A Crust with Nuggets

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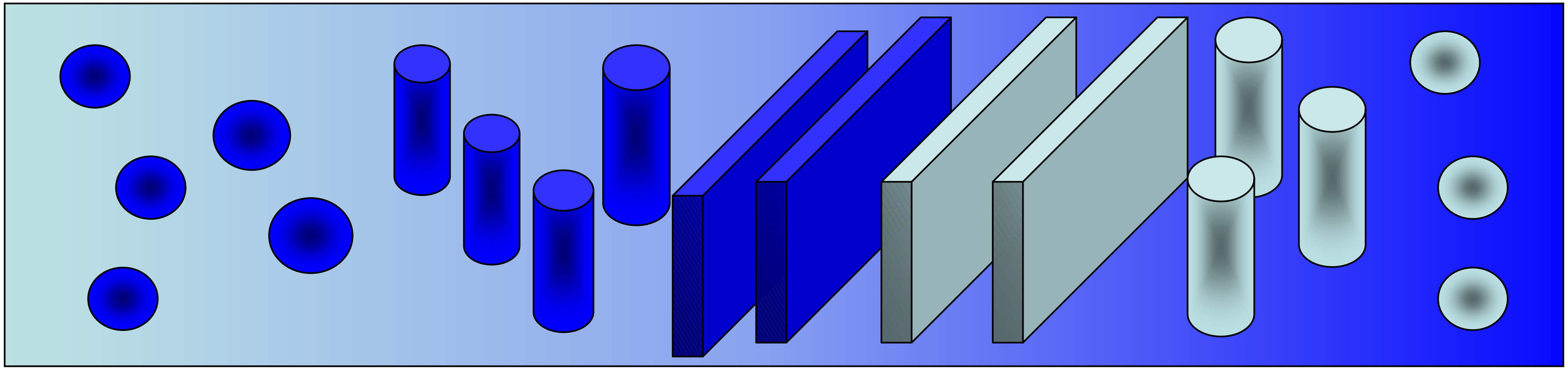

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Jaikumar, Reddy & Stiener, **nucl-th/0507055**

Is dense matter homogeneous ?

Not always: Matter at sub-nuclear density is heterogeneous - charged nuclei are embedded in negatively charge background of electrons.

- In general strong interactions and Fermi degeneracy energy favor an electrically charged state.
- Debye screening will ensure that charged regions have finite spatial extent (set by the λ_{Debye})
- Surface to Volume ratio of the charge neutral unit-cell is finite - consequently a large surface tension can disfavor the heterogeneous state.

Frustration in Dense matter:Enforcing neutrality costs energy

Two Conserved Charges: Baryon number and electric charge

Baryon number fixed by μ and μ_e ensures total charge is zero

It is useful to analyze of pressure (=-free energy) changes with μ_e for fixed μ:

