Neutron Star Structure and Neutron-Rich Matter

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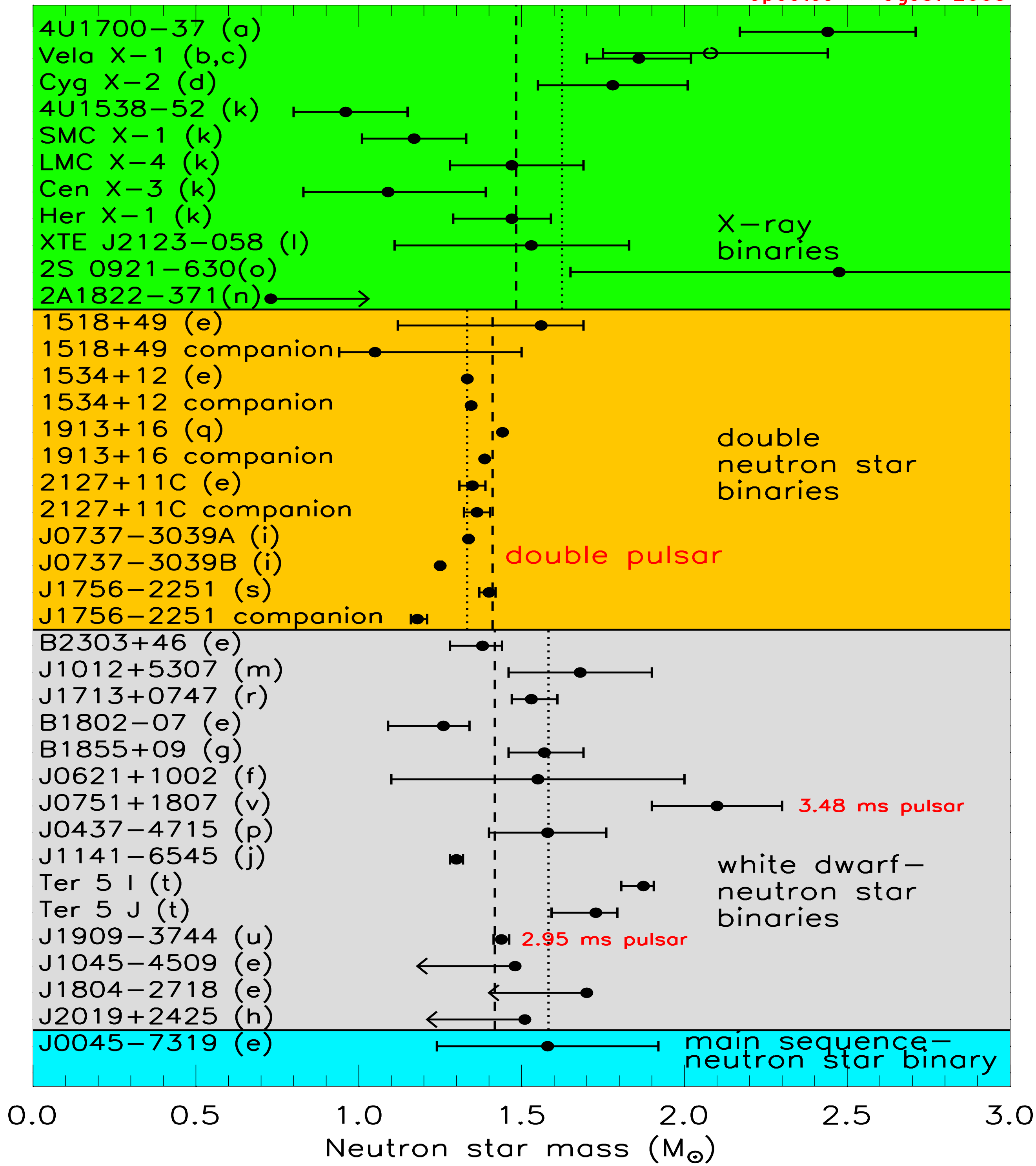
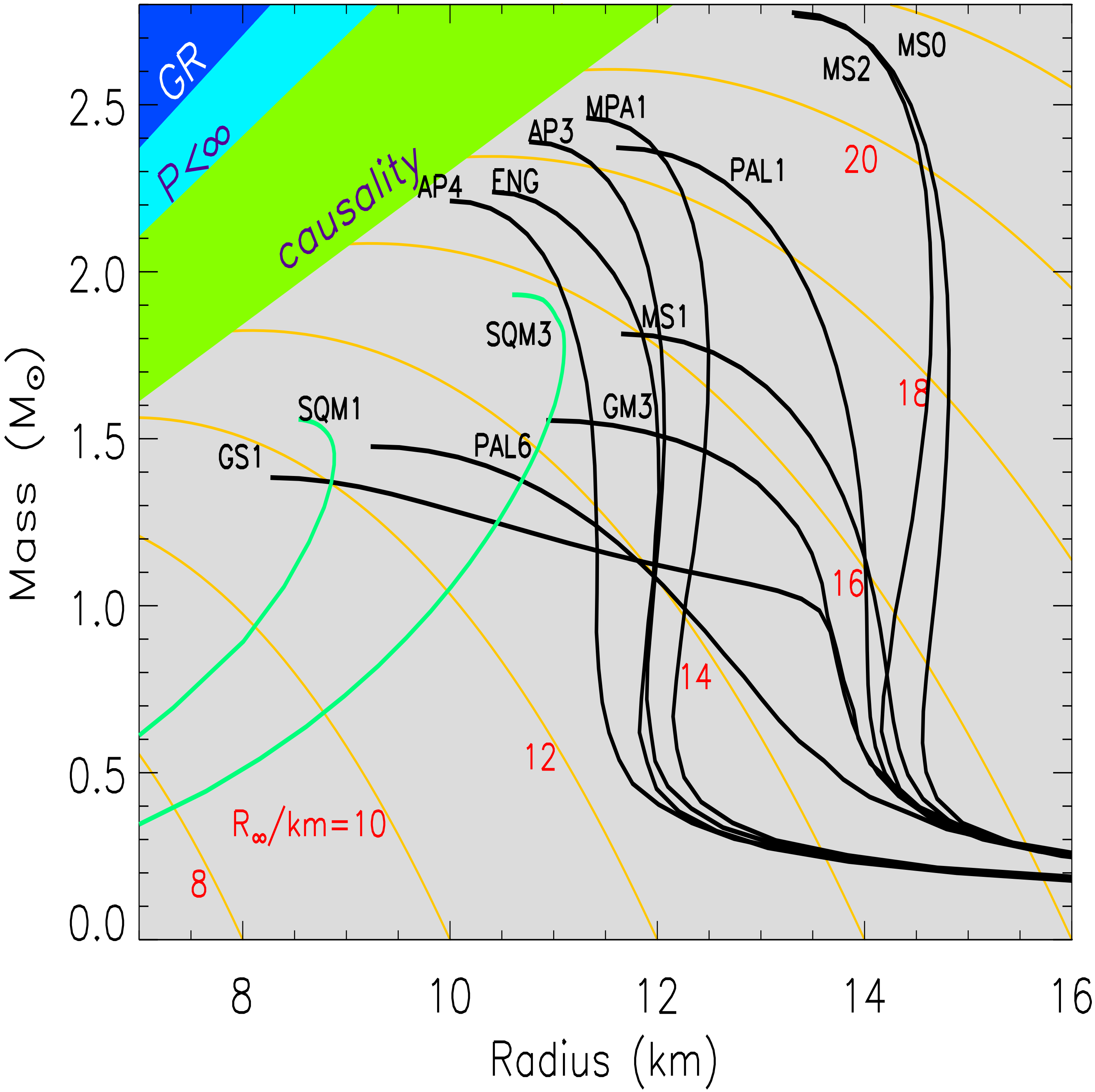

4U1700−37 (a)
Vela X−1 (b,c)
Cyg X−2 (d)
4U1538−52 (k)
SMC X−1 (k)
LMC X−4 (k)
Cen X−3 (k)
Her X−1 (k)
XTE J2123−058 (l)
2S 0921−630(o)
2A1822−371(n)
X−ray binaries
1518+49 (e)
1518+49 companion
1534+12 (e)
1534+12 companion
1913+16 (q)
1913+16 companion
2127+11C (e)
2127+11C companion
J0737−3039A (i)
J0737−3039B (i)
J1756−2251 (s)
J1756−2251 companion
double neutron star binaries
double pulsar
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (g)
J0621+1002 (f)
J0751+1807 (v)
3.48 ms pulsar
J0437−4715 (p)
J1141−6545 (j)
Ter 5 I (t)
Ter 5 J (t)
J1909−3744 (u)
2.95 ms pulsar
J1045−4509 (e)
J1804−2718 (e)
J2019+2425 (h)
white dwarf−neutron star binaries
J0045−7319 (e)
main sequence−neutron star binary
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass (M⊙)


GR
P<∞
causality
MS0
MS2
MPA1
AP3
PAL1
AP4
ENG
20
MS1
SQM3
18
SQM1
GM3
PAL6
GS1
16
14
12
R∞/km=10
8
2.5
2.0
1.5
1.0
0.5
0.0
Mass (M⊙)
8
10
12
14
16
Radius (km)

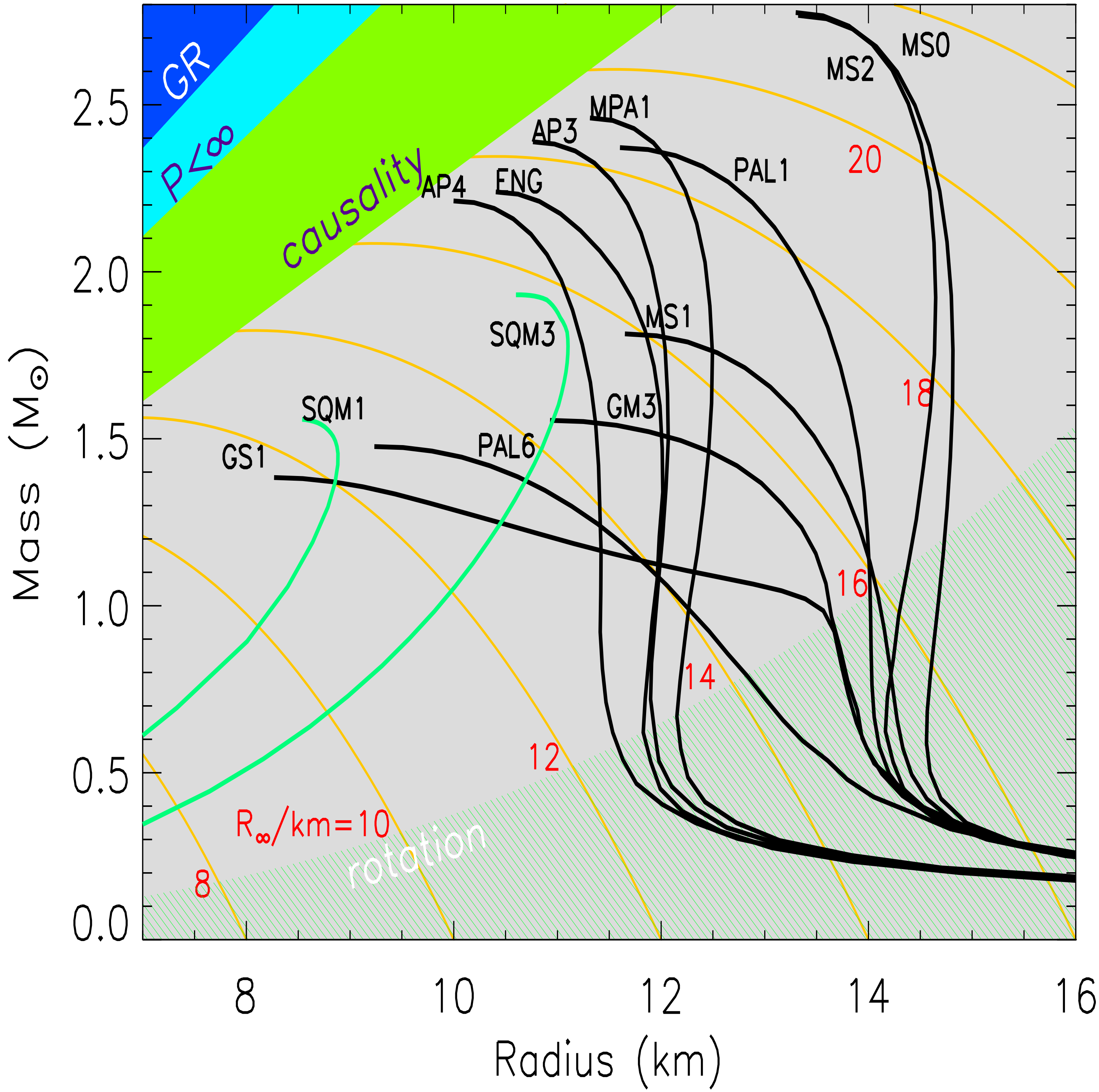
Observable Quantities

- Pulsar Rotation Periods
  - Rotation rate limited by Keplerian velocity (mass-shedding limit):

$$\nu_K \propto \sqrt{\frac{GM}{R^3}}$$

  - Limits R for a given M, independently of EOS (Lattimer & Prakash 2004).

