So what is the weather like on Aquila X-1?

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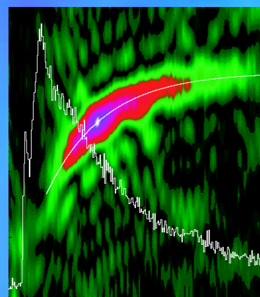
Introduction

If material accretes onto the surface of a neutron star sufficiently slowly, a layer of fuel develops and then suddenly ignites producing a burst of X-rays known as a Type I X-ray burst. These thermonuclear flashes each release about 10^{39} ergs and repeat on a timescale from hours to days. Although only one source (SAX J1808.4-3658) exhibits periodic variation in its quiescent emission, during the bursts themselves the emission is often quasiperiodic, apparently due to rotational modulation of the inhomogeneities in the thermonuclear burning. The oscillations in the cooling tails of the X-ray bursts from 4U 1702-429 and 4U 1728-34 are nearly coherent with Q ~ 4000 and consistent with an increase in the modulation frequency as the burning layer cools.

The change in the modulation frequency may be depicted in a dynamic power spectra like that to the right. Time runs from left to run and frequency increases from the bottom to the top.



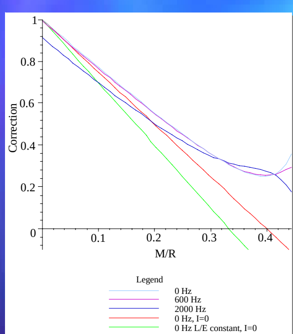
Courtesy: Tod Strohmayer (NASA/GSFC)

The standard explanation for this frequency shift is that the expanding hotspot conserves angular momentum as it moves higher in the atmosphere; consequently just as a ice skater slows as she moves her arms outward and accelerates as she moves her arms in, the angular frequency of the hotspot decreases as the atmosphere expands and increases as it cools.

To use the conservation of angular momentum to explain the change in frequency of the emission as the burning region of the atmosphere (or hotspot) expands and then contracts, one assumes that the specific angular momentum of a fluid element is conserved during the burst. The magnetic fields of the neutron stars are likely to be weak (otherwise there would be periodicities in the persistent emission); consequently, this is a viable assumption. However, the relationship between the specific angular momentum of a fluid element and its position is more complicated in general relativity than in Newtonian theory.

Angular Momentum in General Relativity

In General Relativity, conservation laws are intimately connected with the symmetries of the spacetime. Killing vectors trace the symmetries. The spacetime surrounding a rotating neutron star exhibits a time translation symmetry (it will look the same a minute from now as it does now) and a rotational symmetry (if you walk around the star at a fixed time it looks the same). A hotspot traces a Killing trajectory which means that its velocity can be expressed as a linear combination of the two Killing vectors.



The curvature of the spacetime near the neutron star introduces several types of correction:

- The surface gravity is larger than a Newtonian calculation by a factor of $1+z$.
- If a parcel of gas moves up Δz, its radial coordinate only changes by $(1+z)^{-1}\Delta z$.
- The angular momentum carried by a parcel of rotating gas at a given radius and angular frequency is larger due to the curvature of the space near the neutron star.

For a typical value of M/R of 0.2 (a neutron star with M=1.4 $M_\odot$ and R=10 km) the observed change in angular frequency is a factor of two smaller than a purely Newtonian calculation would indicate.

Furthermore, revised Newtonian calculations indicate that even before the relativistic corrections are included, the atmosphere does not expand sufficiently to explain the change in angular frequency by angular momentum conservation alone.

We must seek an alternative explanation for the oscillations and their change in frequency.

Related Challenges

- From the quick rise time of the burst, one can infer that the nuclear burning envelopes the entire stellar surface within a second; therefore, it is surprising that the emission would be still be variable several seconds into the burst.
- During the cooling portion of the burst, the hotspot apparently lags behind the lower portion of the atmosphere by ten revolutions. The atmosphere suffers a shearing of ~ 100 km over ~ 10 m - a factor of 10^4; the shear develops over about ten seconds, so over 10 m the velocity varies by 10 km/s. What instabilities could such a sheared layer suffer and what forces would resist the shearing?
- Angular momentum does not tend to be conserved when gas flows vertically in the Earth's atmosphere. For example, the tops of thunder clouds do not drift westward relative to their bases.

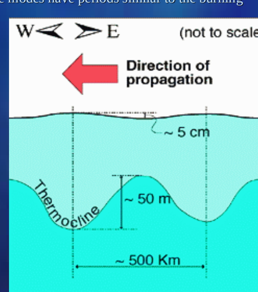
An Alternative Explanation - Let's talk about the weather!

The burning on the surface is complex. It proceeds much more slowly than the sound speed; it is not a detonation wave. On the other hand, the burning front travels significantly faster than thermal conduction; it is not a simple deflagration front. Consequently, the burning front is probably complicated. Subsonic flows mix unburned material into the burning region speeding up the flame.

It takes about a "year" (300 rotations) for the burning front to envelope the entire surface. Stellar rotation is surely important. The surface burning is essentially a long-lasting weather system on the star. Furthermore, one would expect that the burning could exciting the natural modes of the atmosphere if these modes have periods similar to the burning timescale, one second.

Waves on a Rotating Sphere

Rotation strongly affects the propagation of waves on the surface of a sphere. On a non-rotating sphere, it is natural to expand the waves into spherical harmonics. Rotation mixes waves with different values of l and removes the degeneracy among waves with the same value of m. If the rotation frequency of the sphere is sufficiently large compared to the frequency of the wave, a new regime appears in which most waves are restricted to a narrow band near the equator whose width depends on the properties of the mode. However, a wide class of waves, known as the Rossby waves, occupy a large portion of the sphere even when it is rotating quickly. These waves travel opposite to the sphere's rotation.



