Module 4, Lesson 4

Driving at a constant speed: Air and Rolling Resistance.

Objective: If a body in motion tends to stay in motion, why do we need to burn gas to travel at highway speeds? You will learn how to account for air resistance and rolling resistance for moving objects.

Big Ideas

- Two forces are introduced: drag and rolling resistance. Drag is a very complex force that depends on the velocity of the object moving through a fluid. Rolling resistance is simpler conceptually, but many aspects contribute to it.

Introduction

Why do we need to burn gas to keep travelling at the same speed? The basic answer is "because if we didn't, eventually the car would stop." In everyday life, there is always friction and air resistance that opposes any motion, and if you leave a moving object alone, this friction and drag will eventually cause it to stop. This lecture looks at how this drag impacts a car.

First, a quick review: When you are driving a car, the energy from the fuel goes into four main places:

1. Accelerating the bus up to its cruising speed. A moving car has kinetic energy, and it needs to get this energy from the engine. Once we are at a constant speed, we don't need to spend any more energy accelerating but we still need our foot on the gas because of the next two issues.
2. Air resistance. Driving a bus makes the air around it swirl around, and this takes energy. Driving faster makes the air swirl much more.
3. Rolling resistance. This accounts for all of the small bits of friction within the bus, as well as resistance due to the tires on the road.
4. Heat. Burning fuel doesn't make the bus move directly; it creates a lot of heat, and then the engine has to convert that heat into motion. However there is still a lot of heat in the exhaust gases that gets pumped out the back of the bus, so not ALL of it gets converted into motion.

So if we are travelling at a constant speed, we don't need to worry about #1 on the list. We have already accelerated up to speed, so that part is taken care of. However we need to figure out how to understand the other ways that energy is used.

Item number 4 is taken care of by the notion of the efficiency of the car's engine. For a typical gasoline engine, only around 25% of the heat energy from the fuel gets converted into mechanical energy which gets used for the first three items on this list[1].

The energy content of gasoline is about 32 x 106 J / litre, but because of the engine efficiency only 25% of this chemical energy gets converted to mechanical energy[2].

1. Air Resistance

When we drive a car we leave behind us a big tube of air that is swirling around (See Figure 1). The passage of the car is what makes the air swirl around, so our car engine needs to provide all the energy for all of that swirling. Figuring out all the details of exactly which air is swirling where is not important; we just want to make a reasonably accurate estimate of how much energy this will cost us, so we'll develop the following model.

The swirling air is confined to some region near the path of the car. Let's imagine this region is a long tube, with a cross sectional area Atube, and that the passage of the car makes it swirl with velocity v, which is the same velocity as the car. The area Atube is similar to the frontal area of the car, but not exactly the same. A more streamlined car will have Atube slightly smaller than the frontal area of the car. The ratio of Atube / Acar is called the Drag Coefficient (CD). For a typical family sedan, CD= 0.33 and for a cyclist, CD= 0.9[3]

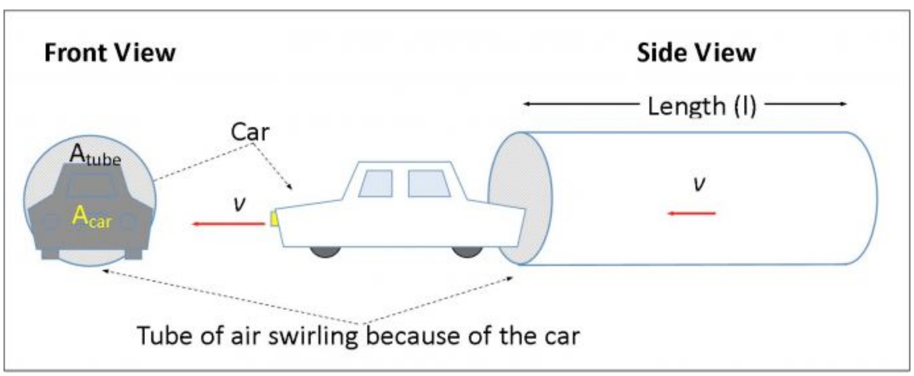

Figure 1. Tube of air swirling around a moving car.

We want use this idea to figure out how much energy it costs the car per kilometre travelled. We can figure out how much energy the car loses to the air by figuring out the kinetic energy of this tube of moving air. To figure out kinetic energy we just need the mass and the volume of the tube of air. A car travelling at speed v will also make the air travel at speed v, so all we need to do is get the mass.