$$\nu_K \simeq \ 1045 \pm 30 \sqrt{\frac{M}{\mathrm{M}_\odot}} \left(\frac{10 \ \mathrm{km}}{R_o}\right)^{3/2} \mathrm{Hz}$$


GR
P<∞
causality
MS0
MS2
MPA1
AP3
PAL1
AP4
ENG
20
MS1
SQM3
18
GM3
SQM1
PAL6
GS1
16
14
12
R∞/km=10
rotation
8
2.5
2.0
1.5
1.0
0.5
0.0
Mass (M☉)
8
10
12
14
16
Radius (km)

Observable Quantities

- ## Pulsar Rotation Periods
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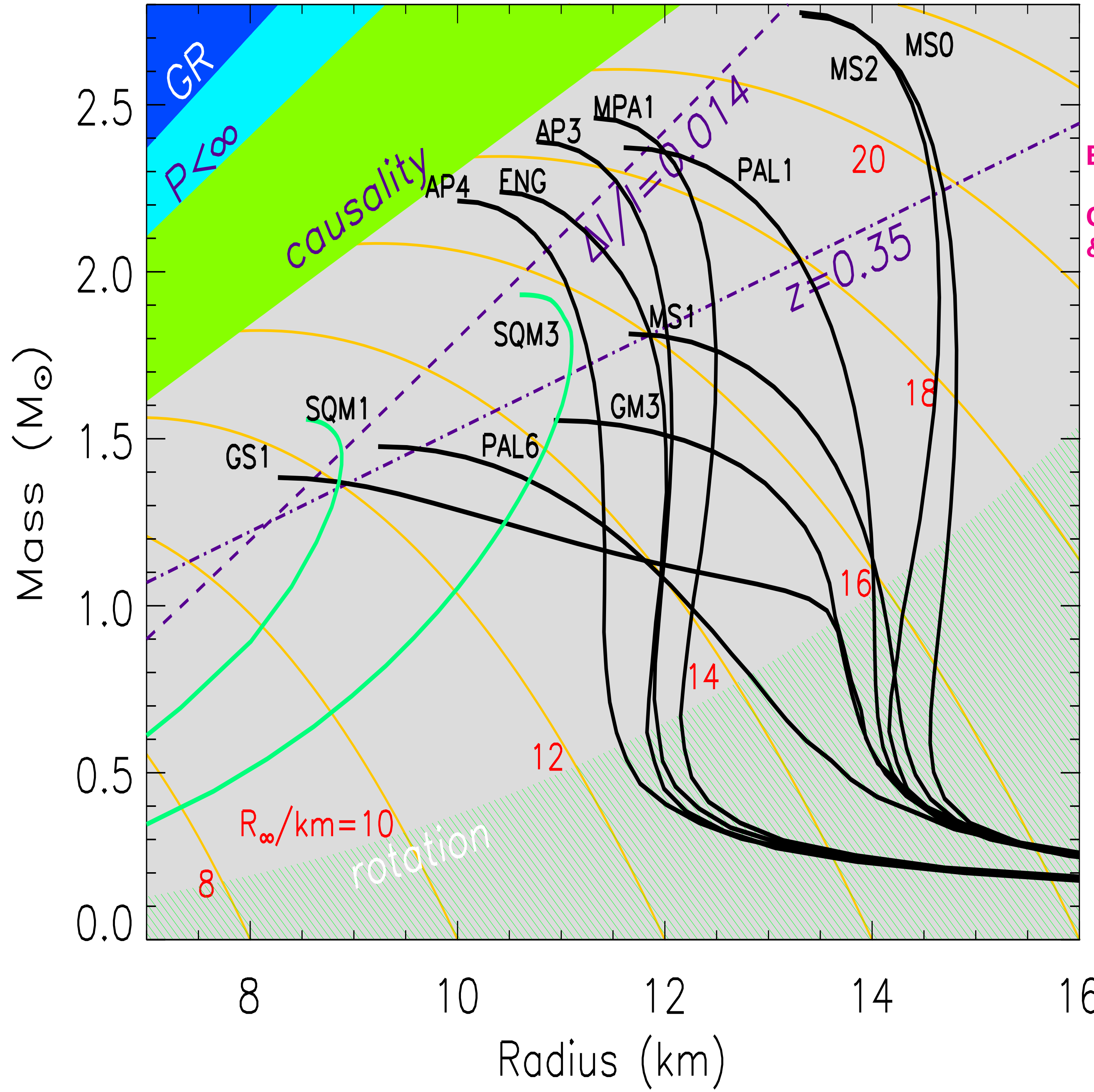
- ## Pulsar Glitches
  - Global transfer of angular momentum.
  - Possible weak coupling between crustal n superfluid and star.
  - Vela implies $I_{crust}/I_{star} > 0.014$

$$\frac{I_{crust}}{I_{star}} \propto \frac{P_t R^4}{GM^2}$$

where P_t is pressure at the core-crust interface (Link, Epstein & Lattimer 1999).

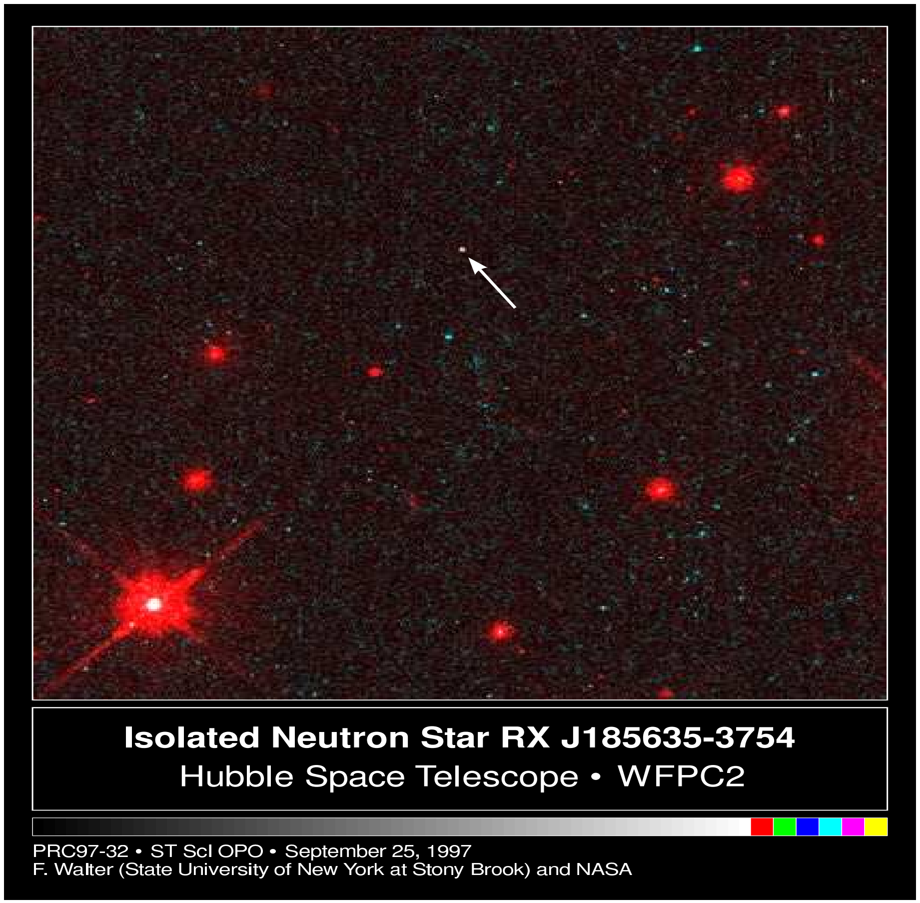
  - Limits R for a given M.

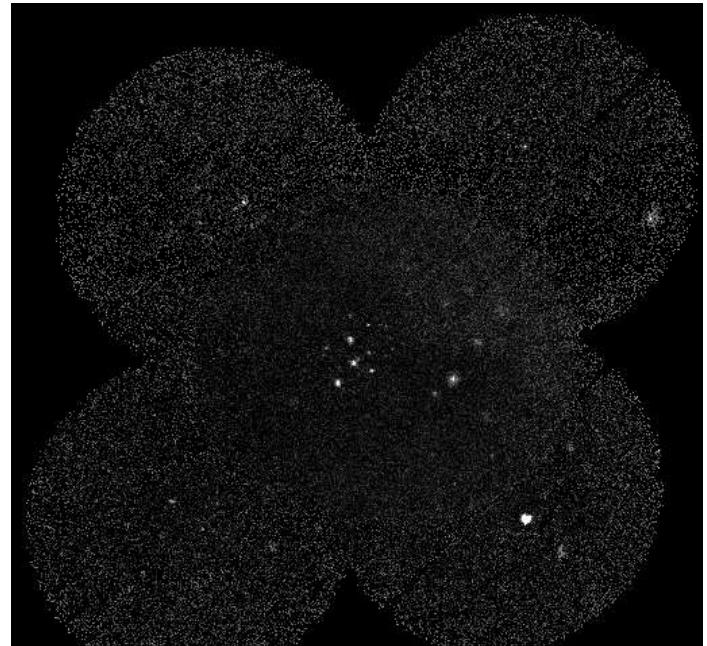


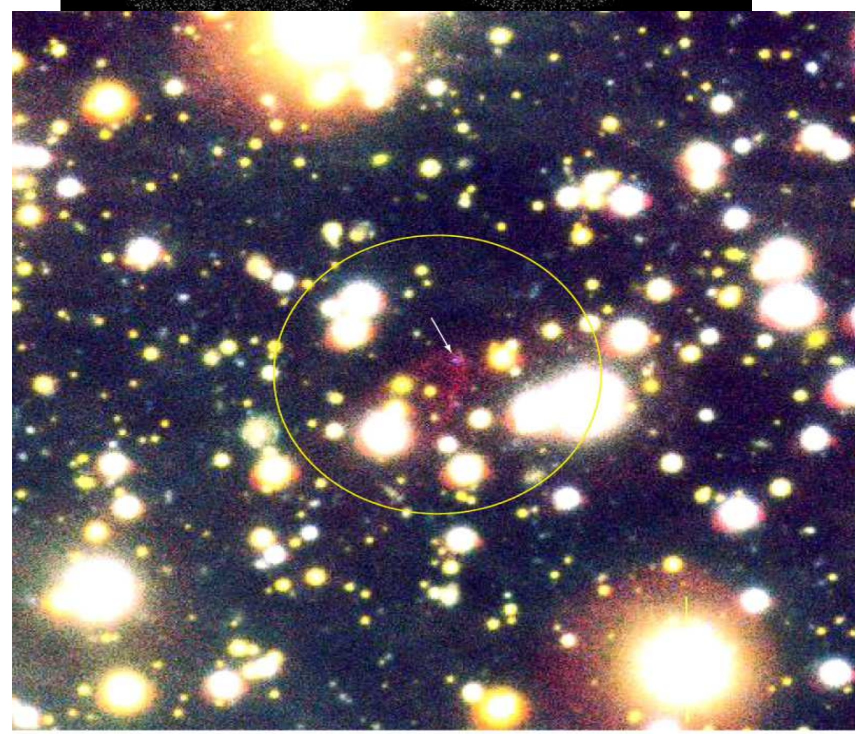
EXO 0748-676:

Cottam, Paerels & Mendez 2002

RX J1856-3754

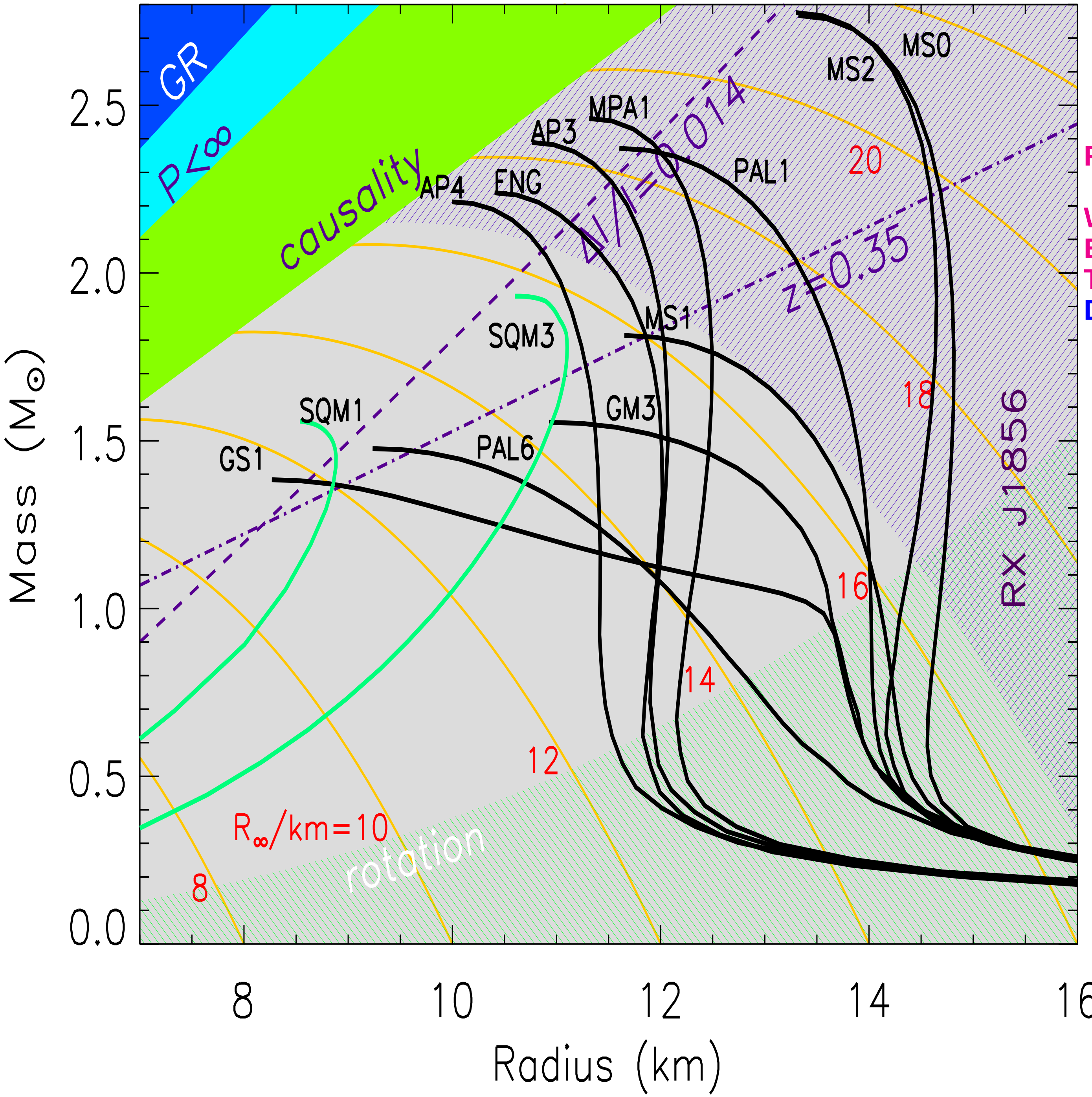






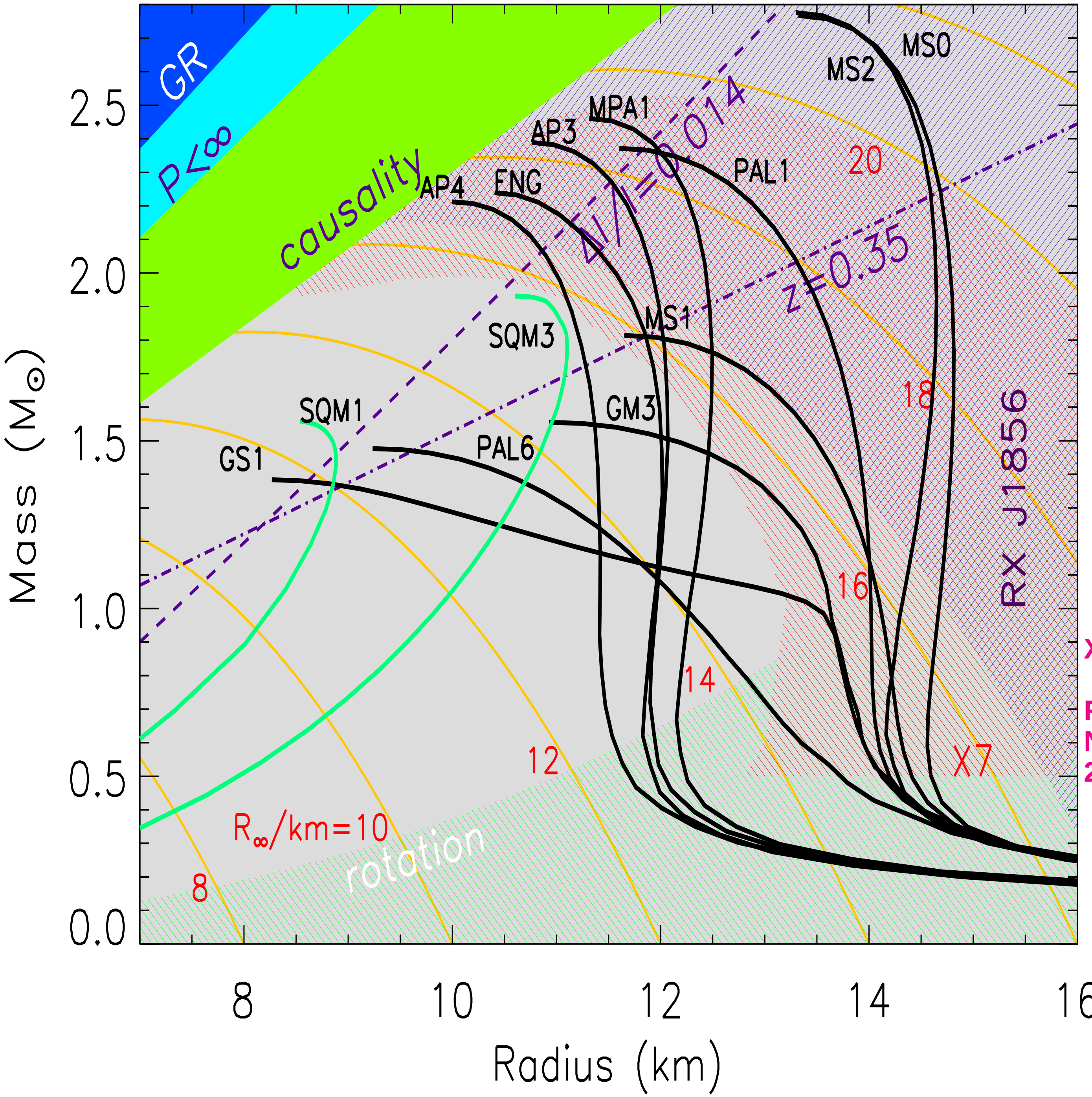
A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

ESO PR Photo 23b/00 (11 September 2000)



RX J1856-3754:

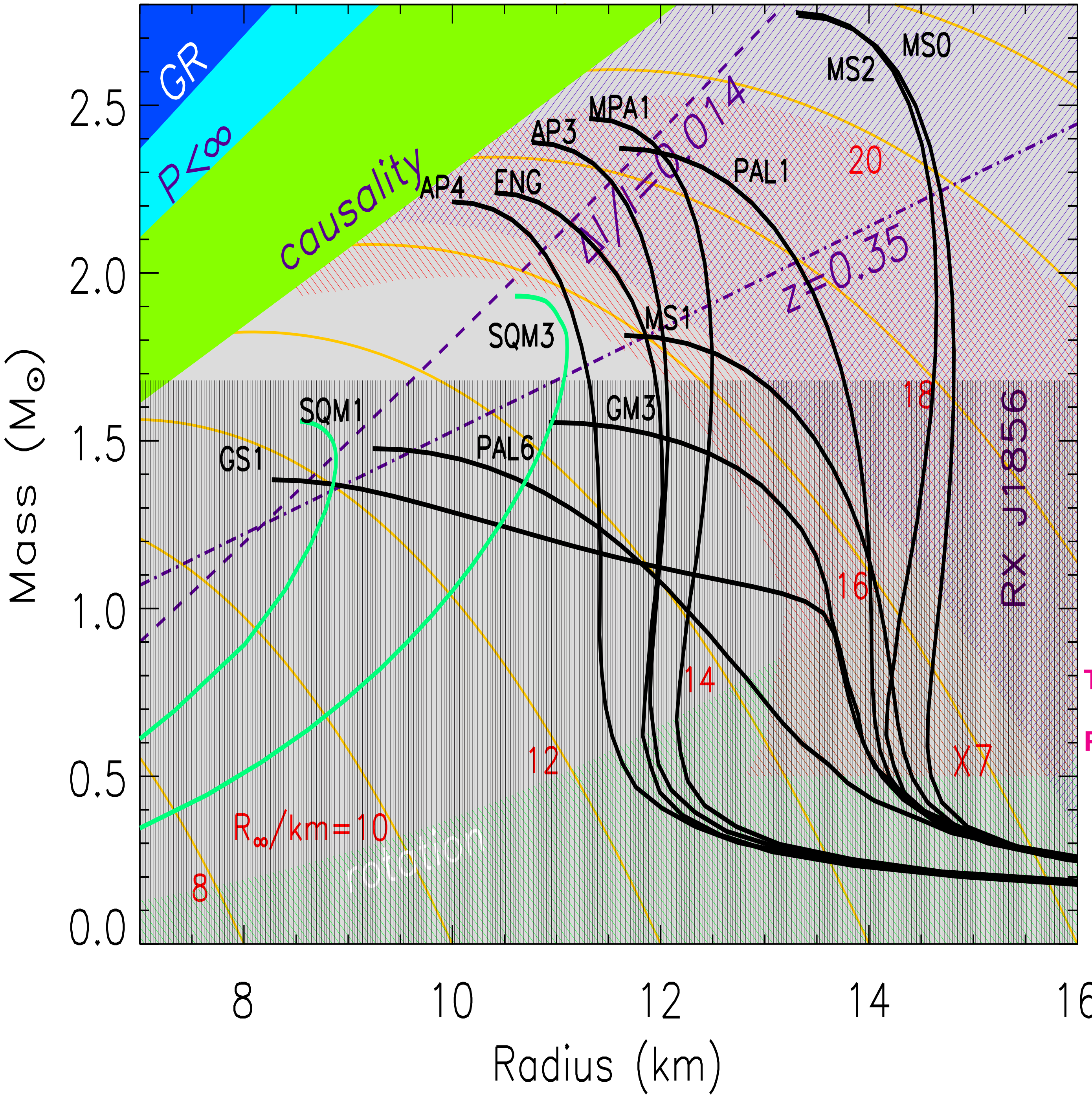
Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=117 pc



X7 in 47 Tucanae:

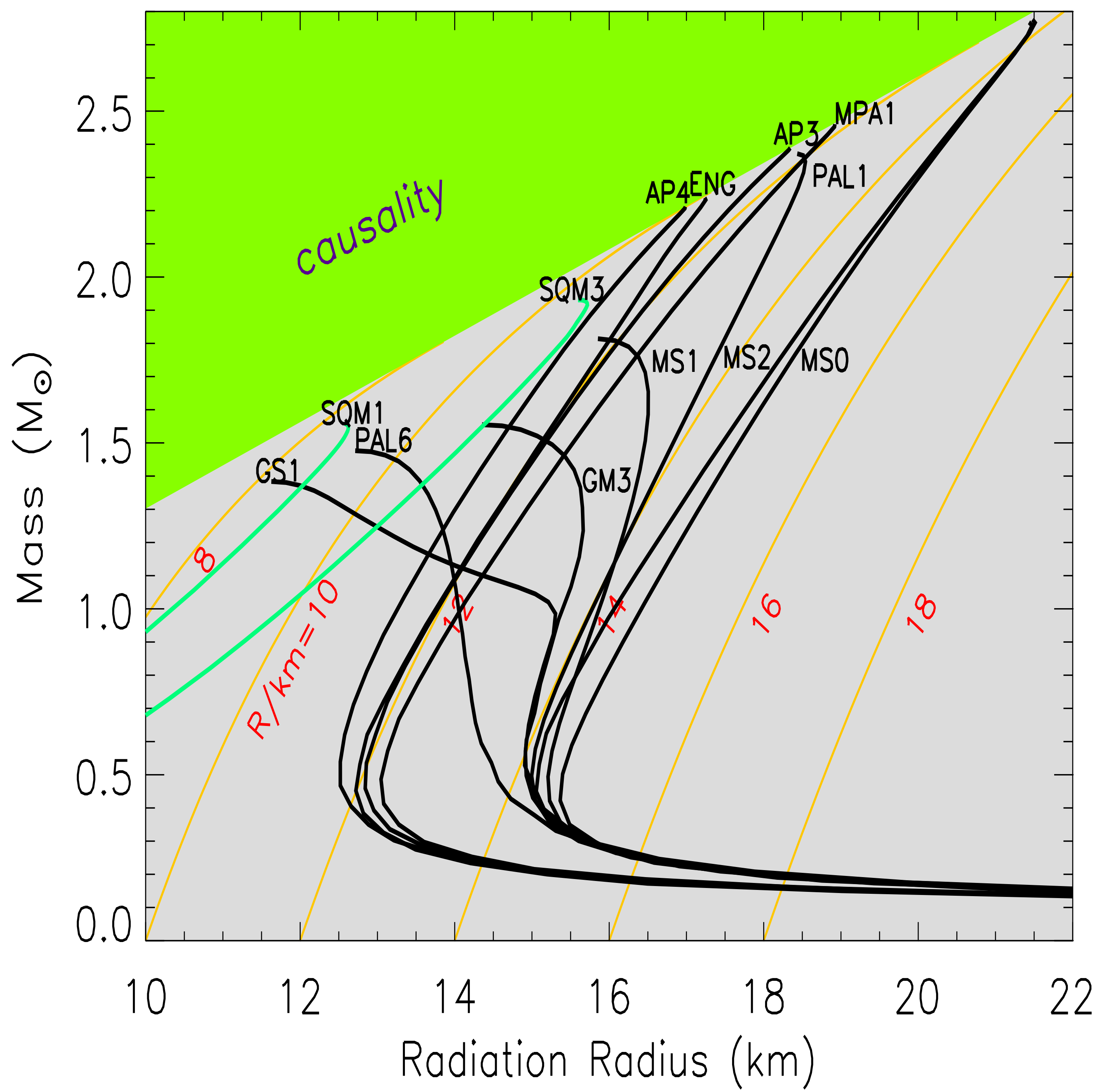
Rybicki, Heinke, Narayan & Grindlay 2005

