Phase diagram of dense QCD with and without neutrino trapping

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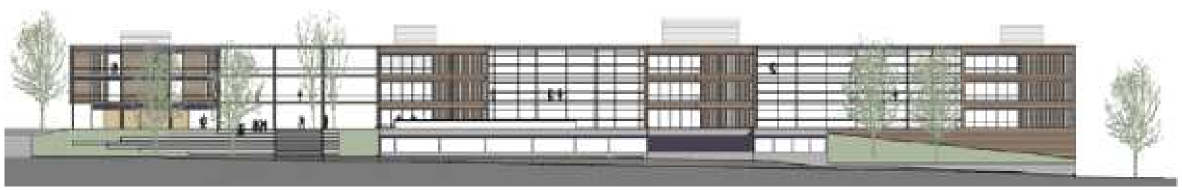

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Outline

- Introduction and motivation

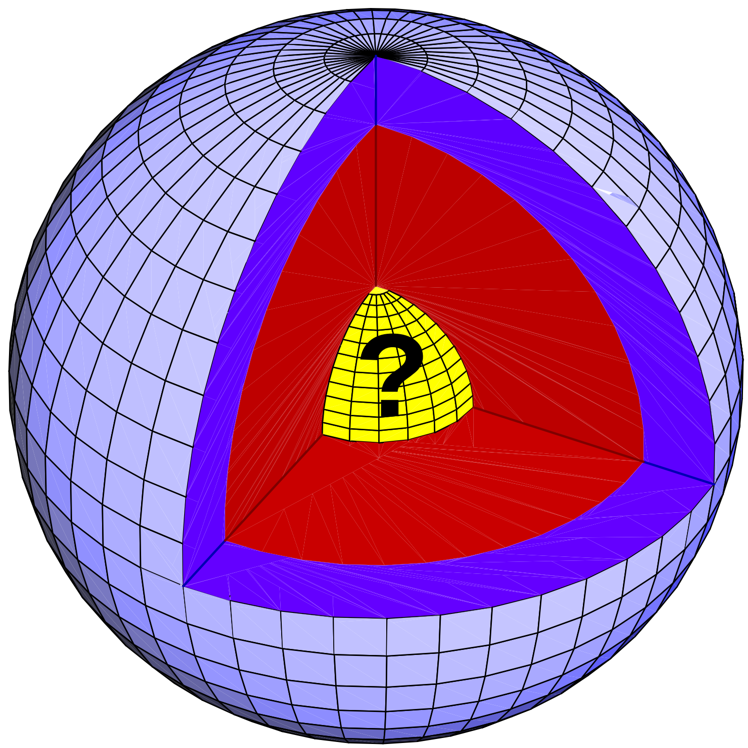
- Results for the phase diagram(s) of dense QCD
  - Self-consistent treatment of quark masses
  - Effect of neutrino trapping
  - Problems, limitations, etc.
- Current "state of the art"

- Conclusions
- Outlook

Dense baryonic matter in Nature

Compact (neutron) stars

- Radius:
  $R \simeq 10$ km
- Mass:
  $1.25 M_{\odot} \lesssim M \lesssim 2 M_{\odot}$
- Core temperature:
  10 keV $\lesssim T \lesssim 10$ MeV
- Surface magnetic field:
  10^{8} G $\lesssim B \lesssim 10^{14}$ G