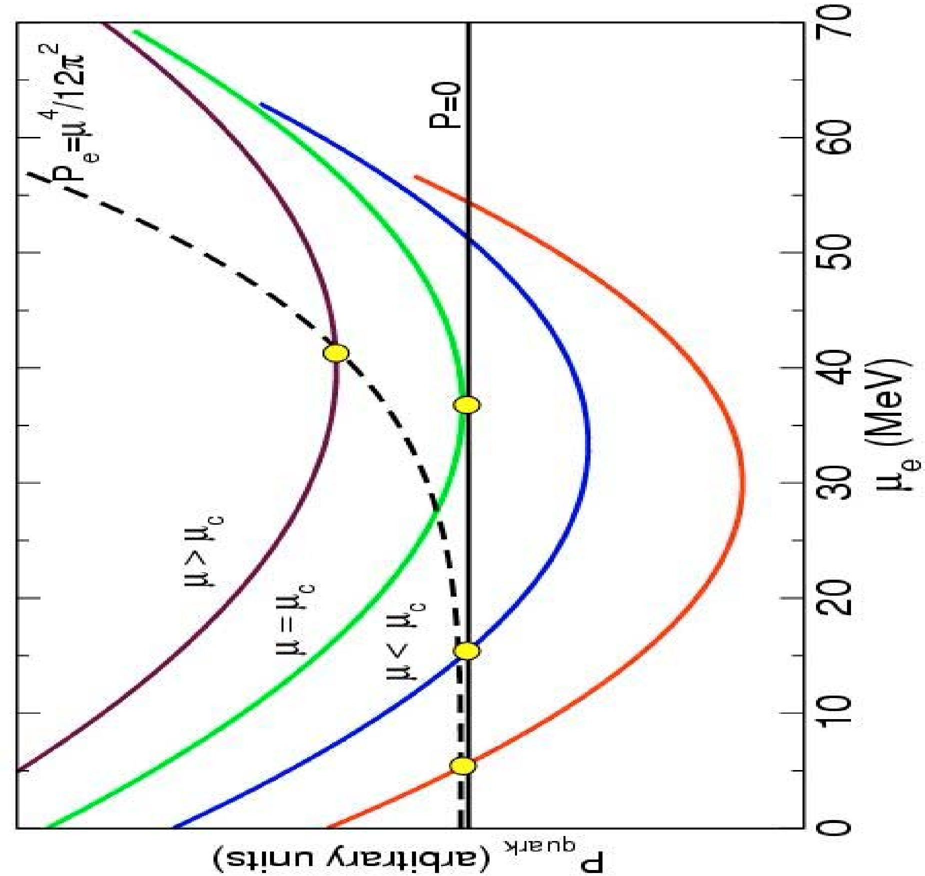
$$P(\mu,\mu_e) = P_0(\mu) - n_Q\ \mu_e + \frac{1}{2}\chi_Q\ \mu_e^2 + K$$

$$\rho_Q = -\frac{\partial P}{\partial \mu_e} = n_Q - \chi_Q\ \mu_e + K$$

Locally neutral state has the lowest pressure !

Quark Matter at Low Pressure: Nuggets & Voids ?

For small surface tension, a mixed phase of quark matter embedded in an electron background is possible and is favored near P_{total}=0.



Jaikumar, Reddy & Stiener, **nucl-th/0507055**

Gibbs Equilibrium: Mixed Phase

Ravenhall, Pethick & Wilson, Phys. Rev. Lett. 50, 2066 (1983) (nuclei -> nuclear matter)
Glendenning, Phys. Rev. D46, 1274 (1992), (nuclear -> quark)
Alford, Rajagopal, Reddy & Wilczek, Phys. Rev. D64, 074017 (2001) (nuclear ->CFL)

$$P_{\text{high}}(\mu,\mu_e) = P_{\text{low}}(\mu,\mu_e)$$

$$\frac{\partial P_{\text{high}}}{\partial \mu_e} \times \frac{\partial P_{\text{low}}}{\partial \mu_e} \leq 0$$

(oppositely charged phases)

$$\frac{\partial P_{\text{high}}}{\partial \mu_e} x + \frac{\partial P_{\text{low}}}{\partial \mu_e} (1-x) = 0$$

$$\varepsilon_{\text{mix}} = x\,\varepsilon_{\text{high}} + (1-x)\,\varepsilon_{\text{low}}$$

$x \equiv$ Volume Fraction of Dense Phase

Quark Matter - Electron Gas Mixed Phase

Assume quark matter pressure is of the form:

$$P_q(\mu, \mu_e) = P_0(\mu) - n_Q(\mu)\ \mu_e + \frac{1}{2}\ \chi_Q(\mu)\ \mu_e^2 ;$$

In the Bag model: $n_Q = m_s^2 \mu / 2\pi^2$ and $\chi_Q = 2\ \mu^2/\pi^2$

Electrons exist in both phases:
Hence Gibbs Equilibrium needs
$P_{quark}(\mu, \mu_e) = 0$

Quark pressure is zero when:

$$\tilde{\mu}_e = \frac{n_Q}{\chi_Q}\ (1 - \sqrt{1-\xi}) \quad \text{where } \xi = \frac{2 P_0 \chi_Q}{n_Q^2}$$