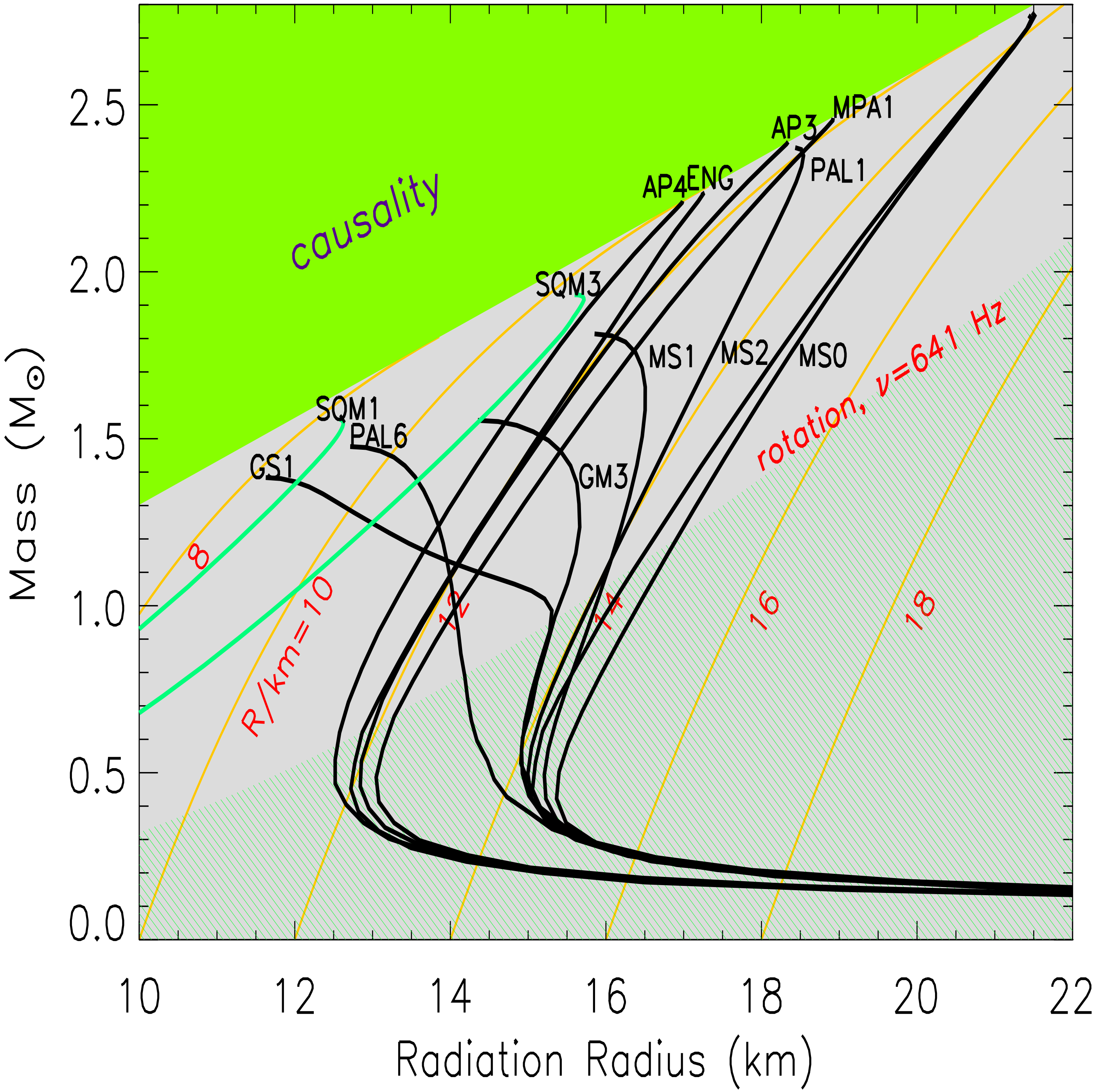
Minimum mass limit from J0751+1807, Ter 5 I & J

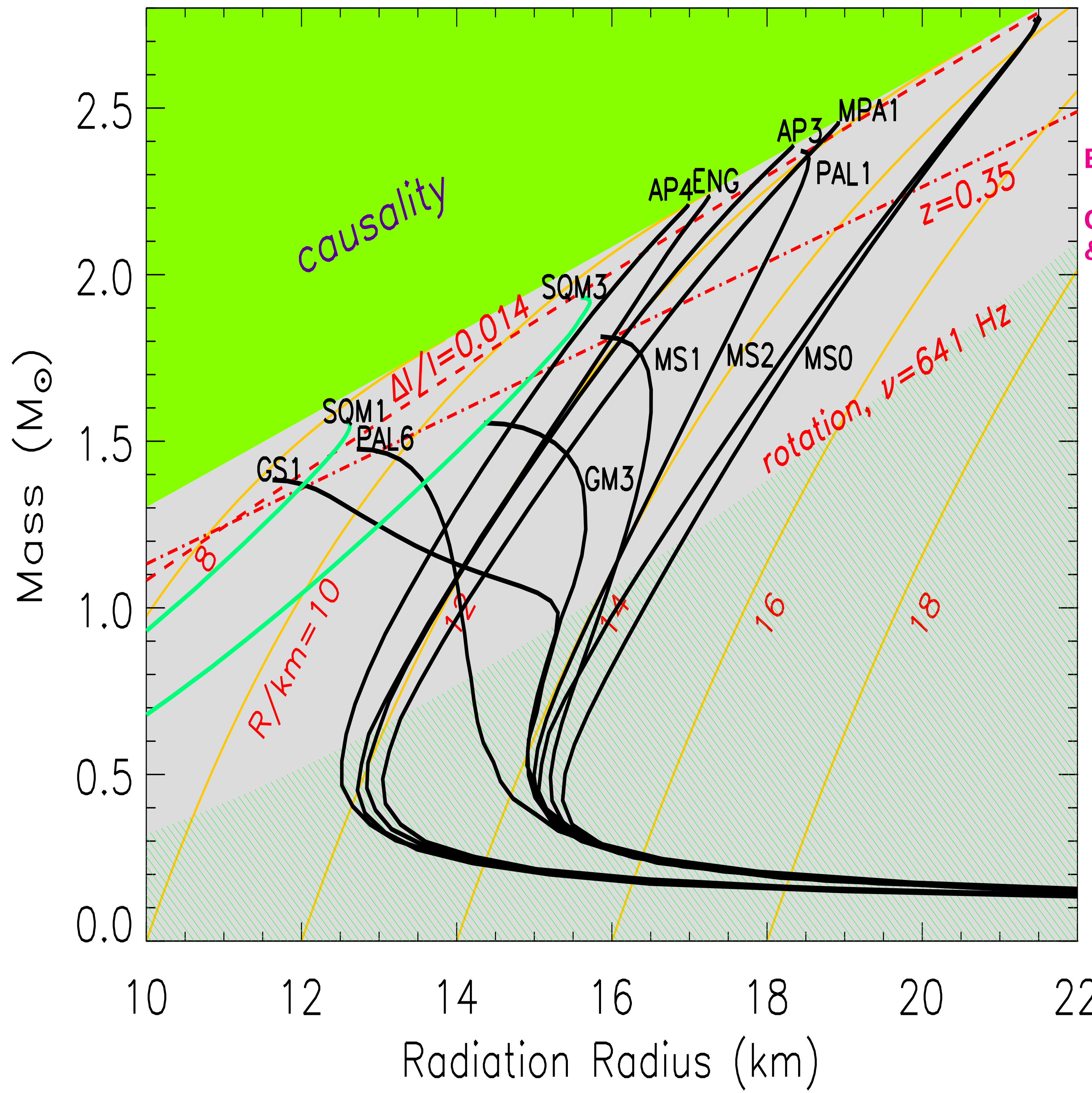


Terzan 5 I & J:

Ransom et al. 2005


causality
Mass (M⊙)
Radiation Radius (km)
2.5
2.0
1.5
1.0
0.5
0.0
10
12
14
16
18
20
22
MPA1
AP3
PAL1
AP4
ENG
SQM3
MS1
MS2
MS0
SQM1
PAL6
GS1
GM3
8
R/km=10
12
14
16
18


causality
SQM3
SQM1
PAL6
GS1
AP4
ENG
AP3
MPA1
PAL1
MS1
MS2
MS0
GM3
rotation, ν=641 Hz
R/km=10
8
12
14
16
18
Mass (M⊙)
Radiation Radius (km)
2.5
2.0
1.5
1.0
0.5
0.0
10
12
14
16
18
20
22



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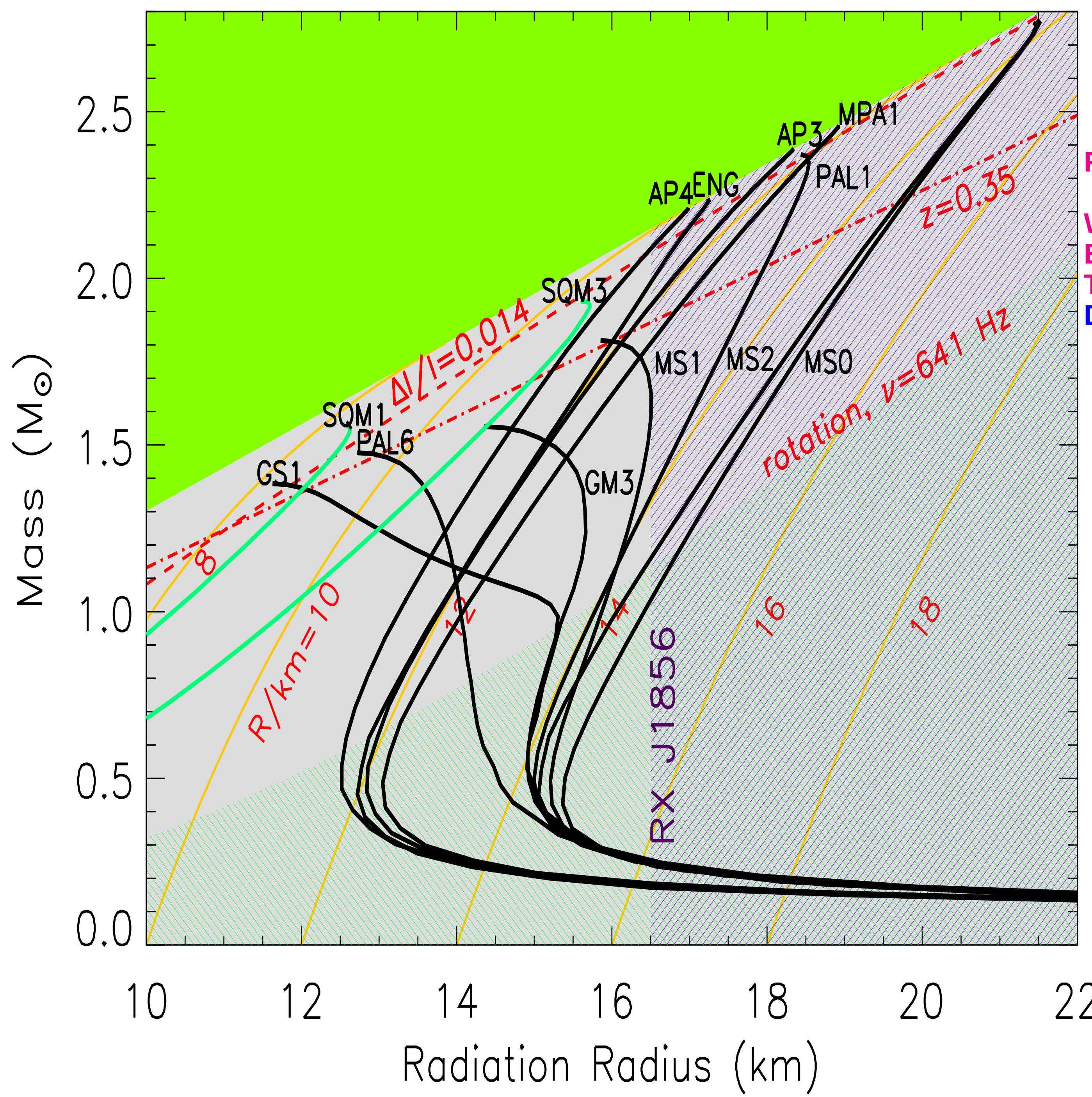