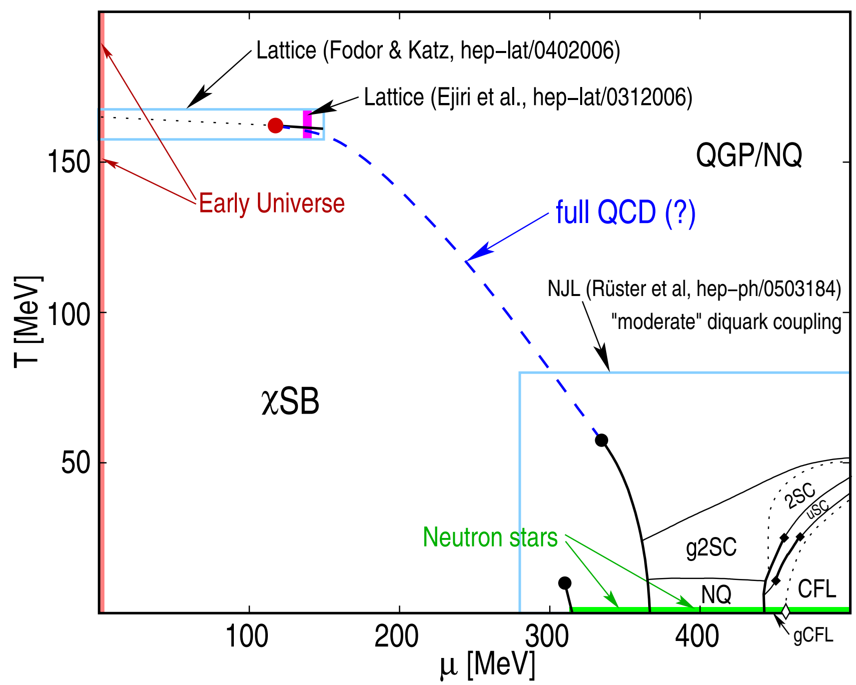
What is the state of matter at the highest stellar densities, $\rho_c \gtrsim 5\rho_0$?

Phase diagram of QCD

The region of prime interest is

$$\mu \gtrsim \Lambda_{QCD} \gtrsim T$$

- So far, there are no reliable lattice results at $\mu \gtrsim \Lambda_{QCD}$
- Effective models have a limitted predictive power
- ⊕ Effects of charge neutrality and β equilibrium are not under control
- ⊕ Difficulties in determining stable ground states



So, how hard could it be?

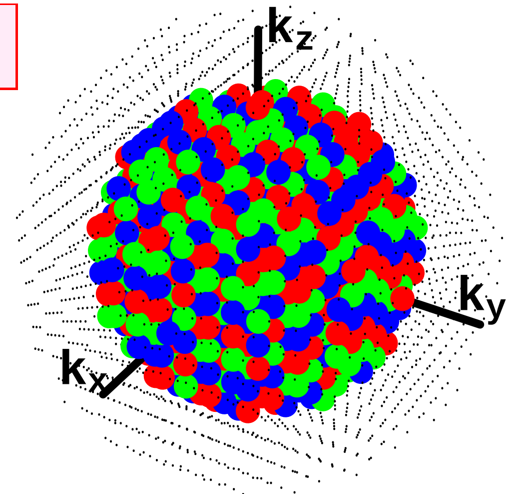
Ground state of dense quark matter

Noninteracting quarks:

(i) Deconfined quarks ($\mu \gg \Lambda_{QCD}$)

(ii) Pauli principle ($s = \frac{1}{2}$)

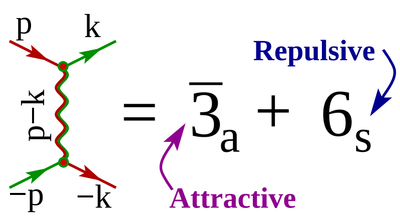
$\Rightarrow$



Interacting quarks:

(i) Effective models ($\mu \gtrsim \Lambda_{QCD}$)

(ii) One-gluon exchange ($\mu \gg \Lambda_{QCD}$)



$\Rightarrow$ Cooper instability

$\Downarrow$

Color superconductivity

$$\langle \left(\bar{\Psi}^C\right)_i^\alpha \gamma_5 \Psi_j^\beta \rangle \neq 0$$

