonations of nuclear bombs. Because it beta decays with a slow 30.2-y half-life into ^{137}Ba, releasing considerable energy in the process, it is an environmental concern. The atomic masses of the Cs and Ba are 136.907084 u and 136.905821 u, respectively. Calculate the total energy released in one decay.

36. A free neutron decays according to Eq. 50-16. Calculate the maximum energy K_{max} of the beta spectrum. Needed atomic masses are:

n 1.008665 u; ^{1}H 1.007825 u.

37. An electron is emitted from a middle-mass nuclide ($A = 150$, say) with a kinetic energy of 1.00 MeV. (*a*) Find its de Broglie wavelength. (*b*) Calculate the radius of the emitting nucleus. (*c*) Can such an electron be confined as a standing wave in a "box" of such dimensions? (*d*) Can you use these numbers to disprove the argument (long since abandoned) that electrons actually exist in nuclei?

38. The radionuclide ^{32}P decays to ^{32}S as described by Eq. 50-14. In a particular decay event, a 1.71-MeV electron is emitted, the maximum possible value. Find the kinetic energy of the recoiling ^{32}S atom in this event. The atomic mass of ^{32}S is 31.97 u. (*Hint:* For the electron it is necessary to use the relativistic expressions for the kinetic energy and the linear momentum. Newtonian mechanics may safely be used for the relatively slow-moving ^{32}S atom.)

50-6 Measuring Ionizing Radiation

39. The nuclide ^{198}Au, half-life = 2.693 d, is used in cancer therapy. Calculate the mass of this isotope required to produce an activity of 250 Ci.

40. A Geiger counter records 8722 counts in 1 min. Calculate the activity of the source in Ci, assuming that the counter records all decays.

41. A typical chest x-ray radiation dose is 25 mrem, delivered by x rays with a quality factor of 0.85. Assuming that the mass of the exposed tissue is one-half the patient's mass of 88 kg, calculate the energy absorbed in joules.

42. A 75-kg person receives a whole-body radiation dose of 24 mrad, delivered by α particles for which the quality factor is 12. Calculate (*a*) the absorbed energy in joules and (*b*) the equivalent dose in rem.

43. An activity of 3.94 μCi is needed in a radioactive sample to be used in a medical procedure. One week before treatment, a nuclide sample with a half-life of 1.82×10^5 s is prepared. What should be the activity of the sample at the time of preparation in order that it have the required activity at the time of treatment?

44. The plutonium isotope ^{239}Pu, atomic mass 239.05 u, is produced as a by-product in nuclear reactors and hence is accumulating in reactor fuel elements. It is radioactive, decaying by alpha decay with a half-life of 2.411×10^4 y. However, plutonium is also one of the most toxic chemicals known; as little as 2.00 mg is lethal to a human. (*a*) How many nuclei constitute a chemically lethal dose? (*b*) What is the decay rate of this amount? (*c*) Its activity in curies?

45. Cancer cells are more vulnerable to x and gamma radiation than are healthy cells. Though linear accelerators are now replacing it, in the past the standard source for radiation therapy has been radioactive ^{60}Co, which beta decays into an excited nuclear state of ^{60}Ni, which immediately drops into the ground state, emitting two gamma-ray photons, each of approximate energy 1.2 MeV. The controlling beta-decay half-life is 5.27 y. How many radioactive ^{60}Co nuclei are present in a 6000-Ci source used in a hospital? The atomic mass of ^{60}Co is 59.93 u.

46. An airline pilot spends an average of 20 h per week flying at 12,000 m, at which altitude the dose equivalent rate due to cosmic and solar radiation is 12 μSv/h (1 Sv = 1 sievert = 100 rem; the sievert is the SI unit of dose equivalent). Calculate the annual equivalent dose in mrem.

47. After long effort, in 1902, Marie and Pierre Curie succeeded in separating from uranium ore the first substantial quantity of radium, 1 decigram (dg) of pure $RaCl_2$. The radium was the radioactive isotope ^{226}Ra, which decays with a half-life of 1600 y. (*a*) How many radium nuclei had they isolated? (*b*) What was the decay rate of their sample, in Bq? (1 Bq = 1 becquerel = 1 decay/s.) (*c*) In curies? The molar mass of Cl is 35.453 g/mol; the atomic mass of the radium isotope is 226.03 u.