RX J1856-3754:

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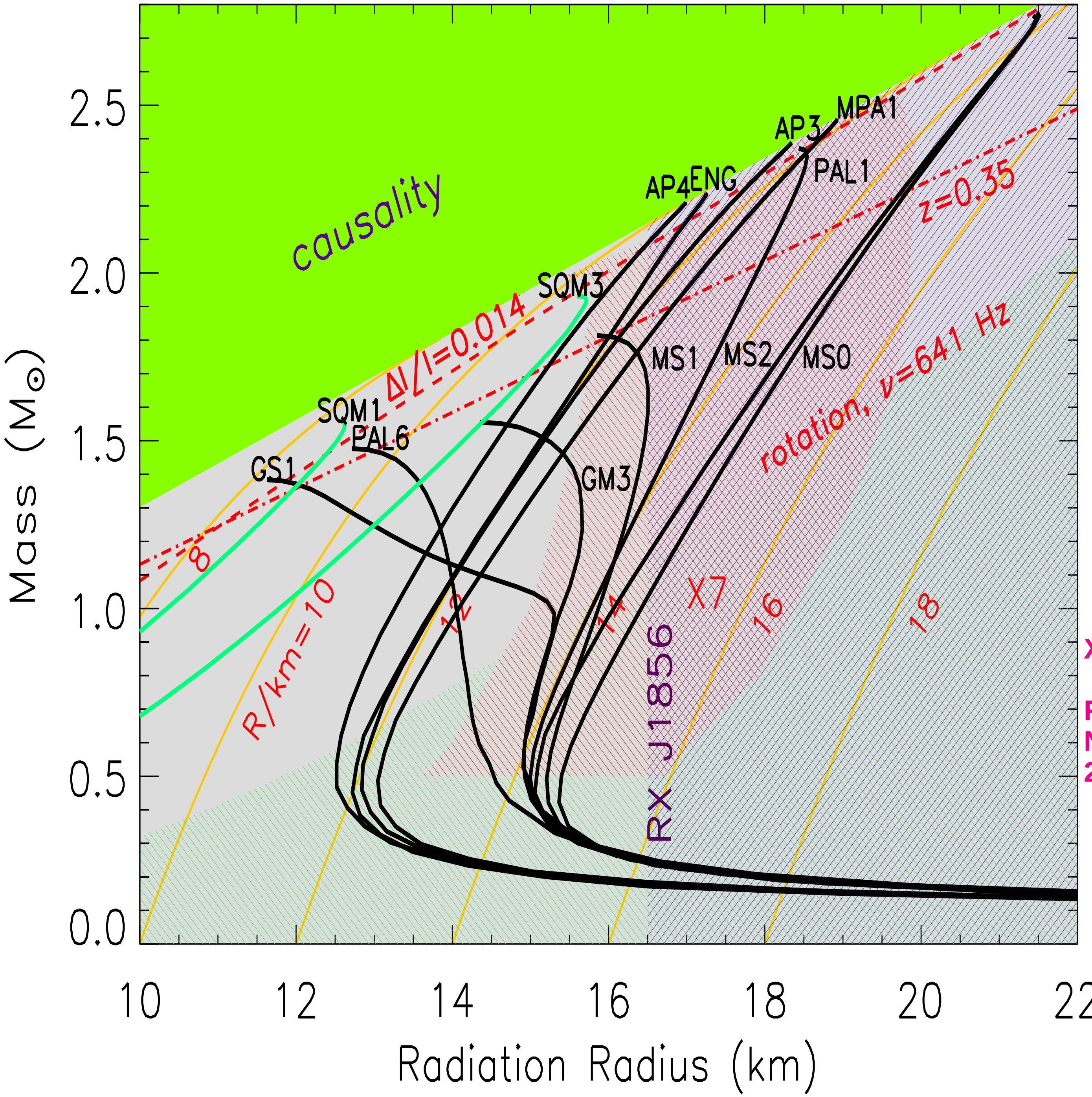

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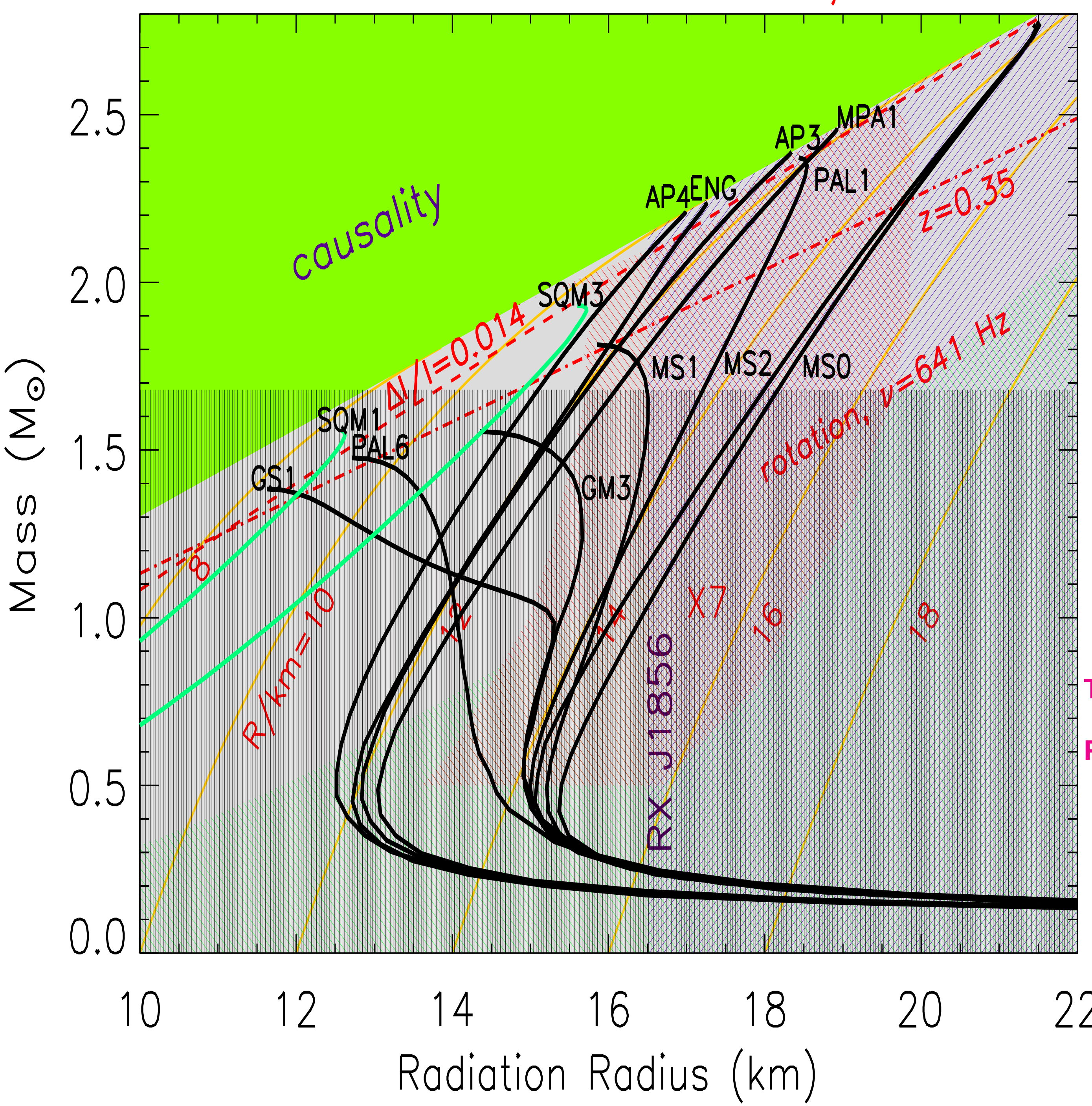


Terzan 5 I & J:

Ransom et al. 2005

Structural Constraints

$n = d \ln P / d \ln \rho - 1 \sim 1$

$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$

$R \propto P_0^{1/2} \rho_0^{-1} M^0$

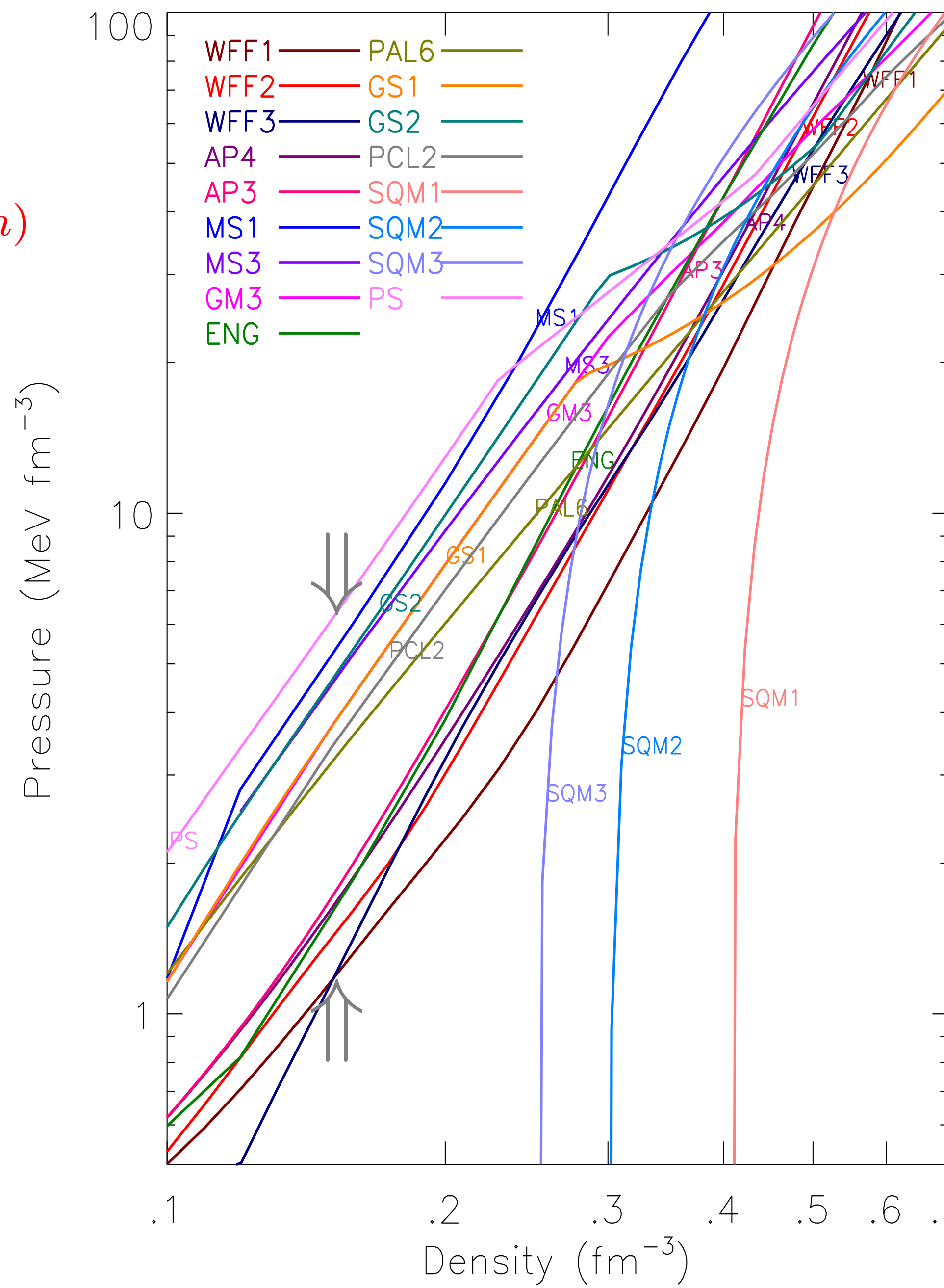
$(1 < \rho_0/\rho_s < 2)$

Wide variation:

$1 < \frac{P(n_s)}{\text{MeVfm}^{-3}} < 6$

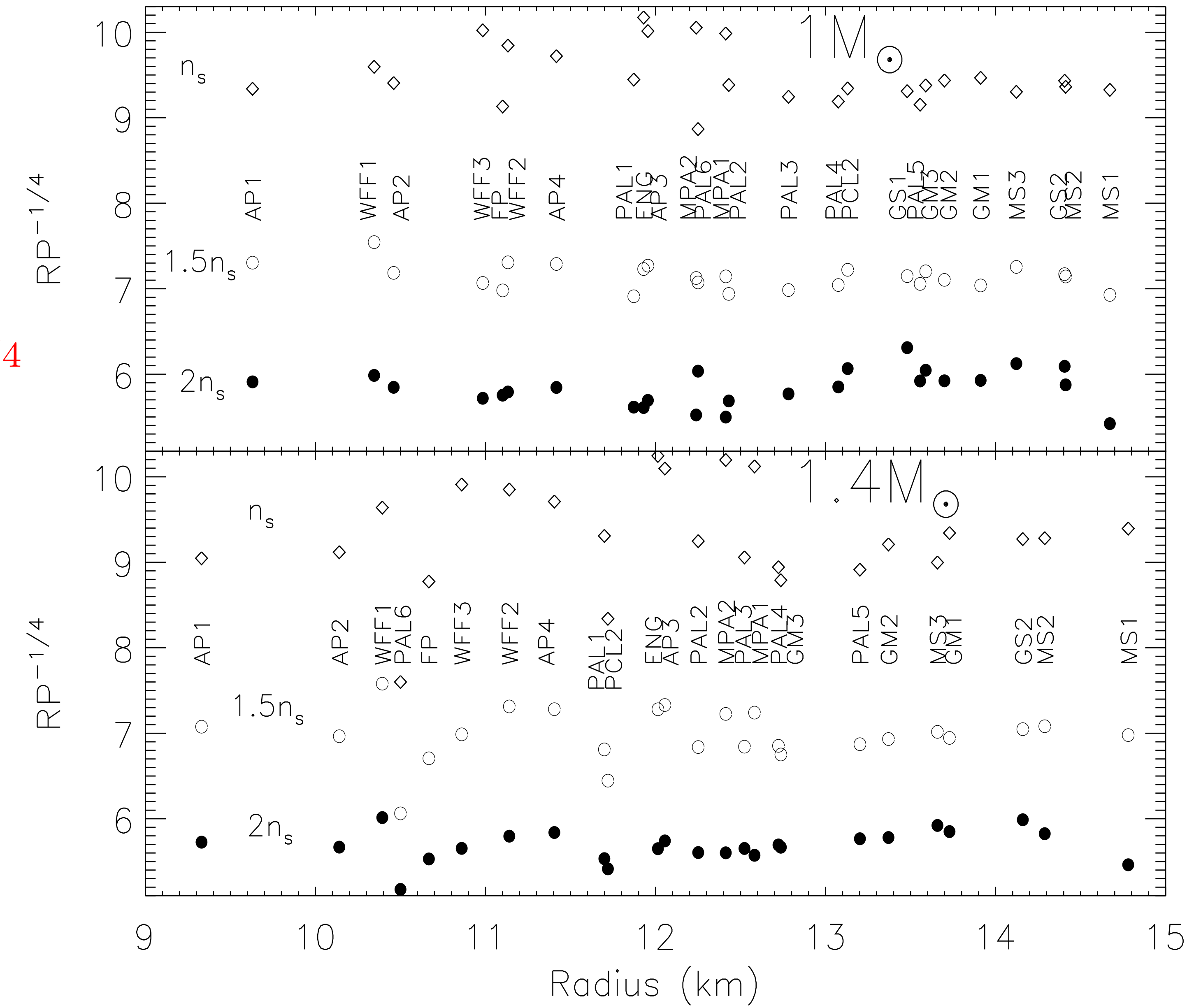
GR phenomenological result (L & Prakash 01)

$R \propto P^{1/4} \rho^{-1/2}$



GR Phenomenology

$R \propto P^{1/4}$



The Pressure of Neutron Star Matter

Expansion of cold nucleonic matter near n_s and isospin symmetry $x = 1.2$:

$$
\begin{aligned}
E(n,x) &\simeq E(n,1/2) + E_{sym}(n)(1-2x)^2 + \frac{3\hbar c}{4}x(3\pi^2 nx)^{1/3}\,, \\
P(n,x) &\simeq n^2\left[\frac{dE(n,1/2)}{dn}\Big|_{n_s} + \frac{dE_{sym}(n)}{dn}\Big|_{n_s}(1-2x)^2\right] + \frac{\hbar c}{4}x(3\pi^2 nx)^{1/3}\,, \\
\mu_e &= \hbar c(3\pi^2 nx)^{1/3} = \mu_n - \mu_p = -\left(\frac{\partial E}{\partial x}\right)_n\,,
\end{aligned}
$$

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$$\mu_e = \hbar c(3\pi^2 nx)^{1/3} = \mu_n - \mu_p = -\left(\frac{\partial E}{\partial x}\right)_n\,,$$

$$P_s = n_s(1-2x_s)\left[n_s\frac{dE_{sym}}{dn}\Big|_{n_s}(1-2x_s) + E_{sym}(n_s)x_s\right]\,,$$

In β equilibrium:

$$x_s \simeq (3\pi^2 n_s)^{-1}\left(\frac{4E_{sym}(n_s)}{\hbar c}\right)^3 \simeq 0.04\,,$$

Thus, $P \propto dE_{sym}/dn$ for $n \sim n_s$.

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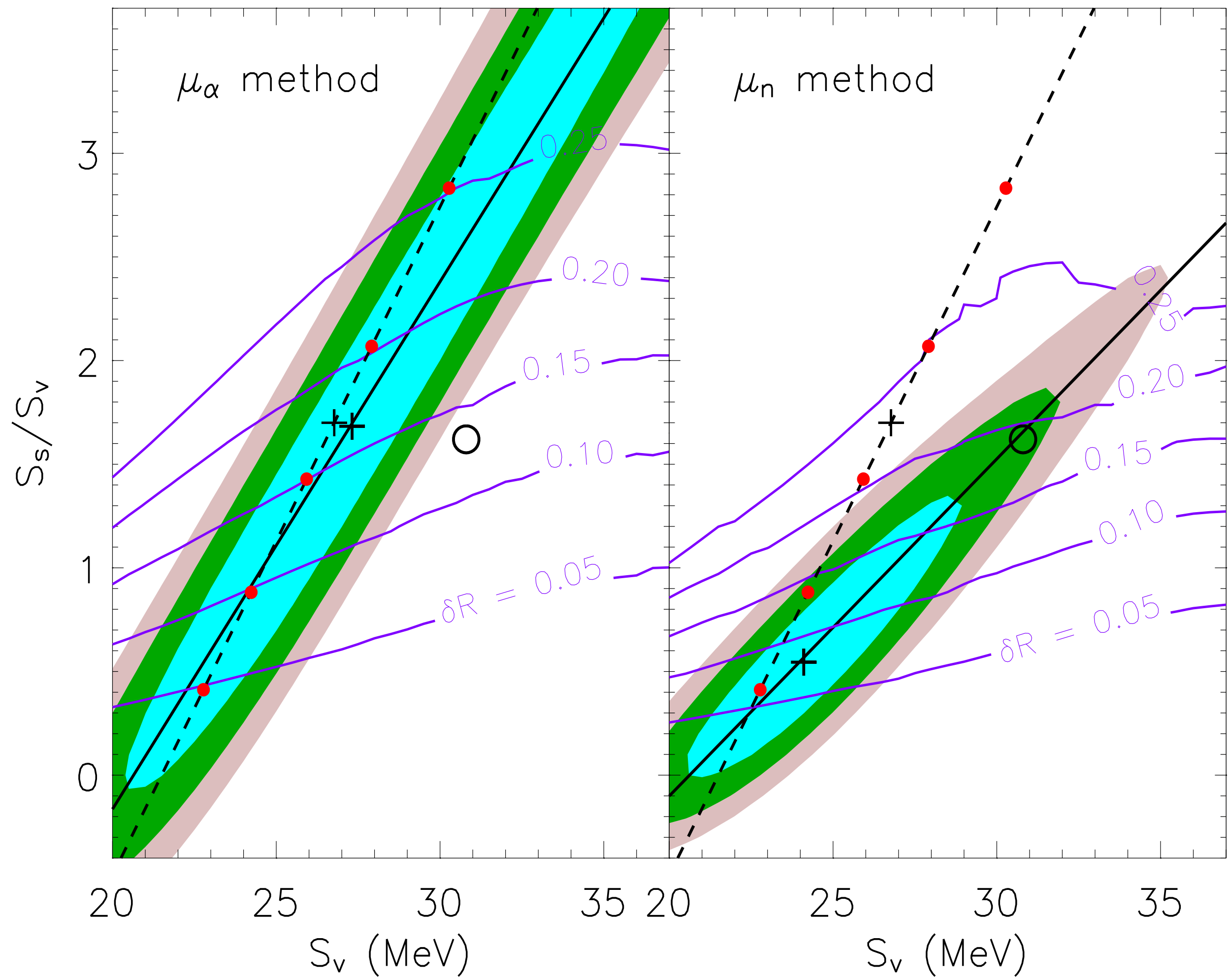
Thus, $P \propto dE_{sym}/dn$ for $n \sim n_s$. Surface and volume symmetry coefficients are highly correlated from nuclear mass measurements.

$$S_v \equiv E_{sym}(n_s)\,, \quad \frac{S_s}{S_v} = \frac{E_{surf}}{2}\frac{\int_0^{n_s}\sqrt{\frac{n}{E(n)-E(n_s)}}\left[\frac{S_v}{E_{sym}(n)} - 1\right]dn}{\int_0^{n_s}\sqrt{n(E(n)-E(n_s))}dn}$$

Fits to Nuclear Masses



Blue: $\Delta E < 0.01$ MeV/b
Green: $\Delta E < 0.02$ MeV/b
Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)
Plus: Best fits
Dashes: Danielewicz (2004)
Solid: Steiner et al. (2004)

$R_n - R_p$ contours

Interacting Fermi Gas

- Non-relativistic Potential Model

$$E_t(p) = \frac{\hbar^2}{2m_t^*}p^2 + V_t, \qquad f_t = \left[\exp\left(\frac{E_t - \mu_t}{T}\right) + 1\right]^{-1}$$

$$n_t = \frac{1}{2\pi^3\hbar^2}\int f_t d^3p, \qquad \tau_t = \frac{1}{2\pi^3\hbar^2}\int f_t p^2 d^3p$$

$$\epsilon - nmc^2 = \sum_t \frac{\hbar^2\tau_t}{2m_t^*} + U, \quad V_t = \sum_t \frac{\hbar^2\tau_t}{2}\left(\frac{\partial m_t^*}{\partial n_t}\right) + \frac{\partial U}{\partial n_t}, \quad P = \sum_t \left(n_t V_t + \frac{\hbar^2\tau_t}{3m_t^*}\right) - U$$

Interacting Fermi Gas

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- Relativistic Field-Theoretical Model

$$E_t = \sqrt{p^2 + M^{*2}} + g_\omega\omega + (\delta_{t,p} - 1/2)g_\rho\rho$$

$$n = n_n + n_p = \frac{1}{\pi^3\hbar^3}\sum_t \int f_t d^3p = \frac{\partial U_1}{\partial g_\omega\omega}, \qquad n_p - n_n = \frac{\partial U_1}{\partial g_\rho\rho}$$

$$n_s = \frac{1}{4\pi^3\hbar^3}\sum_t \int f_t \frac{M^*}{E_t} d^3p = \frac{\partial U_\sigma}{g_\sigma\sigma}, \qquad M^* = M - g_\sigma\sigma$$

$$U_1 = \left(\frac{m_\omega}{g_\omega}\right)^2 \frac{(g_\omega\omega)^2}{2} + \left(\frac{m_\rho}{g_\rho}\right)^2 \frac{(g_\rho\rho)^2}{2}, \quad U_\sigma = \left(\frac{m_\sigma}{g_\sigma}\right)^2 \frac{(g_\sigma\sigma)^2}{2} + \frac{\kappa}{6}(g_\sigma\sigma)^3 + \frac{\lambda}{24}(g_\sigma\sigma)^4$$

$$\epsilon = \frac{1}{4\pi^3\hbar^3}\sum_t \int E_t f_t d^3p + U_1 + U_\sigma, \qquad P = \frac{1}{4\pi^3\hbar^3}\sum_t \int \frac{p^2}{E_t} f_t d^3p + U_1 - U_\sigma$$

Phase Coexistence

Schematic energy density

$$\epsilon = n\left[B + \frac{K}{18}\left(1 - \frac{n}{n_0}\right)^2 + S_v\frac{n}{n_0}(1-2x)^2 + a\left(\frac{n_0}{n}\right)^{2/3}T^2\right]$$

$$P = \frac{n^2}{n_0}\left[\frac{K}{9}\left(\frac{n}{n_0} - 1\right) + S_v(1-2x)^2\right] - \frac{2an}{3}\left(\frac{n_0}{n}\right)^{2/3}T^2$$

$$\mu_n = B + \frac{K}{18}\left(1 - \frac{n}{n_o}\right)\left(1 - 3\frac{n}{n_o}\right) + 2S_v\frac{n}{n_o}(1-4x^2) - \frac{a}{3}\left(\frac{n_0}{n}\right)^{2/3}T^2$$

$$\hat{\mu} = \mu_n - \mu_p = 4S_v\frac{n}{n_0}(1-2x)\,, \qquad s = 2a\left(\frac{n_0}{n}\right)^{2/3}T$$

