Quantum Anomalies and Topological Currents in Dense Matter*

Ariel Zhitnitsky
University of British Columbia
Vancouver

August 9-13 /2005,
Neutron Stars at the Crossroads of Fundamental Physics

* *(Based on works with M.Metlitski and D.T.Son.)*

I. Introduction. Motivation.

1. It is well-known that there are NO topological solitons in the Standard model.
(like monopoles, domain walls, strings...)

2. This statement follows from some simple topological arguments based on the symmetry breaking pattern in the standard model,

$$SU(3)_c \times SU(2) \times U(1) \times (global)$$

$$\Downarrow$$

$$SU(3)_c \times U(1)_{EM} \times (global)$$

3. However! In dense matter ($neutron\ stars?$) the situation could be very different ...

II. Color Superconductivity

($\mu\bar{\Psi}\gamma_0\Psi$ - term in the Lagrangian)

1. If there is a channel in which the quark-quark interaction is attractive, than the true ground state of the system will be a complicated coherent state of Cooper pairs like in **BCS** theory (ordinary superconductor).

2. Diquark condensates break color symmetry (CFL phase, $N_c = N_f = 3$):

$$\langle\psi^{ia}_{L\alpha}\psi^{jb}_{L\beta}\rangle^* \sim \epsilon_{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{abc}X^{\gamma}_{c},$$
$$\langle\psi^{ia}_{R\alpha}\psi^{jb}_{R\beta}\rangle^* \sim \epsilon_{\alpha\beta\gamma}\epsilon^{ij}\epsilon^{abc}Y^{\gamma}_{c}$$

3. $SU(3)_c \times U(1)_{EM} \times SU(3)_L \times SU(3)_R \times U(1)_B$

$$\Downarrow$$

$$SU(3)_{c+L+R} \times U(1)^*_{EM}$$

a) Color gauge group is completely broken;
b) $U(1)_B$ is spontaneously broken;
c) $U(1)_{EM}$ is not broken;
d) $U(1)_A$ is broken spontaneously and explicitly (by instantons)