Unconventional Cooper pairing

- Wave function of a spin-0 Cooper pair:

$$(|\color{red}{\bullet}\color{green}{\bullet}\rangle - |\color{green}{\bullet}\color{red}{\bullet}\rangle)_{\bar{3}} \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)_{J=0} \otimes \boxed{(|\mathrm{u}_{\mathbf{p}}, \mathrm{d}_{-\mathbf{p}}\rangle - |\mathrm{d}_{\mathbf{p}}, \mathrm{u}_{-\mathbf{p}}\rangle)_{1,\bar{3}}}$$

- In β-equilibrium, quarks have non-equal Fermi momenta:

$$p_F^{(u)} \neq p_F^{(d)} \neq p_F^{(s)}$$

Charge neutrality ($N_f = 2$)

$$\mu_d = \mu_u + \mu_e$$

$$\frac{2}{3} n_u - \frac{1}{3} n_d - n_e = 0$$

$$\delta\mu \equiv \frac{p_F^{(d)} - p_F^{(u)}}{2} = \boxed{\frac{\mu_e}{2}}$$

Color neutrality ($N_f = 3$)

$$m_s \gg m_u, m_d$$

CFL: strange $\Leftrightarrow$ blue

$$\delta\mu \equiv \frac{p_F^{(bd)} - p_F^{(gs)}}{2} \approx \boxed{\frac{m_s^2}{2\mu}}$$

[Alford,Kouvaris&Rajagopal,hep-ph/0311286]

How does the mismatch $\delta\mu \neq 0$ affect Cooper pairing?

Model

NJL model: [Rehberg,Klevansky&Hüfner, Phys.Rev.C **53**, 410 (1996)]

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\,(i\partial\!\!\!/ - \hat{m}\,)\psi + G_S \sum_{a=0}^{8} \left[\left(\bar{\psi}\lambda_a \psi\right)^2 + \left(\bar{\psi} i \gamma_5 \lambda_a \psi\right)^2 \right] \\
&+ G_D \sum_{\gamma,c} \left[\bar{\psi}^a_\alpha i\gamma_5 \epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta\right] \left[(\bar{\psi}_C)^r_\rho i\gamma_5 \epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma\right] \\
&- K \left\{ \det_f \left[\bar{\psi}\,(1+\gamma_5)\,\psi\right] + \det_f \left[\bar{\psi}\,(1-\gamma_5)\,\psi\right] \right\}
\end{aligned}
$$

Parameters:

$$
\begin{aligned}
m_{u,d} &= 5.5 \text{ MeV} \\
m_s &= 140.7 \text{ MeV} \\
G_S\Lambda^2 &= 1.835 \\
K\Lambda^5 &= 12.36 \\
\Lambda &= 602.3 \text{ MeV}
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
m_\pi &= 135.0 \text{ MeV} \\
m_K &= 497.7 \text{ MeV} \\
m_{\eta'} &= 957.8 \text{ MeV} \\
f_\pi &= 92.4 \text{ MeV} \\
m_\eta &= 514.8 \text{ MeV}
\end{aligned}
$$

General approach

Quark chemical potentials:

$$\mu_{ab}^{\alpha\beta} = \left(\mu\delta^{\alpha\beta} + \mu_Q Q_f^{\alpha\beta}\right)\delta_{ab} + \left[\mu_3 \left(T_3\right)_{ab} + \mu_8 \left(T_8\right)_{ab}\right]\delta^{\alpha\beta}$$

Dynamically generated quark masses:

$$\hat{M} = \mathrm{diag}_f(M_u, M_d, M_s), \quad \text{with} \quad M_\alpha = m_\alpha - 4G_S\sigma_\alpha + 2K\sigma_\beta\sigma_\gamma$$

Allowed condensates:

$$\Delta_c \sim \epsilon^{\alpha\beta c}\epsilon_{abc}\langle(\bar{\psi}_C)_\alpha^a i\gamma_5\psi_\beta^b\rangle \qquad \text{(no sum over color “}c\text{”)}$$

$$\sigma_\alpha \sim \langle\bar{\psi}_\alpha^a\psi_\alpha^a\rangle \qquad \text{(no sum over flavor “}\alpha\text{”)}$$