Energetics of the Mixed Phase

Gain in Gibbs energy per quark:

$$\Delta g = \frac{n_Q^2}{2\chi_Q n} \left(1 - \frac{2\chi_Q \tilde{\mu}_e}{n_Q} + \frac{\chi_Q^2 \tilde{\mu}_e^2}{n_Q^2}\right)$$

In the bag model: Δg~0.5 MeV/quark for m_s=150 MeV

Δg~4 MeV/quark for m_s=250 MeV

Surface and Coulomb energy cost per quark:

$$\epsilon_{s+C} = \frac{6\pi}{n\,(16\pi^2)^{1/3}} \left[(e^2\,\sigma\,\mathrm{d}\,n_Q)^2\, f_d(x)\right]^{1/3}$$

Mixed Phase wins when:

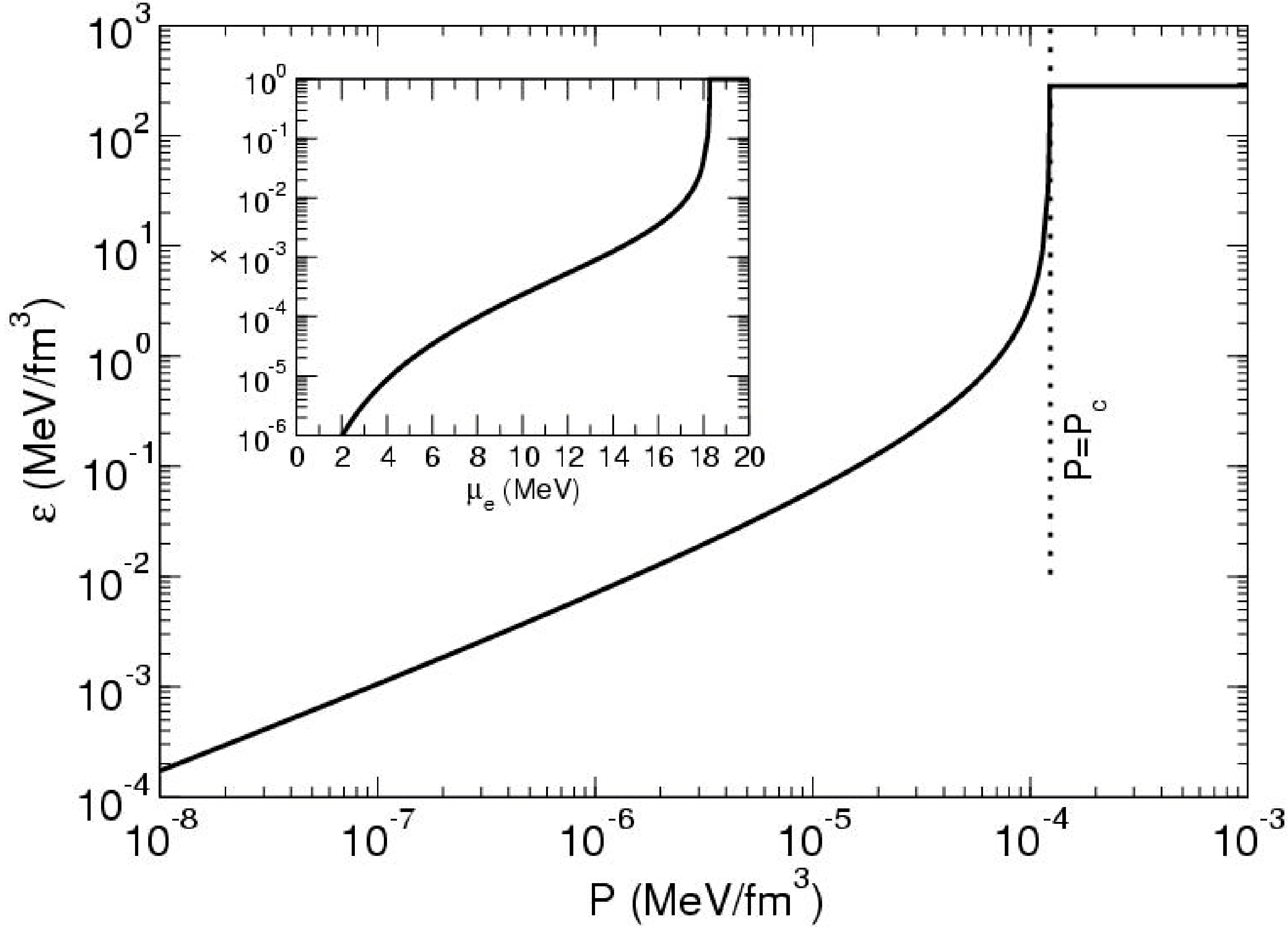
$$\sigma \leq \frac{n_Q^2}{6\sqrt{3\pi\, f_d(x)}\; e^2\; \mathrm{d}\; \chi_Q^{3/2}}$$