Say the car travels for some distance d. The length of the tube of air that the car encounters in that distance will be the same d:

$$\text{Length} = d$$

So the total volume of this tube will be:

$$\text{Volume} = (\text{Area})(\text{Length}) = A_{tube}d$$

And the mass of the tube will be:

$$\text{Mass} = (\text{Density})(\text{Volume}) = \rho A_{tube}d$$

So now the kinetic energy of the tube will be:

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}A_{tube}dv^2 \\ &= \frac{1}{2}\rho A_{car}C_D dv^2 \end{aligned}$$

In the last line you'll notice that we introduced the constant C_D. This is called the drag coefficient and it relates the volume of the tube of air that's being moved around to volume of air that the car passes through. The drag coefficient depends on the shape of the object moving through the air. More aerodynamic objects have lower drag coefficients, and many are less that one, which means that the tube of air that being pushed around is *smaller* than if we just considered the tube made by the cross section of the car.

Question: Let's use this formula. Given that the area of a typical family sedan is

$$A = (2\ \text{m})(1.5\ \text{m}) = 3\ \text{m}^2$$

and the drag coefficient for a car is CD = 0.33, calculate how much work is done against air resistance for each kilometre the car travels when driving at 50 km/h (14 m/s).

Answer:

$$\begin{aligned} \text{Work done against air resistance} &= \frac{1}{2}\rho A_{car}C_D dv^2 \\ &= \frac{1}{2}(1.3\ \text{kg/m}^3)(3\ \text{m}^2)(0.33)(1000\ \text{m})(14\text{m/s})^2 \\ &= 126,126\text{kg}\bullet\text{m}^2/\text{s}^2 \\ &= 126\ \text{kJ} \end{aligned}$$

So, for each kilometre travelled, 126 kJ of work is done against air resistance.

We can figure out how much fuel is required for each kilometre travelled using the efficiency formula:

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Work Output}}{\text{Work Input}} \\ &= \frac{\text{Work Output}}{\text{Fuel Energy Input}} \\ \text{Fuel Energy Input} &= \frac{\text{Work Output}}{\text{Efficiency}} \\ &= \frac{126\ \text{kJ}}{25\%} \\ &= 505\ \text{kJ} \end{aligned}$$

And to provide this amount of energy we need to use

$$\begin{aligned} \text{Energy per litre} &= \frac{\#\ \text{of joules}}{\#\ \text{of litres}} \\ \#\ \text{of litres} &= \frac{\#\ \text{of joules}}{\text{Energy per litre}} \\ &= \frac{505\ \text{kJ}}{32\ \text{MJL}} \\ &= 0.016\ \text{L} \end{aligned}$$

So, 0.016 L of fuel is required to drive 1 km.

If we compare this with our earlier rule of thumb that the typical fuel consumption of a car is 0.076 L/km4. We see that air resistance is only accounting for 21% of the energy cost. This is because we did the calculation at 50 km/h. At this speed, air friction is really a very small part of the fuel requirements of a car, which is why sometimes we choose to neglect it in our calculations. However, because the fuel consumption depends on the velocity squared, air resistance becomes much more important at higher speeds.

At 100 km/h, the fuel consumption will be FOUR times higher, or 0.064 L/km. This is much closer to 0.076 L/km. To get an even better understanding of energy consumption in cars, we can also take into account the rolling resistance of the car. We'll get into that in the next mini-lecture [Energy Use in Cars 3: Rolling Resistance].

2. Rolling Resistance

Why does Natural Resources Canada recommend keeping your tires inflated to conserve gasoline?

Natural Resources Canada recommends keeping your tires inflated to their maximum pressure to conserve gasoline[1]. Why does this matter? A rough calculation of rolling resistance in cars explores the impact of having underinflated tires.

What is Rolling Resistance?

In cars rolling resistance comes from the fact that the tires are soft, and get deformed as we drive forward, costing the car some energy. The effect of this depends on the inflation of the tire, what kind of tire you have, and how fast you are going, BUT a common approximation which is reasonably accurate is just that the rolling resistance is a constant frictional force that depends on the weight of the car (similar to any other kind of friction).

Force due to Rolling Resistance (FRR)= (Coefficient of Rolling Resistance (μRR)) (Mass of vehicle) (Acceleration of gravity (g))

The Coefficient of Rolling Resistance is usually written as μRR, and it has different values for different types of vehicles. Some example values of rolling resistance are given in the table below[4].