Gap equations and neutrality constraints

Pressure:

$$p \quad = \quad p_L - \frac{1}{4G_D}\sum_{c=1}^{3}|\Delta_c|^2 - 2G_S\sum_{\alpha=1}^{3}\sigma_\alpha^2 + 4K\sigma_u\sigma_d\sigma_s + \frac{1}{2}\ln\det\frac{S^{-1}}{T}$$

Coupled set of 9 equations:

$$\frac{\partial p}{\partial \sigma_\alpha} \quad = \quad 0$$

$$\frac{\partial p}{\partial \Delta_c} \quad = \quad 0$$

$$n_Q \quad \equiv \quad \frac{\partial p}{\partial \mu_Q} = 0$$

$$n_3 \quad \equiv \quad \frac{\partial p}{\partial \mu_3} = 0$$

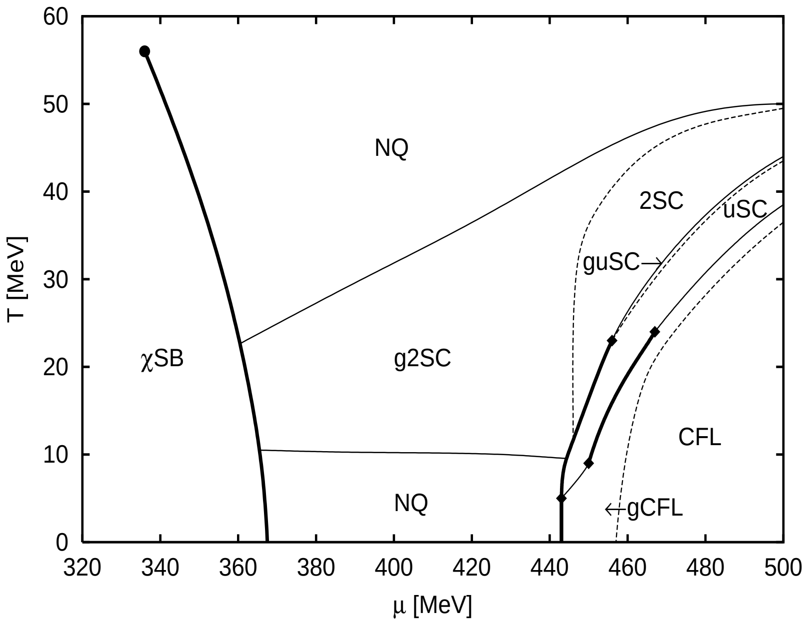
$$n_8 \quad \equiv \quad \frac{\partial p}{\partial \mu_8} = 0$$

Note: charge neutrality is enforced locally (no mixed phases allowed)