In the bag model this implies:

$$\sigma \lesssim 36 \left(\frac{m_s}{150\ \mathrm{MeV}}\right)^3 \frac{m_s}{\mu}\ \mathrm{MeV/fm}^2$$

If Surface Tension is Small then ..

$$P_{mix} = \frac{\tilde{\mu}_e^4}{12\pi^2} \qquad \varepsilon_{mix} = x\,\varepsilon_{quark}$$

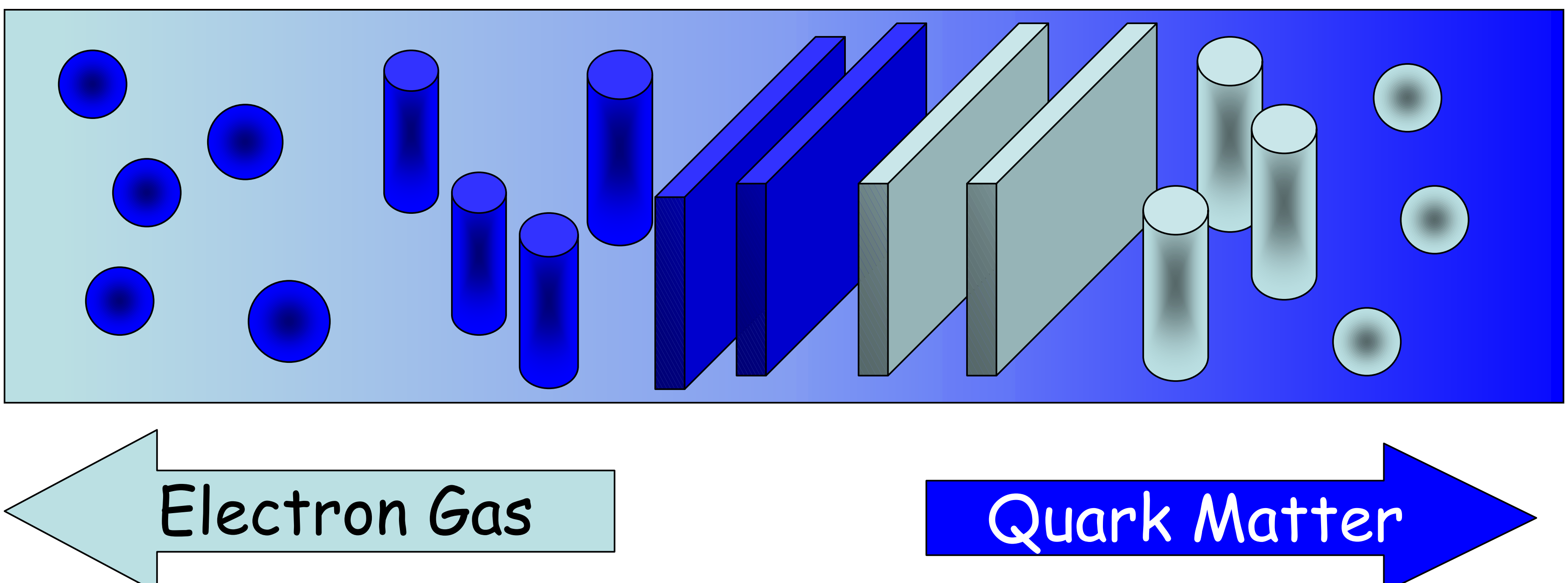