Two-Phase Free Energy Minimization

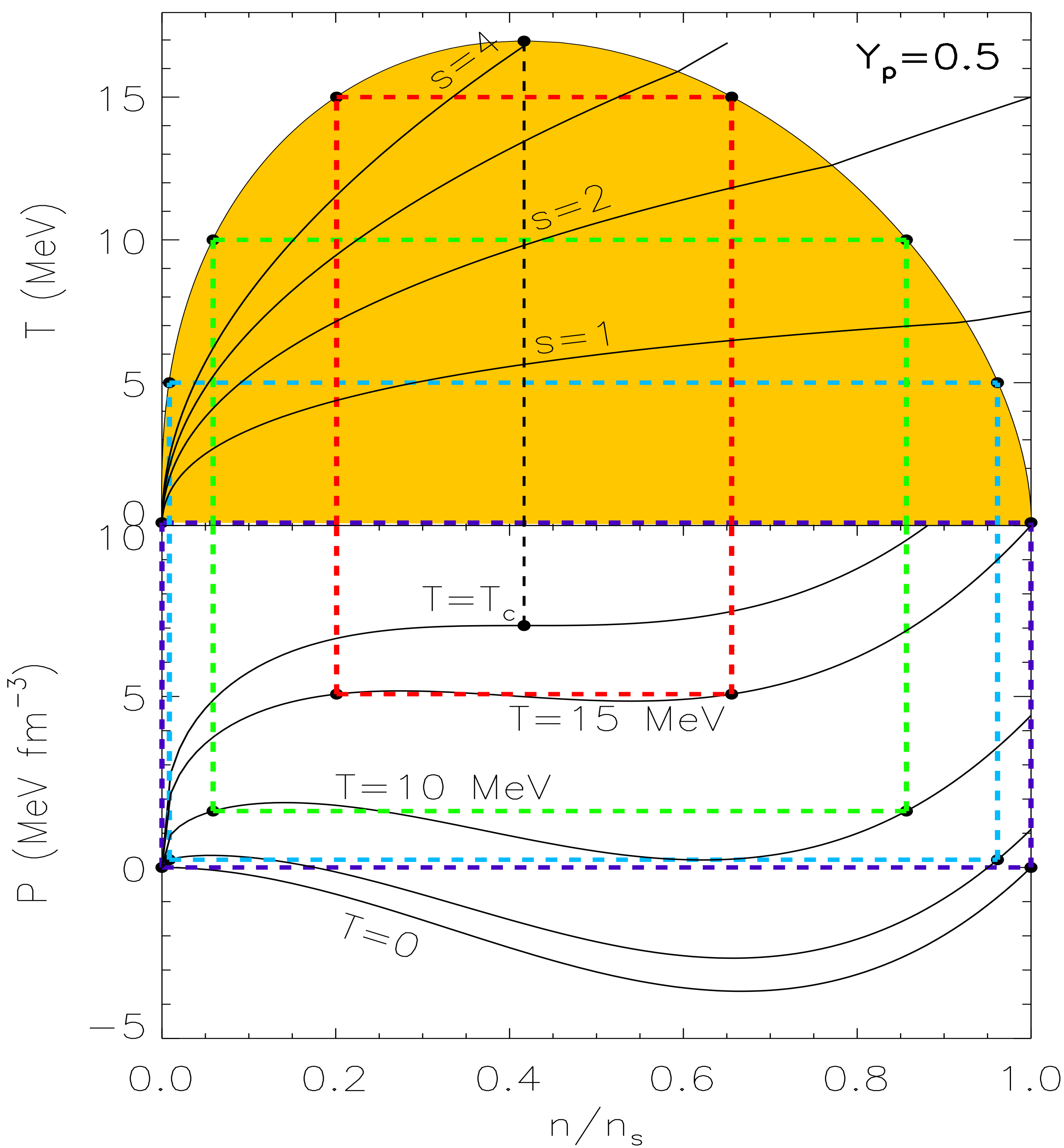
$$F = uF_I + (1-u)F_{II}, \qquad n - un_I + (1-u)n_{II}\,, \qquad \frac{\partial F}{\partial n_I} = 0\,, \qquad \frac{\partial F}{\partial u} = 0$$

$$\mu_I = \mu_{II}\,, \qquad P_I = P_{II}$$

Critical Point

$$\left(\frac{\partial P}{\partial n}\right)_T = \left(\frac{\partial^2 P}{\partial n^2}\right)_T = 0$$

$$n_c = \frac{5}{12}n_0\,, \qquad T_c = \left(\frac{5}{12}\right)^{1/3}\left(\frac{5K}{32a}\right)^{1/2}\,, \qquad s_c = \left(\frac{12}{5}\right)^{1/3}\left(\frac{5Ka}{8}\right)^{1/2}$$



Y_p=0.5
s=4
s=2
s=1
T (MeV)
T=T_c
T=15 MeV
T=10 MeV
T=0
P (MeV fm^-3)
n/n_s
0.0
0.2
0.4
0.6
0.8
1.0

Nuclei in Dense Matter

Liquid Droplet Model

$$F = u(F_I + f_{LD}/V_N) + (1-u)F_{II}\,, \qquad f_{LD} = f_S + f_C + f_T$$

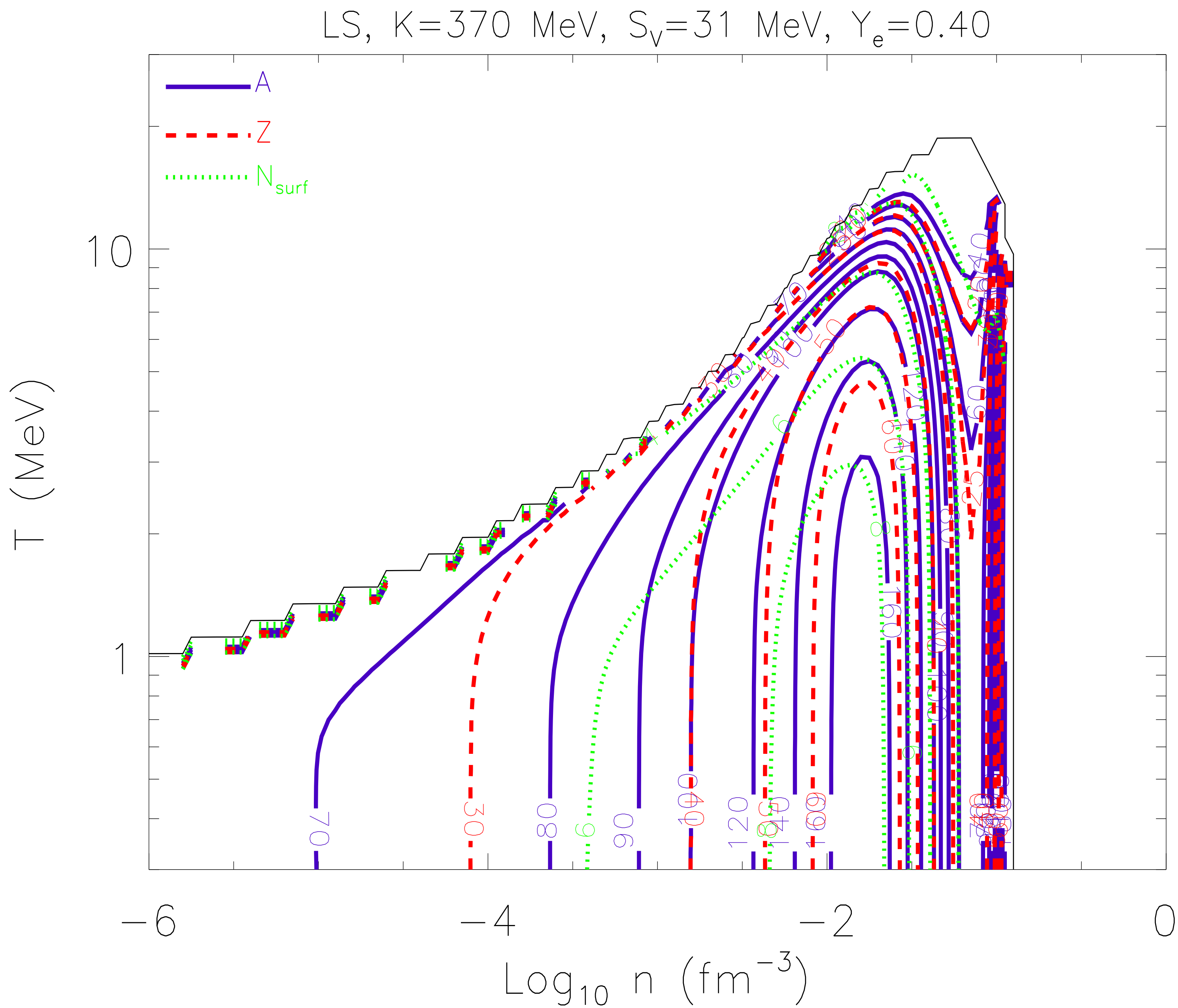
$$\begin{aligned} f_C &= \frac{3}{5}\frac{Z^2e^2}{R_N}\left(1 - \frac{3}{2}u^{1/3} + \frac{u}{2}\right) = \frac{3}{5}\frac{Z^2e^2}{R_N}D(u) \\ f_T &= T\ln\left(\frac{u}{n_Q V_N A^{3/2}}\right) - T = \mu_T - T\,, \qquad n_Q = \left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} \\ f_S &= 4\pi R_N^2 \sigma(\mu_s) \end{aligned}$$

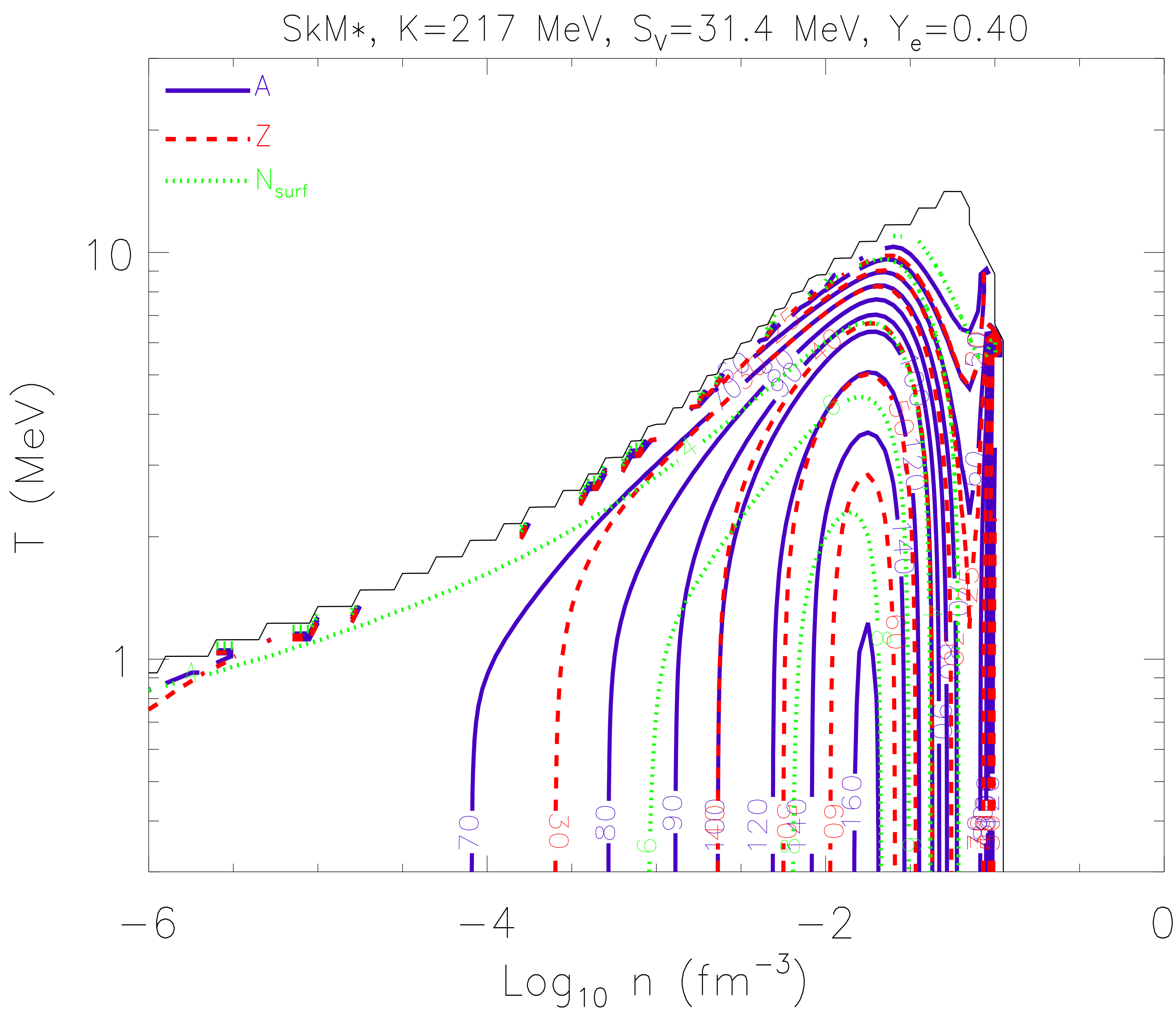
$$n = un_I + (1-u)n_{II}\,, \qquad nY_e = un_I x_I + (1-u)n_{II}x_{II} + u\frac{N_s}{V_N}$$

