What's so unusual about high temperature superconductors?
Everything...

1. ‘Normal’ State - *doped Mott insulator*

2. Pairing Symmetry - *d-wave*

2. Short Coherence Length - *superconducting fluctuations*

3. Low Phase Stiffness - *Uemura plot, phase fluctuations*

4. Low Dimensionality - *Kosterlitz-Thouless transition vs 3D*

5. Competing Order and Phase Separation
Transition temperature

The leap up to a very different range of superconducting transition temperature was the first sign that new physics had been discovered.
YBa$_2$Cu$_3$O$_{7-x}$ - first compound with $T_c > 77$ K
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Square planar array of CuO$_2$ is the basic building block of the high temperature superconductors.

Essential physics is thought to be a square planar array of atoms with one unpaired spin electron per site.
Superconductivity arising from an insulator

RVB state is a superposition of pairs of electrons in singlet states

We now know that the groundstate is an antiferromagnetic insulator driven by strong Coulomb repulsion - strongly correlated electron problem
hole doping

CuO$_2$ plane (AF)

CuO$_2$ plane (doped)

T

T*  AF

SC  hole doping
The superconducting state consists of bound pairs of electrons with equal, but opposite momentum \((k,-k)\). The pairs are condensed into a state that can be denoted by a complex order parameter

\[
\Delta e^{i\Phi}
\]

The magnitude \(\Delta\) appears as a gap in the spectrum of excitations near the Fermi surface.

‘Conventional’ superconductors have singlet pairs \((\uparrow\downarrow - \downarrow\uparrow)\) and a gap \(\Delta(k)\) that does not depend on the direction of the momentum \(k\).
Flux quanta in a YBCO ring with $T_c = 6.0\,\text{K}$

Flux in YBCO rings is quantised in units of $\Phi_0 = \frac{hc}{2e} \Rightarrow$ Cooper pairs
‘Unconventional’ Pairing

One way that a pairing state can differ from the ‘conventional’ superconducting state is for the pair to be in a spin triplet rather than a spin singlet (happens in superfluid $^3$He, but not in the cuprates).

Another way that pairing can be unconventional is for the gap $\Delta$ to vary.
A conventional (s-wave) superconductor has an energy gap with no zeros. Many properties acquire exponential temperature dependence:

$$\exp(-\Delta/k_B T)$$

States with nodes in the gap display power law temperature dependences instead.
Any property that depends on the thermal excitations out of the groundstate is strongly influenced by the presence of nodes in the energy gap.

Eg. density of the superfluid \( n_se^2/m^* \sim 1/\lambda^2 \) determines the London penetration depth \( \lambda \).

Thermal excitations deplete the superfluid density and cause \( \lambda \) to increase.

First definitive power law observed in the cuprates was linear temperature dependence of the London penetration depth in YBa\(_2\)Cu\(_3\)O\(_{6.95}\) (Hardy et al.)

\[ \Delta \lambda(T) = \lambda(T) - \lambda(0) \text{ linear in } T \] suggested lines of nodes on the fermi surface.
Problem - pairing states with nodes are strongly influenced by impurities.

Pair-breaking by impurities generates a non-zero density of states at the fermi energy and changes the power law in the energy dependence.
Substituting Zn impurities for planar Cu in YBa$_2$Cu$_3$O$_{6.95}$ changes the power law from linear to quadratic in $T$.

Even worse - any power law faster than linear can be mistaken for an exponential if there is noise in the data and/or poor sensitivity.
Many properties of a solids are determined by electrons near $E_F$ (conductivity, magnetoresistance, superconductivity, magnetism)

Only a narrow energy slice around $E_F$ is relevant for these properties ($kT=25$ meV at room temperature)
Photoemission intensity: \( I(k, \omega) = I_0 |M(k, \omega)|^2 f(\omega) A(k, \omega) \)

Single-particle spectral function

\[
A(k, \omega) = - \frac{1}{\pi} \frac{\Sigma''(k, \omega)}{[\omega - \epsilon_k - \Sigma'(k, \omega)]^2 + [\Sigma''(k, \omega)]^2}
\]

\( \Sigma(k, \omega) \) : the “self-energy” captures the effects of interactions
TI2201 : ARPES Results
Angle-Resolved Photoemission Spectroscopy (ARPES) confirmed the presence of nodes and determined their orientation with respect to the crystal axes, but is unable to show the additional broken symmetry.