.....Strange Stars have Strange Crusts

Radial Extent
ΔR~50-100 m

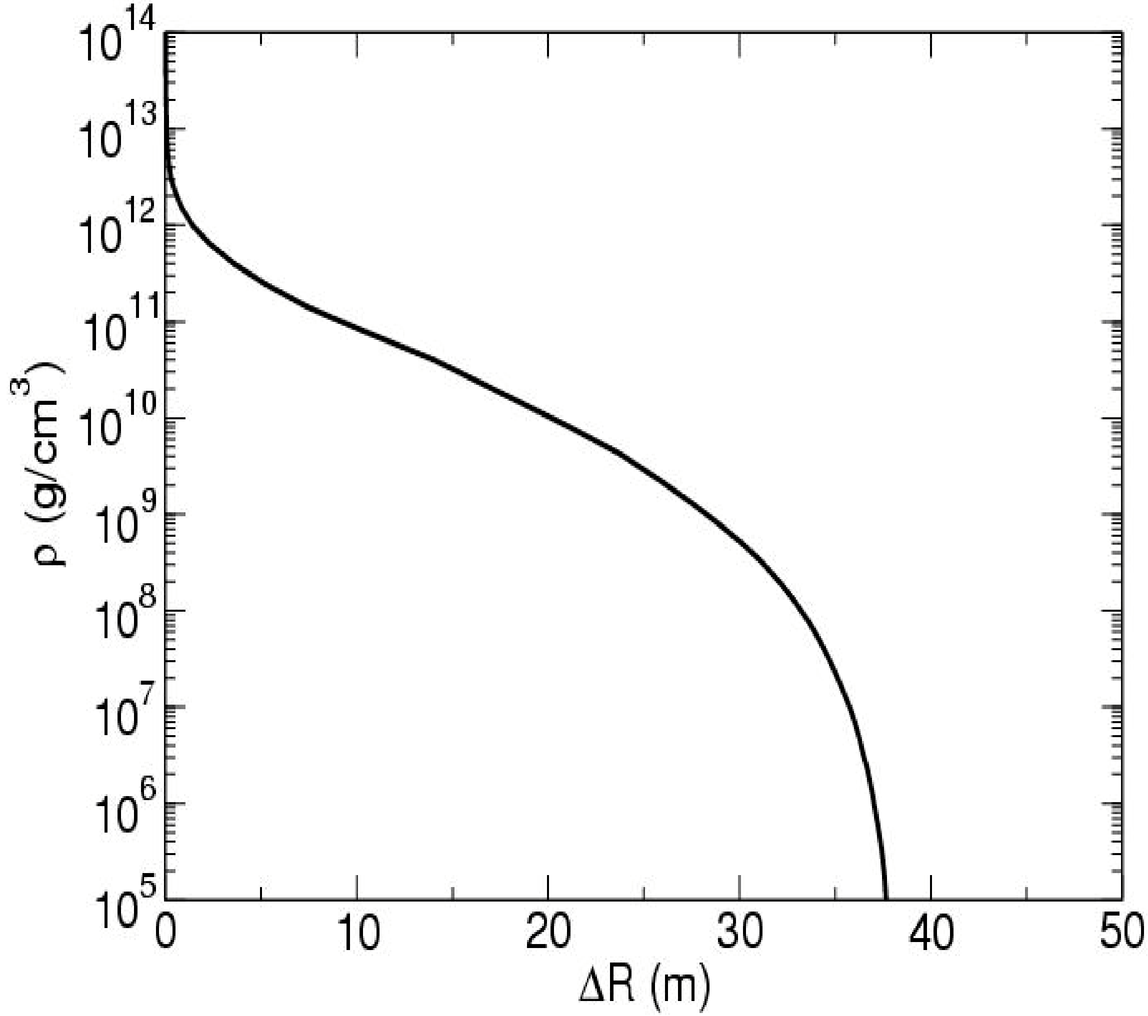
$$\Delta R = \frac{R^2}{GM} \frac{n_Q}{\epsilon_0} \int_0^{\mu_e^c} d\mu_e \left(1 - \frac{\chi_Q \mu_e}{n_Q}\right) = \frac{R}{R_s} \frac{n_Q^2}{\chi_Q \epsilon_0} R\,,$$