Schematic of a Rossby wave

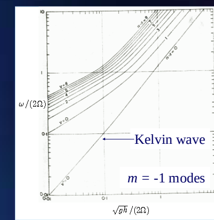
As a baroclinic Rossby wave travels along the interface between more and less buoyant material, both buoyancy and the Coriolis force restore the fluid to its unperturbed state. The frequencies of the Rossby waves approach a constant value (~ 1 Hz for a neutron star) as the spin of the star increases.

Mode Structure

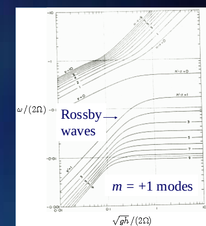
The rotation of the star naturally divides the modes into those that travel with the rotation (eastbound $m<0$), those that travel against the rotation (westbound $m>0$) and those that do neither ($m=0$). Someone who is not rotating with the star sees a oscillation at the frequency,

$$|m\Omega - \omega|.$$

For $m=+1$, this is somewhat less than the spin frequency of the star Ω. The frequency of the mode, ω, generally depends on Ω. For $m=0$, the fundamental mode in the radial direction typically has a frequency ~ 20 Hz for $\Omega \sim 2\pi$ (300 Hz) which is inferred for Type-I burst sources. The frequency of all the $m=0$ modes increases as $\Omega^{0.5}$. For each negative value of m, there is a single wave whose frequency is completely independent of Ω, the Kelvin wave. In the plots to the right, a mode with a constant frequency is traced by a line with a slope of unity.



Looking at the lower figure, we see a family of modes whose frequencies approach constant values as the spin frequency of the star increases (the left side of the diagram). These modes are the Rossby modes. Their frequencies are somewhat smaller than the Kelvin waves, and their footprint on the surface of the star is somewhat larger.



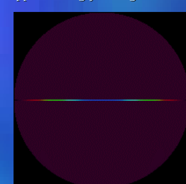
Longuet-Higgins 1968

$$|\mu| < \begin{cases} q^{-1}(2+1/\nu)^{-1/2} & g-\text{mode} \\ |qm|^{-1/2} & \text{Kelvin mode} \\ |qm|^{-1/2}(2\nu^2+\nu)^{1/2} & r-\text{mode} \end{cases}$$

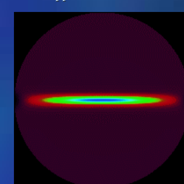
where $q=\Omega/\omega$, $\mu=\cos\theta$, and $\nu>0$ for a Rossby wave and $\nu\geq 0$ for a gravity wave.

Mode Visibility

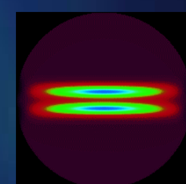
When we measure the flux from the surface of a neutron star, we see a bit more than a entire hemisphere of the surface at a time. This dramatically reduces the observed variability of the mode that occupies a small portion of the stellar surface. If one looks at the three classes of modes for a given observed frequency ~ 1 Hz, which modes will appear strongly for a given level of excitation is apparent.



Gravity wave

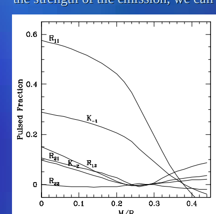


Kelvin wave



Rossby Wave

For neutron stars with larger values of M/R, we observe a larger fraction of the stellar surface, so the total expected variability is smaller. If we assume a simple prescription to translate the strength of the mode at a particular point to the strength of the emission, we can calculate the pulsed fractions from the various modes. The mode with the largest footprint is the Rossby wave with one latitudinal node in the velocity (ν=1) and one azimuthal node (m=1), R_{11}, and this mode exhibits a significantly larger variability than any other mode. The next strongest mode is the K_{-1} mode. Modes with additional latitudinal, azimthal or radial nodes exhibit much less variability.



Observed Frequencies

The natural frequency of the mode depends on the properties of the ocean layers through which it propagates.

$$2.4\,\text{Hz}\;\; m T_8^{1/2} R_6^{-1} A_{56}^{-1/2} (1+0.47n^2)^{-1/2}(2\nu+1)^{-1}$$

As the ocean cools from T~10^9 K down to T~10^8 K the frequency of the mode will change by a few Hertz. The observed frequency for the Rossby wave is the difference between the spin frequency of the star and frequency of the wave, so we would observe a periodic oscillation with Δf ~ 1-2 Hz. As the atmosphere cools the observed frequency will approach the spin frequency of the star from below.

Contrasts with the Standard Model

- It works. For iron, one would expect ω to change by about 1.4 Hz enough to account for the observations.
- Stars with different spin frequencies would exhibit the same absolute change in oscillation frequency.
- The final frequency of the oscillation is not necessarily constant from burst to burst.
- Potentially other waves might be excited and observed:
  - Kelvin waves (Δf ~ 5 Hz) with the observed frequency *decreasing* as the burst cools, and
  - Higher-order Rossby waves (n > 0) with smaller frequency shifts.
  - Each wave would appear in the dynamic power spectrum.
  - Also waves with $|m|$ > 1 could be excited and seen at $|m\Omega - \omega|$.

Prospects

More sensitive observations of Type-I bursts perhaps with Chandra or XMM-Newton should uncover clear evidence for the excitation of additional modes. The spectrum of excited modes will probe how the burning proceeds across the stellar surface. The frequency of the observed modes depends on the temperature and composition of the layer along with the gravitational redshift of the surface; consequently, observations could also probe the properties of the lower layers of the burning atmosphere. Observations of the spectrum of the emission itself only directly constrain the properties of the upper atmosphere.

Regardless further observational data will decide whether we have observed weather on the surface of a neutron star.

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