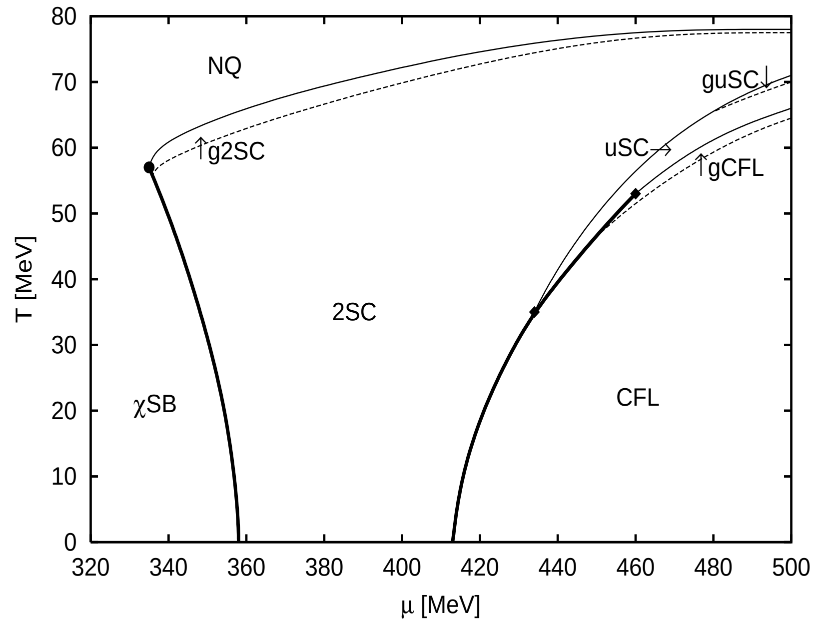
Phase diagram, $\mu_{\nu_e} = 0$

(without neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, hep-ph/0503184]



$G_D = \frac{3}{4} G_S$ (intermediate coupling)



$G_D = G_S$ (strong coupling)

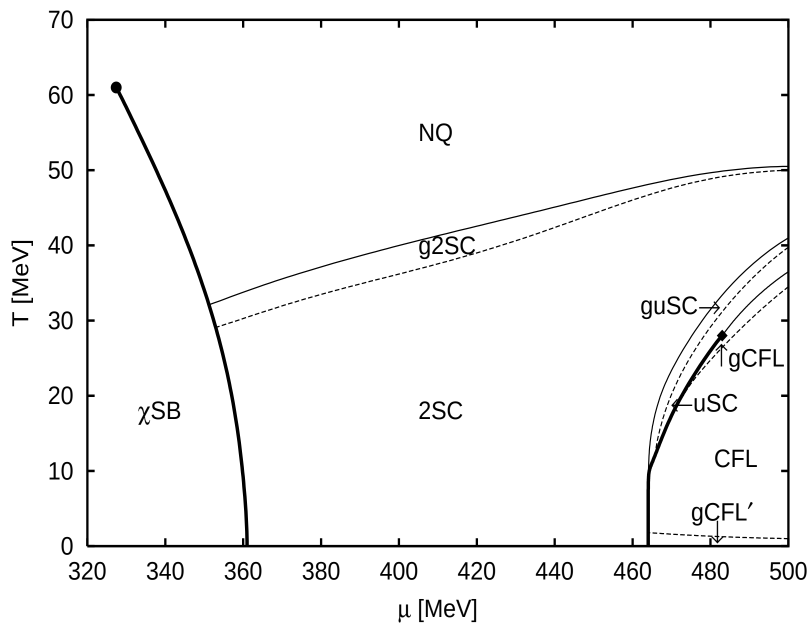
Note: Gapless phases play little role at strong coupling, $G_D = G_S$

See also [Blaschke, Fredriksson, Grigorian, Sandin & Öztaš, hep-ph/0503194]

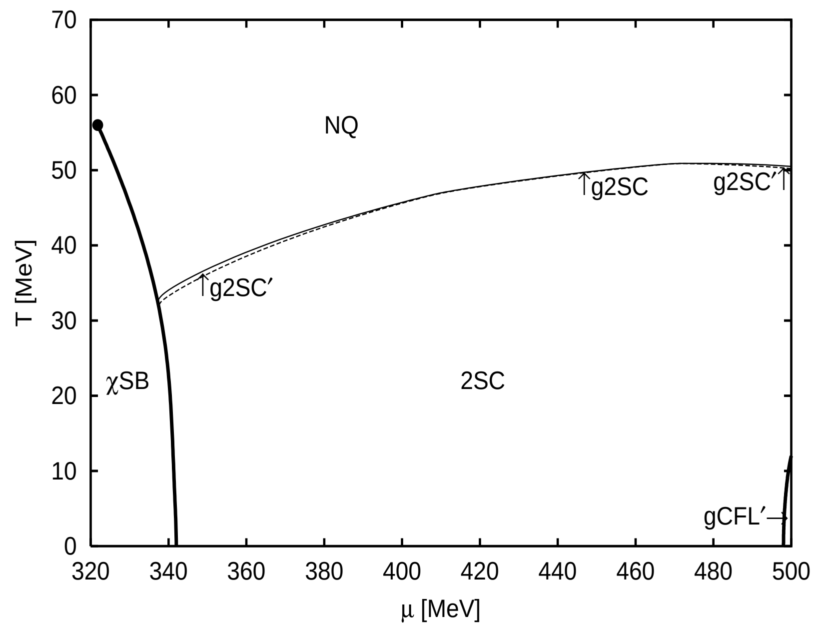
Phase diagram, $\mu_{\nu_e} \neq 0$

(with neutrino trapping)