Tire Type	Coefficient of Rolling Friction
Low rolling resistance car tire	0.006 - 0.01
Ordinary car tire	0.015
Truck tire	0.006 - 0.01
Train wheel	0.001

So what does this tell us? In order to figure out how this force impacts our fuel economy we need to figure out how much energy is required to overcome it. For this we use the Work-Energy principle, which tells us how much energy a force will add to a system.

$$\text{Work} = (\text{Force})(\text{Distance})$$

Because the rolling friction opposes the motion of the car, it actually subtracts energy from the car. This energy needs to be made up by burning more fuel.

A typical sedan has a mass of around 1200 kg. For this car, plus a single driver (70 kg) the force of rolling resistance will be:

$$\begin{aligned} F_{RR} &= \mu_{RR} m g \\ &= (0.015)(1270 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 187 \text{ Newtons} \end{aligned}$$

Over the course of driving one kilometre, this will require extra energy given by:

$$
\begin{aligned}
W &= (F_{RR})(\text{Distance}) \\
&= (187\text{N})(1000 \text{ m}) \\
&= 187,000 \text{ N}\bullet\text{m} \\
&= 187 \text{ kJ for each kilometre driven}
\end{aligned}
$$

Questions: Using our technique from earlier (by fuel energy input), determine how much fuel is required to drive one kilometre. Remember, the energy content of gasoline is about 32 x 106 J / litre, but because of the engine efficiency only 25% of this chemical energy gets converted to mechanical energy 2.

Answer: First we calculate the efficiency and fuel energy input.

$$
\begin{aligned}
\text{Efficiency} &= \frac{\text{Work Output}}{\text{Work Input}} \\
&= \frac{\text{Work Output}}{\text{Fuel Energy Input}} \\
\text{Fuel Energy Input} &= \frac{\text{Work Output}}{\text{Efficiency}} \\
&= \frac{187 \text{ kJ}}{25\%} \\
&= 748 \text{ kJ}
\end{aligned}
$$

And to provide this amount of energy we need to use

$$
\begin{aligned}
\text{Energy per litre} &= \frac{\#\text{ of Joules}}{\#\text{ of litres}} \\
\#\text{ of litres} &= \frac{\#\text{ of Joules}}{\text{Energy per litre}} \\
&= \frac{748 \text{ kJ}}{32 \text{ MJ/L}} \\
&= 0.023 \text{ L}
\end{aligned}
$$

So, 0.023 L of fuel is required to drive 1 km.

Remember, this result is just to overcome the rolling friction. If we add this to the 0.064 L/km highway mileage we calculated in Constant Speed Cruising (taking air drag into account) this comes out to a total of 0.087 L/km. This is a little bit higher than the reported average of 0.076 L/km5, which seems reasonable as the highway mileage was calculated at a speed of 100 km/h which is perhaps a bit fast.

So, now what would be the impact of having low air pressure in our tires? Let's imagine that having your air pressure reduced by 5% would result in a 5% increase in the coefficient of rolling resistance. A typical car's tires are inflated to around 40 psi, so this would correspond to being 2 psi lower than average. A 5% increase in the Coefficient of Rolling Resistance would bring it up to 0.01575, and the associated fuel consumption would increase to 0.024 L/km. This is an extra 0.01 L/km, or approximately an extra 1% of fuel mileage. This corresponds closely with the guidelines published by Natural Resources Canada[1].

This extra drag starts to add up when your tires are really low on air. If they are 10 psi low, that would correspond to an extra 5% fuel mileage!

Summary

Can we use these ideas to figure out how to get more mileage out of our cars? Well it looks like when you're travelling at high speed MOST of the gasoline goes into making the air swirl around.

Resources

References for section 1 on air resistance.

1. MacKay DJC. Sustainable Energy - Without the Hot Air (Online). UIT Cambridge. p. 262.http://www.inference.phy.cam.ac.uk/sustainable/book/tex/ps/253.326.pdf [25 August 2009].
2. Wikimedia Foundation Inc. Gasoline (Online). http://en.wikipedia.org/wiki/Gasoline [25 August 2009].
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1. a. b. Natural Resources Canada. Tire Inflation (online). http://oee.nrcan.gc.ca/transportation/personal/driving/autosmart-maintenance.cfm#h [25 August 2009].

2. MacKay DJC. Sustainable Energy - Without the Hot Air (Online). UIT Cambridge. p. 262.http://www.inference.phy.cam.ac.uk/sustainable/book/tex/ps/253.326.pdf [25 August 2009].

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4. A Discovery Company. How Tires Work (online). http://auto.howstuffworks.com/tire4.htm [25 August 2009].

5. MacKay DJC. Sustainable Energy - Without the Hot Air (Online). UIT Cambridge. p.31.http://www.inference.phy.cam.ac.uk/sustainable/book/tex/ps/1.112.pdf [25 August 2009].