Free Energy Minimization

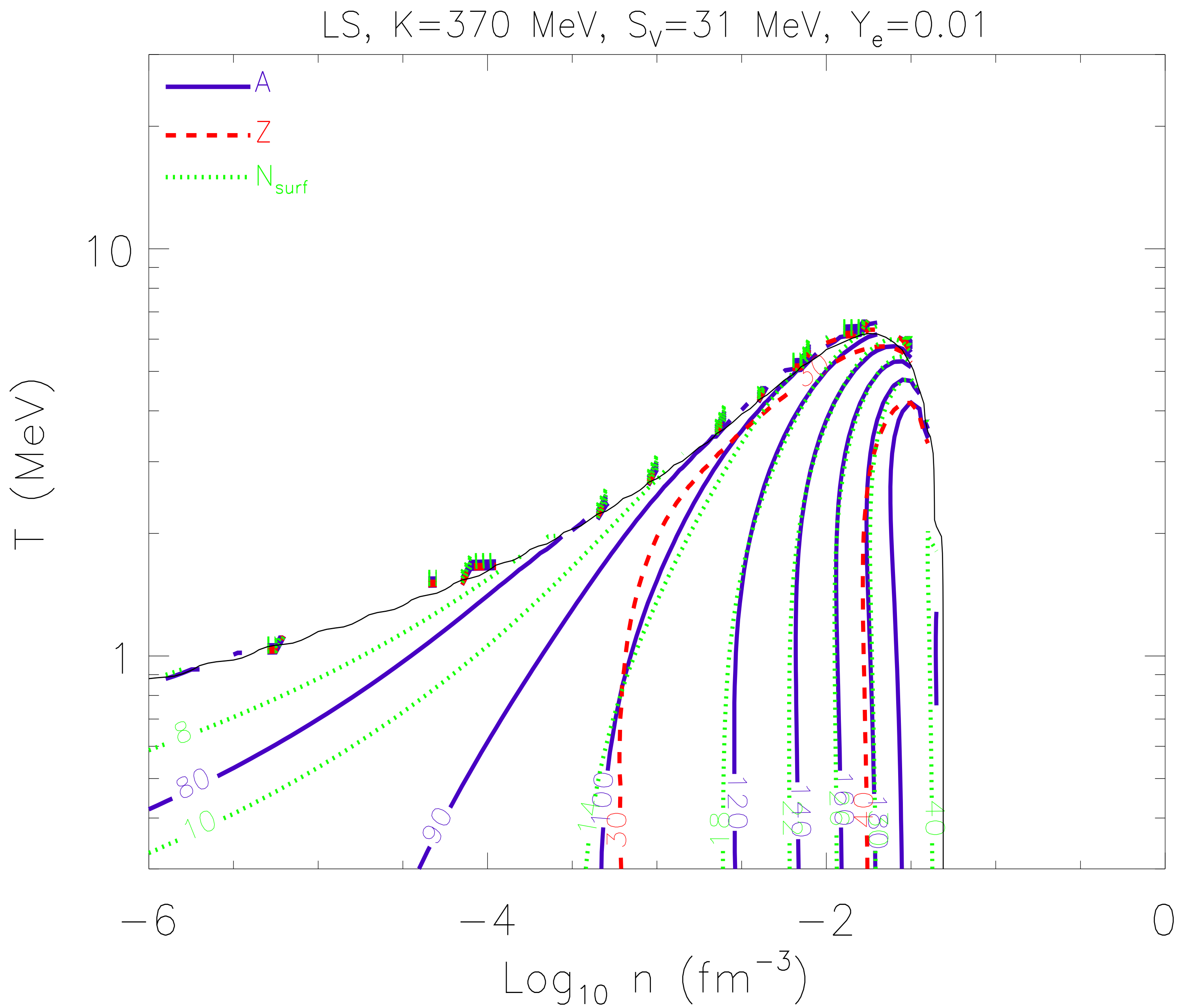
$$\frac{\partial F}{\partial z_i} = 0\,, \qquad z_i = (n_I, x_I, R_N, u, \nu_s, \mu_s)$$

$$\begin{aligned} \mu_{n,II} &= \mu_{n,I} + \frac{\mu_T}{A}\,, \qquad \hat\mu_{II} = \hat\mu_I - \frac{3\sigma}{R_N n_I x_i} = -\mu_s\,, \qquad N_s = -4\pi R_N^2 \frac{\partial\sigma}{\partial\mu_s} \\ P_{II} &= P_I + \frac{3\sigma}{2R_N}\left(1 + \frac{uD'}{D}\right)\,, \qquad R_N = \left(\frac{15\sigma}{8\pi n_I^2 x_I^2 e^2 D}\right)^{1/3} \end{aligned}$$


LS, K=370 MeV, S_V=31 MeV, Y_e=0.40
A
Z
N_surf
T (MeV)
Log10 n (fm^-3)
10
1
−6
−4
−2
0

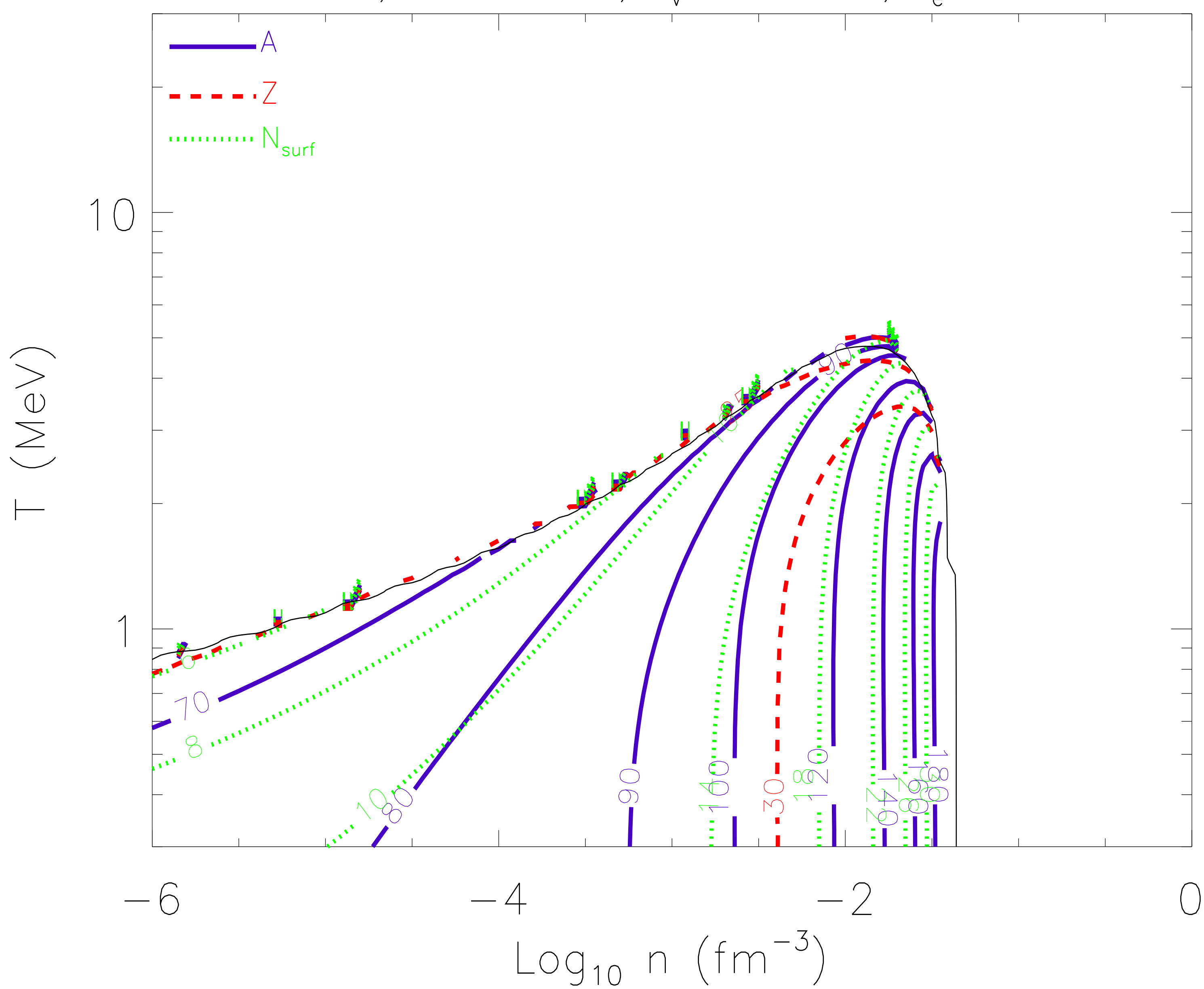


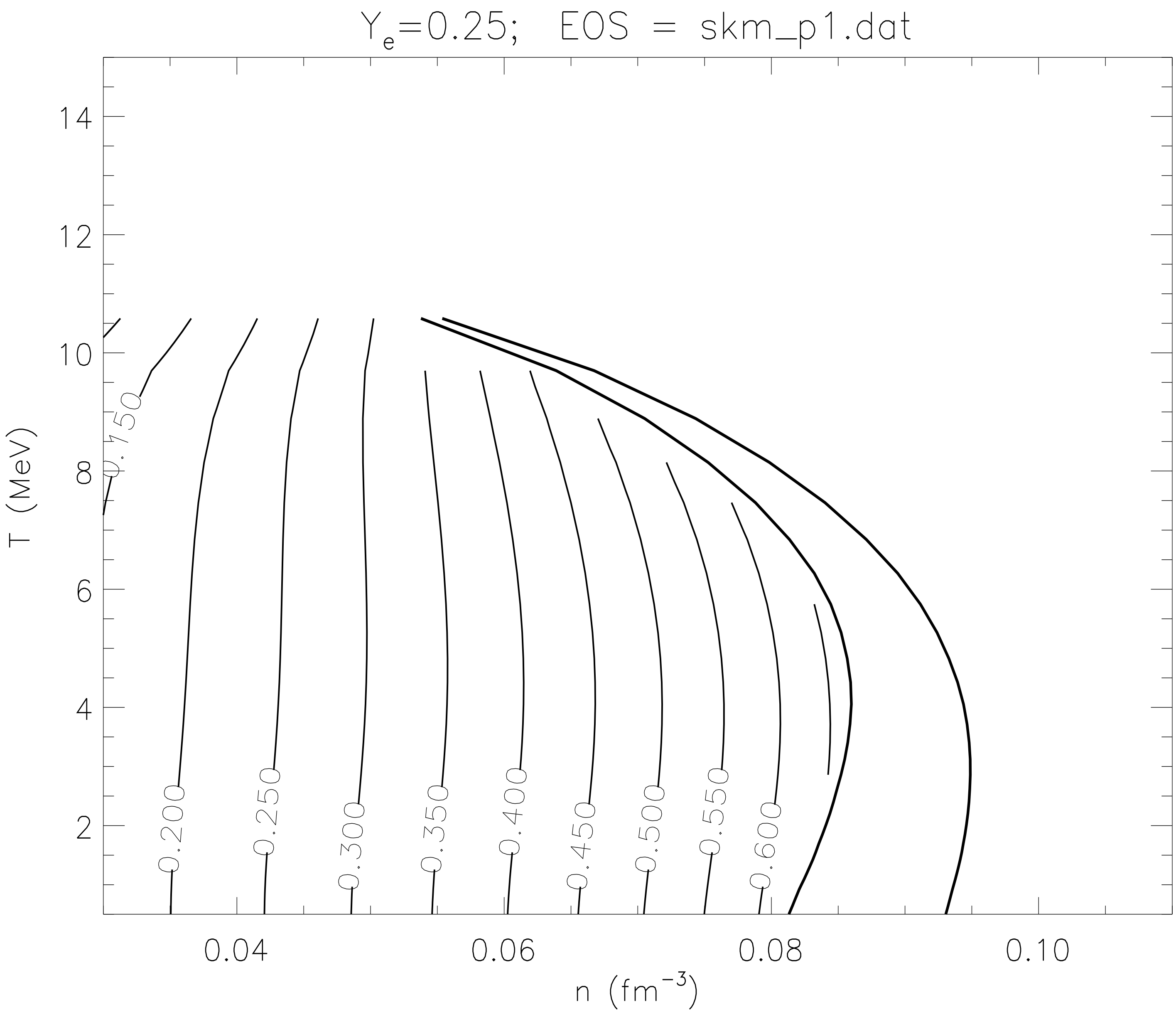
SkM*, K=217 MeV, S_V=31.4 MeV, Y_e=0.40
A
Z
N_{surf}
T (MeV)
Log_{10} n (fm^{-3})



LS, K=370 MeV, S_V=31 MeV, Y_e=0.01
A
Z
N_{surf}
T (MeV)
Log$_{10}$ n (fm^{-3})

SkM*, K=217 MeV, S_V=31.4 MeV, Y_e=0.01




Ye=0.25; EOS = skm_p1.dat
T (MeV)
n (fm−3)
0.04
0.06
0.08
0.10
2
4
6
8
10
12
14
0.150
0.200
0.250
0.300
0.350
0.400
0.450
0.500
0.550
0.600

Beta Equilibrium

$$\hat{\mu} = \mu_e = \hbar c(3\pi^2 n Y_e)^{1/3}$$