4. Goldstone fields are the phases of the condensate

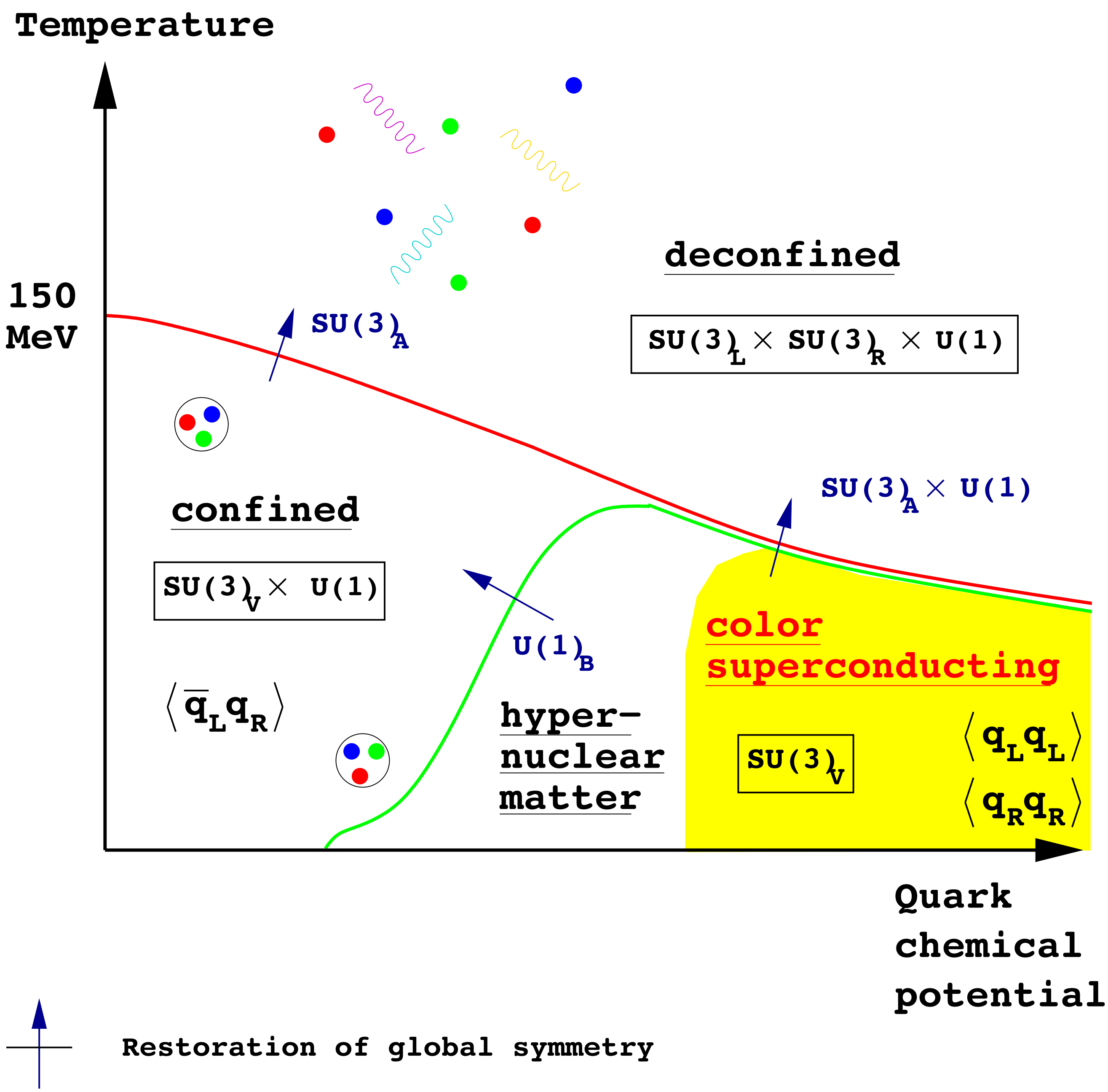
$$\Sigma^{\beta}_{\gamma} = \sum_c X^{\beta}_c Y^{c*}_{\gamma} \sim e^{\lambda^a \pi^a} e^{i\eta'}.$$

5. Superfluid phonon field is the phase of the condensate

$$detX \sim detY \sim e^{i\varphi_B}.$$

6. The gap $\Delta \simeq 100 MeV$ is large.
The critical temperature $T_c \simeq 0.6\Delta$ is also large.

7. The phase structure in QCD (with parameters realized in nature) could be much more complicated. In particular, condensates of different fields: $\langle K^0 \rangle, \langle K^+ \rangle, \langle \eta \rangle$ along with diquark condensate $\langle \psi^{ia}_{L\alpha} \psi^{jb}_{L\beta} \rangle$ and $\langle \psi^{ia}_{R\alpha} \psi^{jb}_{R\beta} \rangle$, could develop.



Conjectured phase diagram for QCD with 3 light flavors. (From Mark Alford's review paper).

III. Topological Defects in Dense Matter

1. $U(1)_B$ is spontaneously broken $\Rightarrow$ there are global vortices, similar to what is observed in He^4. Effective lagrangian is

$$L_B \sim f_B^2[(\partial_0\varphi_B)^2 - u^2(\partial_i\varphi_B)^2]$$

2. $U(1)_A$ is spontaneously broken (by the condensate $\langle\Sigma_\gamma^\beta\rangle \neq 0$) $\Rightarrow$ there are axial global vortices.

3. The symmetry is broken also explicitly by the instantons $\Rightarrow$ there are axial domain walls. Effective lagrangian is

$$L_A \sim f_A^2[(\partial_0\varphi_A)^2 - u^2(\partial_i\varphi_A)^2] + a\cos(\varphi_A - \theta),$$

where $a \sim \int d\rho n(\rho) \sim (\frac{\Lambda_{QCD}}{\mu})^b \ll 1$ with $n(\rho) \sim e^{-\mu^2\rho^2 N_f}$ such that instanton calculations are under complete theoretical control.