[Rüster, Werth, Buballa, Shovkovy & Rischke, work in progress]



$\mu_{\nu_e} = 200$ MeV



$\mu_{\nu_e} = 400$ MeV

Note: gapless phases play little role already at $G_D = \frac{3}{4} G_S$

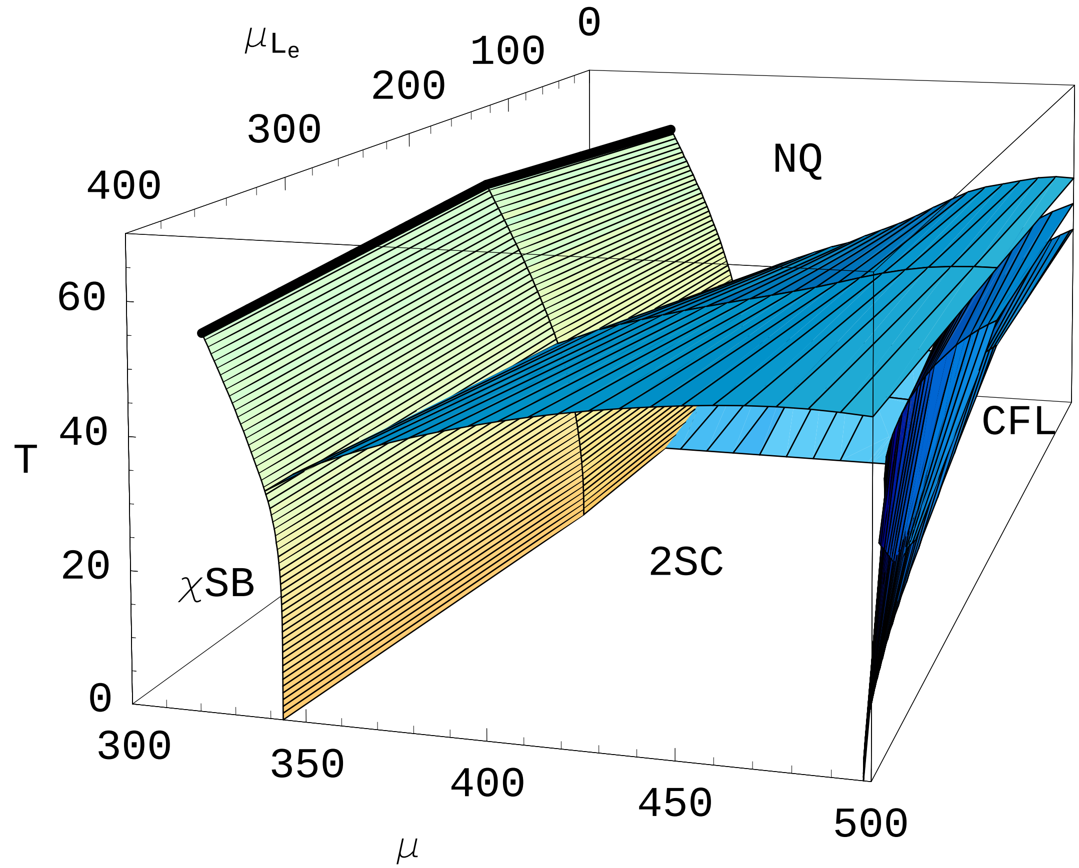
3D phase diagram

Neutrino trapping

$\mu_{L_e} > 0$

inside hot matter