Quark "Nuggets"~ 10-15 fm   Electron Voids ~ 50-100 fm



Density Profile at the Surface

- Density gradient is reduced
- Electron chemical potential drops to zero
- Poor thermal conductor (electron-nugget scattering)
- Solid !



Debye Screening

Large droplets ($R \gg \lambda_D$) are neutral inside
- no energy gain

Surface to Volume ratio of small droplets is large
- surface effects are enhanced

Surface + Coulomb ($\varepsilon_c \approx \varepsilon_s/2$) energy per quark for droplets with radius $R = \lambda_D$:

$$\varepsilon_{s+c} \cong \frac{3}{2} \frac{4\pi\sigma\lambda_D^2}{n_q(4\pi\lambda_D^3/3)} = \frac{9\sigma}{2n_q\lambda_D}$$

$$\cong 2\ \mathrm{MeV}\ \left(\frac{\sigma}{5\ \mathrm{MeV}/\mathrm{fm}^2}\right)\left(\frac{10\ \mathrm{fm}}{\lambda_D}\right)\left(\frac{1\ \mathrm{fm}^{-3}}{n_q}\right)$$

Is the mixed phase favored ?

Recall energy gain per quark :

$$\Delta g \approx \frac{n_Q^2}{2\ n\ \chi_Q}$$

$$\cong 0.5 \left(\frac{1\ fm^{-3}}{n} \right) \left(\frac{m_s}{150\ \ MeV} \right)^4 MeV$$

In the bag model, the mixed is favored when:
$m_s \geq 200$ MeV and $\sigma < 5$ MeV/fm^2

Bag Model Calculations of Surface Tension:
(Berger & Jaffe Phys.Rev. C35, 213 (1987))

For m_s=150 MeV $\quad \sigma \approx 8$ MeV/fm^2
For m_s=200 MeV $\quad \sigma \approx 5$ MeV/fm^2