4. The η' is light: $m^2_{\eta'} \sim \frac{a}{f^2} \sim (\frac{\Lambda_{QCD}}{\mu})^b \rightarrow 0.$

5. The η' domain wall solutions correspond to the transitions between $\varphi_A = 0, 2\pi, 4\pi...$, which are the same physical points.

6. Analytical solution: $\varphi_A = 4\tan^{-1}[\exp(\frac{m_{\eta'}x}{4})]$. Domain wall tension $\sigma \sim \sqrt{a}f \sim f^2 m_{\eta'}$.

7. Few remarks on domain walls at large μ:
a). If states $\varphi_A = 0, 2\pi, 4\pi...$, were physically different states, these domain walls would be absolutely stable (ferromagnetic domain walls);
b). If φ_A is the only degree of freedom of the theory, the soliton would be absolutely stable, like in $2d$ Sine Gordon model;
c). If the heavy degrees of freedom $m \sim \Delta$ are taken into account, the domain walls become metastable (tunneling with the excitations of heavy degrees of freedom is possible) in which case the life time is expected to be very large for large μ when the instanton density is small and η' is light, $\tau(a \rightarrow 0) \sim \exp\left(\frac{\pi^2}{24}\frac{f^6 u^2}{a\Delta^2}\ln^3\frac{1}{\sqrt{a}}\right) \rightarrow \infty.$

IV. Quantum Anomalies in Dense Matter

1. It is known, that the **quantum anomalies** in QFT play very important role in theory and phenomenology,

2. Our goal is to derive a **new** **anomalous effective lagrangian** describing the interaction of light fields: the electromagnetic photons A_μ, neutral light Nambu-Goldstone bosons (π, η, η'), the **superfluid phonon** φ_B and **axion** α in dense matter.

3. In terms of physical fields (A_μ-electromagnetic, φ_B -baryon phase of the diquark condensate , φ_A–NG fields, π, η' or axion α), the effective lagrangian takes the form,

$$
\mathcal{L}_{\rm anom} = \frac{1}{8\pi^2 q_{\rm A}}\partial_\mu\varphi_{\rm A}\Big[e^2 C_{{\rm A}\gamma\gamma}A_\nu\tilde{F}^{\mu\nu}
-eC_{{\rm AB}\gamma}\epsilon^{\mu\nu\alpha\beta}\left(\mu n_\nu - \tfrac{1}{2}\partial_\nu\varphi_{\rm B}\right)F^{\alpha\beta}
-\tfrac{1}{2}C_{\rm ABB}\,\epsilon^{\mu\nu\alpha\beta}\left(\mu n_\nu - \tfrac{1}{2}\partial_\nu\varphi_{\rm B}\right)\partial_\alpha\partial_\beta\varphi_{\rm B}\Big].
$$

V. Applications

1. Magnetization of axial η' domain walls.

a) Consider an axial domain wall in an external magnetic field. The baryon field $B_\nu = (1, \vec{0})$ is considered as a background which is at rest. The following term is present in the anomaly Lagrangian:

$$\mathcal{L}_{\mathrm{AB}\gamma} = -\frac{eC_{\mathrm{AB}\gamma}\mu}{8\pi^2}\partial_\mu \varphi_{\mathrm{A}} n_\nu \tilde{F}_{\mu\nu} = \frac{eC_{\mathrm{AB}\gamma}\mu}{8\pi^2}\vec{B}\cdot\vec{\nabla}\varphi_{\mathrm{A}}$$

b) Anomaly lagrangian $\mathcal{L}_{\mathrm{AB}\gamma}$ implies that the energy is changed by a quantity proportional to BS, where S is the area of the domain wall. This means that the **domain wall is magnetized**, with a finite magnetic moment per unit area equal to $eC_{\mathrm{AB}\gamma}\mu/(4\pi)$.