48. Calculate the mass of 4.60 μCi of ^{40}K, which has a half-life of 1.28×10^9 y and an atomic mass of 40.0 u.

50-7 Natural Radioactivity

49. A rock is found to contain 4.20 mg of ^{238}U and 2.00 mg of ^{206}Pb. Assume that the rock contained no lead at formation, with all the lead now present arising from the decay of the uranium. Find the age of the rock. The half-life of ^{238}U is 4.47×10^9 y.

50. A particular rock is thought to be 260 million years old. If it contains 3.71 mg of ^{238}U, how much ^{206}Pb should it contain?

51. A rock, recovered from far underground, is found to contain 860 μg of ^{238}U, 150 μg of ^{206}Pb, and 1.60 mg of ^{40}Ca. How much ^{40}K will it very likely contain? Needed half-lives are 4.47×10^9 y for ^{238}U and 1.28×10^9 y for ^{40}K.

50-8 Nuclear Reactions

52. Fill in the missing nuclide in each of the following reactions: (*a*) ^{116}Sn(?,p)^{117}Sn; (*b*) ^{40}Ca(α,n)?; and (*c*) ?(p,n)^{7}Be.

53. Calculate Q for the reaction ^{59}Co(p,n)^{59}Ni. Needed atomic masses are

^{59}Co	58.933200 u	^{1}H	1.007825 u
^{59}Ni	58.934352 u	n	1.008665 u.

54. Making mental use of the overlay of Fig. 50-13 applied to Fig. 50-12, write down the reactions by which the radionuclide ^{197}Pt ($t_{1/2}$ = 18.3 h) can be prepared, at least in principle. Except in special circumstances, only stable nuclides can serve as practical targets for nuclear reactions.

55. The radionuclide ^{60}Co ($t_{1/2}$ = 5.27 y) is much used in cancer therapy. Tabulate possible reactions that might be used in preparing it. Limit the projectiles to neutrons, protons, and deuterons. Limit the targets to stable nuclides. The stable nuclides suitably close to ^{60}Co are ^{63}Cu, ^{60,61,62}Ni, ^{59}Co, and ^{57,58}Fe. (Commercially, ^{60}Co is made by bombarding elemental cobalt, which consists only of the isotope ^{59}Co, with neutrons in a reactor.)

56. A beam of deuterons falls on a copper target. Copper has two stable isotopes, ^{63}Cu (69.2%) and ^{65}Cu (30.8%). Tabulate the residual nuclides that can be produced by the reactions (d,n), (d,p), (d,α), and (d,γ). By inspection of Fig. 50-4, indicate which residual nuclides are stable and which are radioactive.

57. Prepare an overlay like that of Fig. 50-13 in which that figure is extended to include reactions involving the light nuclides ^{3}H (tritium) and ^{3}He, considered both as projectiles and as emerging particles.

58. A platinum target is bombarded with cyclotron-accelerated deuterons for several hours and then the element iridium (Z = 77) is separated chemically from it. What radioisotopes of iridium are present and by what reactions are they formed? (*Note:* ^{190}Pt and ^{192}Pt, not shown in Fig. 50-12, are stable platinum isotopes, but their isotopic abundances are so small that we may ignore their presence.)

59. Prepare an overlay like that of Fig. 50-13 in which two nucleons or light nuclei may appear as emerging particles. The reaction ^{63}Cu(α,pn)^{65}Zn is an example. Consider the combinations nn, np, and pd as possibilities.

50-9 Nuclear Models

60. From the following list of nuclides, identify (*a*) those with filled nucleon shells, (*b*) those with one nucleon outside a filled shell, and (*c*) those with one vacancy in an otherwise filled shell. The nuclides are ^{13}C, ^{18}O, ^{40}K, ^{49}Ti, ^{60}Ni, ^{91}Zr, ^{92}Mo, ^{121}Sb, ^{143}Nd, ^{144}Sm, ^{205}Tl, and ^{207}Pb.