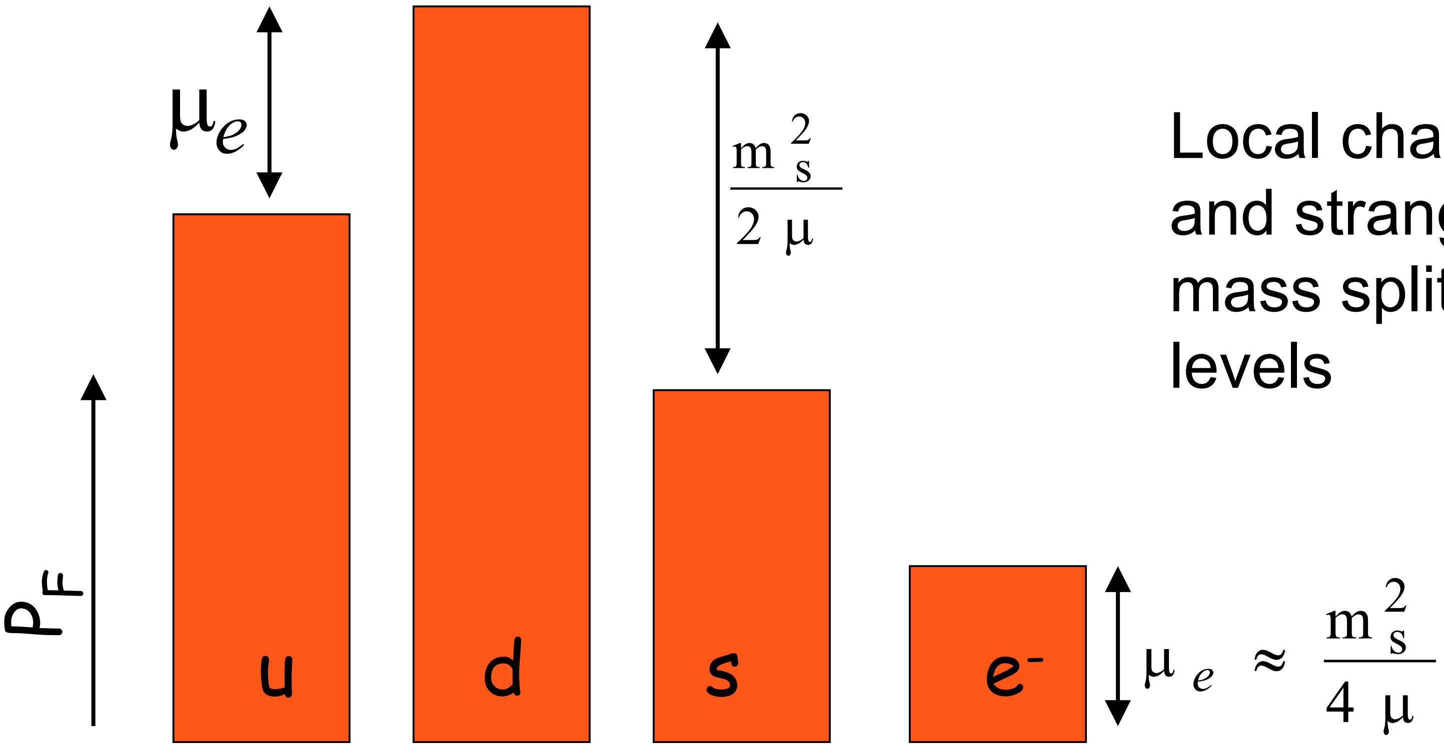
Beyond the Bag Model:

Physics that can change the electric charge density (n_Q) and the charge susceptibility (χ_Q) include:

- Pairing Correlations: Pairing is strongest in the flavor anti-symmetric channel. Pairing energy between up and down quarks $\Delta \sim 100$ MeV. This will act like the symmetry energy in nuclear physics.

- Other strong interaction correlations: Largely unknown. Clearly warrants further work.

Role of Pairing or Color Superconductivity



Local charge neutrality and strange quark mass split the Fermi levels

If pairing energy $\Delta > m_s^2/2\mu$ then the Color-Flavor-Locked (CFL) state is favored. Here $n_u = n_d = n_s$ is enforced by strong interactions. **The consequence is n_Q=0 !**

Rajagopal & Wilczek, PRL 86, 3492 (2001)
Steiner, Reddy & Prakash, PRD 66, 094007 (2002)

Up-Down Pairing (2SC)

When $\Delta \leq m_s^2/4\mu$ pairing does not involve the strange quarks.

However, when we abandon the condition of local neutrality Up-down pairing is inevitable !

This will result in an additional energy gain for the heterogeneous nugget phase (compared to the homogeneous state):

$$\Delta g_{pairing} \approx \Delta \left(\frac{\Delta}{\mu} \right) \approx 3 \left(\frac{\Delta}{30\ MeV} \right)^2 \left(\frac{300\ MeV}{\mu} \right) MeV$$

Pairing will also tend to increase the Debye Screening length. These effects combine to enhance the stability of the nugget phase.

Conclusions

<u>If the heterogeneous “Nugget & Void” phase is favored:</u>

•Strange stars do not have a large electric fields at the surface and are not “bare” - normal radiation and no suspended normal crusts (cannot hide them) !

•Strange stars can potentially mimic surface behavior of normal stars - they can glitch, burst, have an insulating surface layer etc. But, can they mimic this behavior just right ?

<u>Stability of the heterogeneous phase</u>:

•Seems likely if m_s> 200 MeV in the bag model.

•Need to study the role of pairing, Debye screening and curvature energy.

•The phase competition seems to depend sensitively on the (poorly known) details - cannot exclude this possibility.

•This strange crust (with nuggets) needs more baking - when its done not everyone will like it !