ARPES measurements of the low energy excitations in the superconducting state (Ding et al., PRB 54, 9679)
In BCS theory, the coherence length is \( \xi_0 = \frac{\hbar v_F}{\pi \Delta(0)} \).

This lengthscale can be thought of as the spatial extent of a Cooper pair and in conventional superconductors with low \( T_c \) and small energy gap \( \Delta(0) \), \( \xi_0 \sim 10's \) of nanometres or more.

\( \xi_0 \) also sets the lengthscale over which the superconducting order parameter is able to vary. One consequence of long coherence length is that the superconducting transition is well-described by mean field theory. Conversely, the small value of \( \xi_0 \) in the cuprates makes fluctuations important near \( T_c \).
A mean field superconducting transition has a superfluid density $1/\lambda^2 \alpha (T_c-T)$.

Cuprate single crystals frequently exhibit $1/\lambda^3 \alpha (T_c-T)$.

$\lambda \propto (T_c-T)^{-1/3}$ is the critical behaviour expected for a 2-component order parameter in 3-dimensions (3DXY) (Kamal et al. PRL 73, 1845).
Thermal expansion coefficient of YBCO near $T_c$ shows a transition very similar to the superfluid transition in He$^4$.

Background thermal expansion has opposite sign in the $a$ and $b$ directions, allowing a subtraction that more clearly exhibits the critical behaviour.

(Pasler et al. PRL 1994)
Emery and Kivelson suggested that in the cuprates the low superfluid density is responsible for controlling the critical temperature on the underdoped side of the phase diagram.
Uemura Plot for Single Crystal YBa$_2$Cu$_3$O$_{6+x}$

Non-linear relationship between superfluid density and $T_c$, unlike prediction of Kosterlitz-Thouless behaviour in 2D.
Truly low dimensional superconductivity has usually been achieved in thin films. In 2D, the superconducting transition is governed by a Kosterlitz-Thouless transition at which the phase stiffness drops abruptly due to a sudden proliferation of vortices.

Bulk samples appear to behave 3-dimensionally, despite a layered structure and strong anisotropy.

Zuev et al., cond. matt. 0407113
T >> 150°C
- Tetragonal, high temperature phase where oxygen content is set

T ~ 150°C
- Ortho-I phase, orthorhombic, but very short chain fragments
  - No hole-doping, no Tc

T ~ room temp., P ~ 25 kbar
- Ortho-III-inverse phase?
  - Higher Tc, more perfect

T ~ 150°C
- Ortho-II phase, chain fragments lengthen, hole-doping and Tc evolve.

Chain oxygen ordering in YBa$_2$Cu$_3$O$_{6.34}$
Superfluid Density from 6 GHz Cavity Perturbation

Data taken from one sample with room temperature annealing and pressure.

$1/\lambda^2(T=0)$ depends very strongly on doping

No sign of 2D BKT jump or 3DXY fluctuations
(Broun et al., preprint)
Nanoscale inhomogeneity due to dopant atoms in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$

Scanning tunnelling spectroscopy (STS).

Inhomogeneous superconducting gap driven by random oxygen dopant atoms (x).
A definitive understanding of the origin of high temperature superconductivity still eludes us.

However, the path has led to the invention of new ways of looking at materials - many just a short walk away in AMPEL...
The sign change of a d-wave order parameter as it winds around a tricrystal junction is compensated by the generation of a supercurrent with a half flux quantum in the ring.

$\frac{1}{2} \Phi_0$ flux quantum spontaneously generated in a ring of YBCO that encircles a frustrated tricrystal junction. (Kirtley, Tsuei et al.)