61. The nucleus ^{91}Zr (Z = 40, N = 51) has a single neutron outside a filled 50-neutron core. Because 50 is a magic number, this neutron should perhaps be especially loosely bound. (*a*) Calculate its binding energy. (*b*) Calculate the binding energy of the next neutron, which must be extracted from the filled core. (*c*) Find the binding energy per particle for the entire nucleus. Compare these three numbers and discuss. Needed atomic masses are

^{91}Zr	90.905645 u	n	1.008665 u
^{90}Zr	89.904704 u	^{1}H	1.007825 u.
^{89}Zr	88.908889 u		

PROBLEMS

1. When an α particle collides elastically with a nucleus, the nucleus recoils. A 5.00-MeV α particle has a head-on elastic collision with a gold nucleus, initially at rest. What is the kinetic energy (*a*) of the recoiling nucleus and (*b*) of the rebounding α particle? The mass of the α particle may be taken to be 4.00 u and that of the gold nucleus to be 197 u.

2. Because the neutron has no charge, its mass must be found in some way other than by using a mass spectrometer. When a resting neutron and a proton meet, they combine and form a deuteron, emitting a gamma ray whose energy is 2.2233 MeV. The atomic masses of the proton and the deuteron are 1.007825 u and 2.014102 u, respectively. Find the mass of the neutron from these data, to as many significant figures as the data warrant.

3. The spin and the magnetic moment (maximum z component) of ^{7}Li in its ground state (see Table 50-1) are $\frac{3}{2}$ and +3.26 nuclear magnetons, respectively. A free ^{7}Li nucleus is placed in a magnetic field of 2.16 T. (*a*) Into how many levels will the ground state split because of space quantization? (*b*) What is the energy difference between adjacent pairs of levels? (*c*) What is the wavelength that corresponds to a transition between such a pair of levels? (*d*) In what region of the electromagnetic spectrum does this wavelength lie?

4. (*a*) Show that the electrostatic potential energy of a uniform sphere of charge Q and radius R is given by

$$U = \frac{3Q^2}{20\pi\epsilon_0 R}.$$

(*Hint:* Assemble the sphere from thin spherical shells brought in from infinity.) (*b*) Find the electrostatic potential energy for the nuclide ^{239}Pu, assumed spherical; see Table 50-1. (*c*) Compare its electrostatic potential energy per particle with its binding energy per nucleon of 7.56 MeV. (*d*) What do you conclude?

5. A certain radionuclide is being manufactured in a cyclotron, at a constant rate P. It is also decaying, with a disintegration constant λ. Let the production process continue for a time that is long compared to the half-life of the radionuclide. Show that the number of radioactive nuclei present at such times will be constant and will be given by $N = P/\lambda$. Show further that this result holds no matter how many of the radioactive nuclei were present initially. The nuclide is said to be in *secular equilibrium* with its source; in this state its decay rate is equal to its production rate.

6. The radionuclide ^{56}Mn has a half-life of 2.58 h and is produced in a cyclotron by bombarding a manganese target with deuterons. The target contains only the stable manganese isotope ^{55}Mn, and the reaction that produces ^{56}Mn is

$$^{55}\text{Mn} + \text{d} \rightarrow {}^{56}\text{Mn} + \text{p}.$$

After being bombarded for a time >> 2.58 h, the activity of the target, due to ^{56}Mn, is 8.88×10^{10} s^{-1}; see Problem 5. (*a*) At what constant rate P are ^{56}Mn nuclei being produced in the cyclotron during the bombardment? (*b*) At what rate are they decaying (also during the bombardment)? (*c*) How many ^{56}Mn nuclei are present at the end of the bombardment? (*d*) What is their total mass? The atomic mass of ^{56}Mn is 55.94 u.

7. A radium source contains 1.00 mg of ^{226}Ra, which decays with a half-life of 1600 y to produce ^{222}Rn, an inert gas. This radon gas in turn decays by alpha decay with a half-life of 3.82 d. (*a*) Calculate the decay rate of ^{226}Ra in the source. (*b*) At what rate is the radon decaying when it has come to secular equilibrium with the radium source? See Problem 5. (*c*) How much radon is in secular equilibrium with the radium source?

8. There is speculation that the free proton may not actually be a stable particle but may be radioactive, with a half-life of about 1×10^{32} y. If this turns out to be true, about how long would you have to wait to be reasonably sure that one proton in your body has decayed? Assume that you are made of water and have a mass of 70 kg.