2. Axion interaction in dense matter.

Similar interaction can be derived for the axion field in dense matter,

$$\mathcal{L}_{\mathrm{B}\alpha\gamma} \sim \mu\vec{B}\cdot\vec{\nabla}\alpha.$$

It may play an important role in constraints on the axions emitted from dense neutron stars.

3. Currents on axial η' vortices.

a) One can rewrite the same lagrangian $\mathcal{L}_{AB\gamma}$ into the following form, $\mathcal{L}_{AB\gamma} = \frac{eC_{AB\gamma}\mu}{4\pi^2}\epsilon_{ijk}A_i\,\partial_j\partial_k\varphi_A$.

b). Since $\epsilon_{ijk}\partial_j\partial_k\varphi_A \sim 2\pi\delta^2(x_\perp)$ on the vortex core, the action can be written as a line integral along the vortex,

$$\mathcal{S}_{\rm anom} = \frac{eC_{AB\gamma}\mu}{2\pi}\int d\vec{\ell}\cdot\vec{A} \qquad (1)$$

which means that **there is an electric current running along the core of the axial vortex**. The magnitude of the current is $j^{\rm em} = \frac{eC_{AB\gamma}\mu}{2\pi}$

c). The electromagnetic current running along a closed vortex loop generates a **magnetic moment equal to $\frac{1}{2}jS$,** where S is the area of the surface enclosed by the loop.

d). A large vortex loop has a magnetic moment that can be interpreted as created by the current running along the loop, *or* as the total magnetization of the domain wall stretched on the loop.

4. Axial current in the background B field .

a) The following term is present in the anomaly Lagrangian in the presence of the magnetic B field:

$$\mathcal{L} = 4C_{\phi AV} \nabla\phi \cdot \vec{B}$$

.

b). This anomalous term implies the existence of an axial current flowing through the dense matter which is proportional to the magnetic flux

$$J = N_c \sum_a \frac{e_a \mu_a Q_a}{2\pi^2} \Phi$$

where Φ is the total magnetic flux through the cross-section S.

5. Superconducting strings in dense matter.

a) In the presence of the magnetic field and/or magnetic flux (type II superconductor) one can keep only zero modes such that relevant degrees of freedom are described by an effective 2d theory.

b) Effective 2d action can be shown to be,

$$\mathcal{S}_{\rm 2d} = \int dtdz \Big[\bar{\psi} i D_i \gamma_i \psi - m\bar{\psi}\psi + \mu\bar{\psi}\gamma_0\psi\Big] \cdot \Phi$$

where ψ describes the effective 2d zero modes in B background with flux $\Phi = \int d^2 x_\perp \cdot \frac{eB}{2\pi}$

c) This lagrangian takes the form (after bosonization)

$$\mathcal{S}_{\rm 2d} \sim \Phi \int dzdt \Big[\frac{1}{2}(\partial_\mu \varphi)^2 + eA_a \epsilon^{ab}\partial_b\varphi\Big].$$

d)This form implies that $\partial_z\varphi$ can be interpreted as the density of electrical (not axial) charge, while $\partial_0\varphi$ can be interpreted as the electromagnetic current along the string.

e) This effective lagrangian **exactly coincides** with the one introduced by Witten, 1984 to describe the **superconducting strings.**

VI. Conclusion

1. The quantum anomalies, especially those involving the baryon (electric) current, lead to new and extremely unusual effects in matter at high densities. Some of these effects should have consequences for the physics of compact objects, and still to be explored.

2. Of particular interest is the finding that the axial current is induced in the background of magnetic field (which always present in neutron stars).

3. It is quite remarkable that a very nontrivial construction invented by Witten for cosmic strings is automatically realized in dense matter systems. Therefore, many consequences of the Witten's construction, such as there existence of the closed loops of superconducting strings (vortons), may be realized in dense matter.

4. In particular, such superconducting strings can lead to the macroscopically large non dissipating azimuthal B_ϕ field as well as magnetic helicity $\int d^3x \vec{B} \cdot \vec{A}$ in neutron stars. Physical consequences are still to be explored.