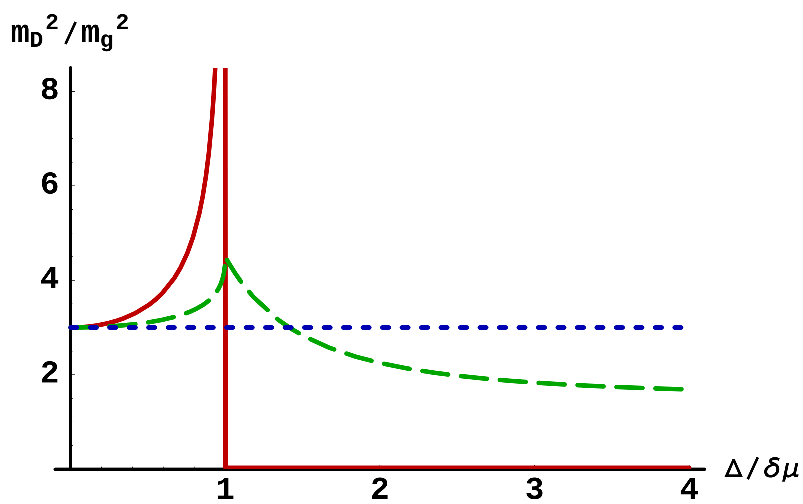
$T \lesssim 40$ MeV

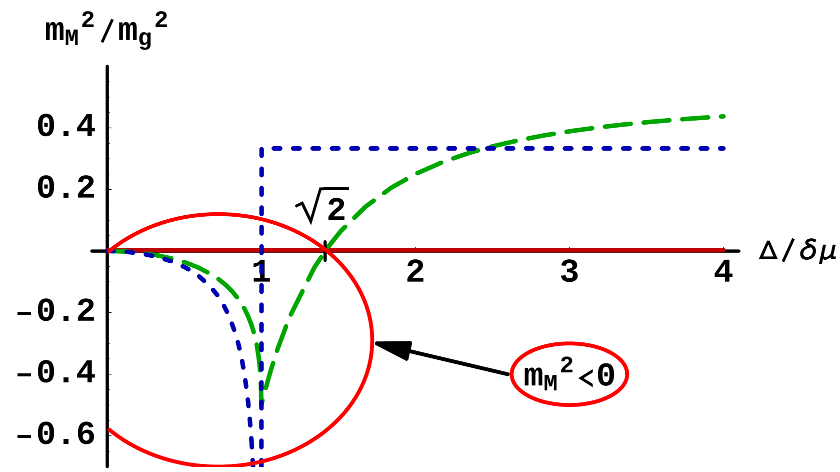


[Rüster, Werth, Buballa, Shovkovy & Rischke, in preparation]

The problem of instabilities

Chromomagnetic instability in the g2SC phase
[Huang & Shovkovy, hep-ph/0407049; hep-ph/0408268]:





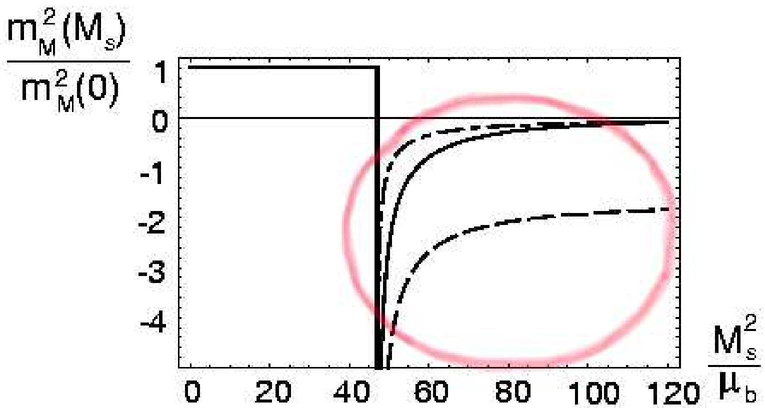
$A = 1, 2, 3$ — red solid line

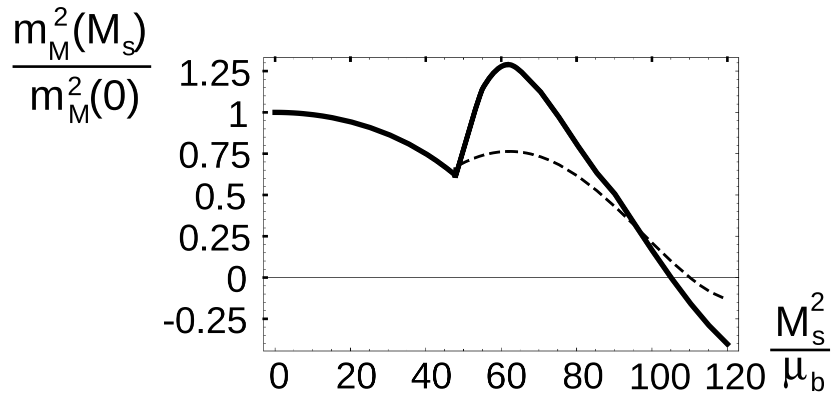
$A = 4, 5, 6, 7$ — green long-dash line

$A = \tilde{8}$ — blue short-dash line

Chromomagnetic instability in gCFL phase

Similar results for Meissner screening masses
[Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401]:





$A = 1, 2$ — solid line

$A = 3$ — short-dashed line

$A = 8$ — long-dashed line

$A = 4, 5$ — dashed line

$A = 6, 7$ — solid line

See also [Wu & Yip, cond-mat/0303185], [Alford & Wang, hep-ph/0501078]

State of the art

(i) g2SC $\to$ mixed phase

[Reddy & Rupak, nucl-th/0405054]

(ii) g2SC/gCFL $\to$ crystalline (LOFF) phase

[Alford, et al. hep-ph/0008208], [Giannakis & Ren, hep-th/0504053]

(iii) g2SC/gCFL $\to$ g2SC/gCFL $\oplus$ secondary pairing

[D.K. Hong, hep-ph/0506097]

(iv) 2SC ($\delta\mu < \delta < \sqrt{2}\delta\mu$) $\to$ "gluonic" phase with broken rotational SO(3) and $U(1)_{em}$ [Gorbar, et al. hep-ph/0507303]

Conclusions

- Neutrality and β-equilibrium strongly affect the properties of dense quark matter
- Phase diagram of neutral dense matter has a very rich structure
- Some features of the QCD phase diagram at $\mu \gtrsim \Lambda_{QCD}$ start to develop
- There is a fundamental problem in current understanding of gapless phases and their instabilities
- Several promising alternatives to gapless phases do exist (e.g., LOFF and mixed phases)

Outlook

- One needs to clarify the precise nature of instabilities
- The "price" of imposing neutrality in the LOFF phase should be studied in detail [Giannakis, hep-ph/0507306]
- The possibility of mixed phases should be subjected to close scrutiny (e.g., along the lines of [Maruyama, et al. nucl-th/0503027])
- The possibility of spontaneously induced currents in gapless phases should be studied (e.g., along the lines of [Huang, hep-ph/0504235])
- One should look into other possible ways of stabilizing phases with unconventional Cooper pairing (e.g., gauge field condensates, or meson condensates, etc.) [Gorbar, et al. hep-ph/0507303]

Collaborator(s)

- Stefan Rüster
- Dirk Rischke
- Verena Werth
- Michael Buballa

References

- S. B. Rüster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, `hep-ph/0503184`, Phys. Rev. D **72** (2005) 034004
- I. A. Shovkovy, S. B. Rüster, D. H. Rischke, J. Phys. G**31** (2005) S725-S732, `nucl-th/0411040`
- S. B. Rüster, I. A. Shovkovy, D. H. Rischke, Nucl. Phys. A **743** (2004) 127, `hep-ph/0405170`