9. A ^{238}U nucleus emits an α particle of energy 4.196 MeV. Calculate the disintegration energy Q for this process, taking the recoil energy of the residual ^{234}Th nucleus into account. The atomic mass of an α particle is 4.0026 u and that of the ^{234}Th is 234.04 u. Compare your result with that of Sample Problem 50-6*a*.

10. Consider that a ^{238}U nucleus emits (*a*) an α particle or (*b*) a sequence of neutron, proton, neutron, proton. Calculate the energy released in each case. (*c*) Convince yourself both by reasoned argument and also by direct calculation that the difference between these two numbers is just the total binding energy of the α particle. Find that binding energy. Needed atomic masses are

^{238}U	238.050783 u	^{4}He	4.002603 u
^{237}U	237.048724 u	^{1}H	1.007825 u
^{236}Pa	236.048674 u	n	1.008665 u.
^{235}Pa	235.045432 u		
^{234}Th	234.043596 u		

11. The radionuclide ^{11}C decays according to

$$^{11}\text{C} \rightarrow {}^{11}\text{B} + e^+ + \nu, \qquad t_{1/2} = 20.3 \text{ min.}$$

The maximum energy of the positron spectrum is 960.8 keV. (*a*) Show that the disintegration energy Q for this process is given by

$$Q = (m_C - m_B - 2m_e)c^2,$$

where m_C and m_B are the atomic mass of ^{11}C and ^{11}B, respectively and m_e is the electron (positron) mass. (*b*) Given that $m_C = 11.011434$ u, $m_B = 11.009305$ u, and $m_e = 0.0005486$ u, calculate Q and compare it with the maximum energy of the positron spectrum, given above. (*Hint:* Let m'_C and m'_B be the nuclear masses and proceed as in Sample Problem 50-7 for beta decay. Note that positron decay is an exception to the general rule that, if atomic masses are used in nuclear decay processes, the mass of the emitted electron is automatically taken care of.)

12. Some radionuclides decay by capturing one of their own atomic electrons, a *K*-electron, say. An example is

$$^{49}\text{V} + e^- \rightarrow {}^{49}\text{Ti} + \nu, \qquad t_{1/2} = 331 \text{ d.}$$

(*a*) Show that the disintegration energy Q for this process is given by

$$Q = (m_V - m_{Ti})c^2 - E_K,$$

where m_V and m_{Ti} are the atomic masses of ^{49}V and ^{49}Ti, respectively, and E_K is the binding energy of the vanadium *K*-electron. (*Hint:* Put m'_V and m'_{Ti} as the corresponding nuclear masses and proceed as in Sample Problem 50-7; see the footnote in that sample problem.) (*b*) Find the disintegration energy Q for the decay of ^{49}V by *K*-electron capture. The needed data are $m_V = 48.948517$ u, $m_{Ti} = 48.947871$ u, and $E_K = 5.47$ keV.

13. One of the dangers of radioactive fallout from a nuclear bomb is ^{90}Sr, which beta decays with a 29-y half-life. Because it has chemical properties much like calcium, the strontium, if eaten by a cow, becomes concentrated in its milk and ends up in the bones of whoever drinks the milk. The energetic decay electrons damage the bone marrow and thus impair the production of red blood cells. A 1-megaton bomb produces approximately 400 g of ^{90}Sr. If the fallout spreads uniformly over a 2000-km^2 area, what area would have radioactivity equal to the allowed bone burden for one person of 0.002 mCi? The atomic mass of ^{90}Sr is 89.9 u.

14. An 87-kg worker at a breeder reactor plant accidentally ingests 2.5 mg of ^{239}Pu dust. ^{239}Pu has a half-life of 24,100 y, decaying by alpha decay. The energy of the emitted α particles is 5.2 MeV, with a quality factor of 13. Assume that the plutonium resides in the worker's body for 12 h, and that 95% of the emitted α particles are stopped within the body. Calculate (*a*) the number of plutonium atoms ingested, (*b*) the number that decay during the 12 h, (*c*) the energy absorbed by the body, (*d*) the resulting physical dose in rad, and (*e*) the equivalent biological dose in rem.

15. Two radioactive materials that are unstable to alpha decay, ^{238}U and ^{232}Th, and one that is unstable to beta decay. ^{40}K, are sufficiently abundant in granite to contribute significantly to the heating of the Earth through the decay energy produced. The alpha-unstable isotopes give rise to decay chains that stop at stable lead isotopes. ^{40}K has a single beta decay. Decay information follows:

Parent Nuclide	*Decay Mode*	*Half-life* (y)	*Stable Endpoint*	Q (MeV)	f (ppm)
^{238}U	α	4.47×10^9	^{206}Pb	51.7	4
^{232}Th	α	1.41×10^{10}	^{208}Pb	42.7	13
^{40}K	β	1.28×10^9	^{40}Ca	1.32	4

Q is the total energy released in the decay of one parent nucleus to the final stable endpoint and f is the abundance of the isotope in kilograms per kilogram of granite; ppm means parts per million. (*a*) Show that these materials give rise to a total heat production of 987 pW for each kilogram of granite. (*b*) Assuming that there is 2.7×10^{22} kg of granite in a 20-km thick, spherical shell around the Earth, estimate the power this will produce over the whole Earth. Compare this with the total solar power intercepted by the Earth, 1.7×10^{17} W.

16. Consider the reaction $X(a,b)Y$, in which X is taken to be at rest in the laboratory reference frame. The initial kinetic energy in this frame is

$$K_{lab} = \tfrac{1}{2} m_a v_a^2.$$

(*a*) Show that the initial velocity of the center of mass of the system in the laboratory frame is

$$V = v_a \left(\frac{m_a}{m_X + m_a} \right).$$

Is this quantity changed by the reaction? (*b*) Show that the initial kinetic energy, viewed now in a reference frame attached to the center of mass of the two particles, is given by

$$K_{cm} = K_{lab}\left(\frac{m_X}{m_X + m_a}\right).$$

Is this quantity changed by the reaction? (*c*) In the reaction $^{90}Zr(d,p)^{91}Zr$ the kinetic energy of the deuteron, measured in the laboratory frame, is 15.9 MeV. Find v_a $(= v_d)$, V, and K_{cm}. Ignore the small relativistic effects.

17. In an endothermic reaction ($Q < 0$), the interacting particles a and X must have a kinetic energy, measured in the center-of-mass reference frame, of at least $|Q|$ if the reaction is to "go." Show, using the result of Problem 16, that the *threshold energy* for particle a, measured in the laboratory reference frame, is

$$K_{th} = |Q|\frac{m_X + m_a}{m_X}.$$

Is it reasonable that K_{th} should be greater than $|Q|$?

18. The nuclide ^{208}Pb is "doubly magic" in that both its proton number Z (= 82) and its neutron number N (= 126) represent filled nucleon shells. An additional proton would yield ^{209}Bi and an additional neutron ^{209}Pb. These "extra" nucleons should be easier to remove than a proton or a neutron from the filled shells of ^{208}Pb. (*a*) Calculate the energy required to remove the "extra" proton from ^{209}Bi and compare it with the energy required to remove a proton from the filled proton shell of ^{208}Pb. (*b*) Calculate the energy required to remove the "extra" neutron from ^{209}Pb and compare it with the energy required to remove a neutron from the filled neutron shell of ^{208}Pb. Do your results agree with expectation? Use these atomic mass data:

Nuclide	*Z*	*N*	*Atomic Mass* (u)
^{209}Bi	82 + 1	126	208.980383
^{208}Pb	82	126	207.976636
^{207}Tl	82 − 1	126	206.977408
^{209}Pb	82	126 + 1	208.981075
^{207}Pb	82	126 − 1	206.975881

The atomic masses of the proton and the neutron are 1.007825 u and 1.008665 u, respectively.

COMPUTER PROBLEM

1. After a brief neutron irradiation of silver, two activities are present: ^{108}Ag ($t_{1/2}$ = 2.42 min) with an initial decay rate of 3.1×10^5/s, and ^{110}Ag ($t_{1/2}$ = 24.6 s) with an initial decay rate of 4.1×10^6/s. Make a plot similar to Fig. 50-8 showing the total combined decay rate of the two isotopes as a function of time from $t = 0$ until $t = 10$ min. In Fig. 50-8, the extraction of the half-life for simple decays was illustrated. Given only the plot of total decay rate, can you suggest a way to analyze it in order to find the half-lives of both isotopes?