Neutron Rich Matter
I) In the Laboratory
II) In the Inner Crust of NS (Pasta)
III) In the Neutrinosphere of SN

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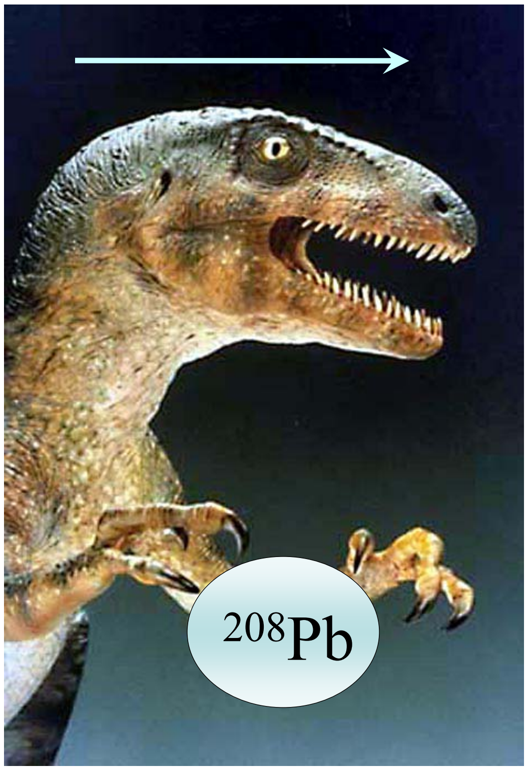
with J. Piekarewicz, and A. Schwenk

Neutron Stars at the Crossroads of
Fundamental Physics, Vancouver, 8/05

I) Neutron Rich Matter in the Laboratory



Parity Radius Experiment (PReX) uses parity violating electron scattering to measure the neutron radius of ^{208}Pb

PReX references:

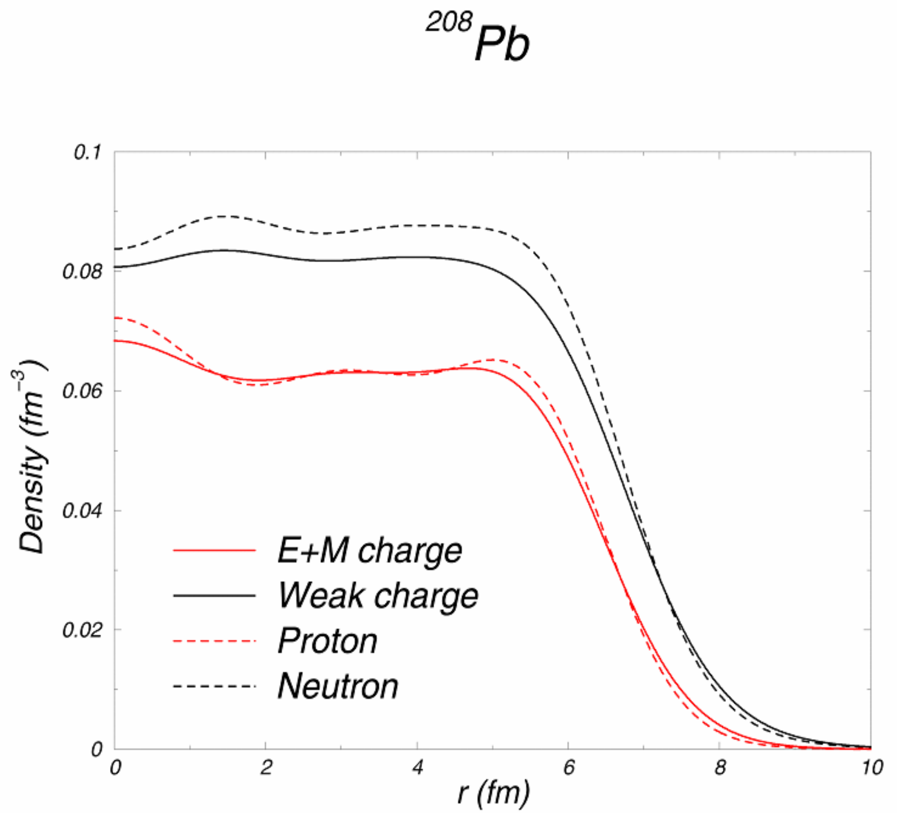
http://cecelia.physics.indiana.edu/prex

Parity Radius Experiment

- Parity violation probes neutrons because weak charge of a n $\gg$ p.
- Elastic scattering of 850 MeV e from ^{208}Pb at 6°.

$$A = \frac{\frac{d\sigma}{d\Omega_+} - \frac{d\sigma}{d\Omega_-}}{\frac{d\sigma}{d\Omega_+} + \frac{d\sigma}{d\Omega_-}}$$

- Measure A $\approx$ 0.6 ppm to 3%. This gives neutron radius to 1% ($\pm$ 0.05 fm).
- Purely electroweak reaction is model independent
- Spokespersons: P. Souder, R. Michaels, G. Urciuoli



- Two day test of full PREX experiment during this years HAPPEX II run.
- Plan for full PREX to run in 2007

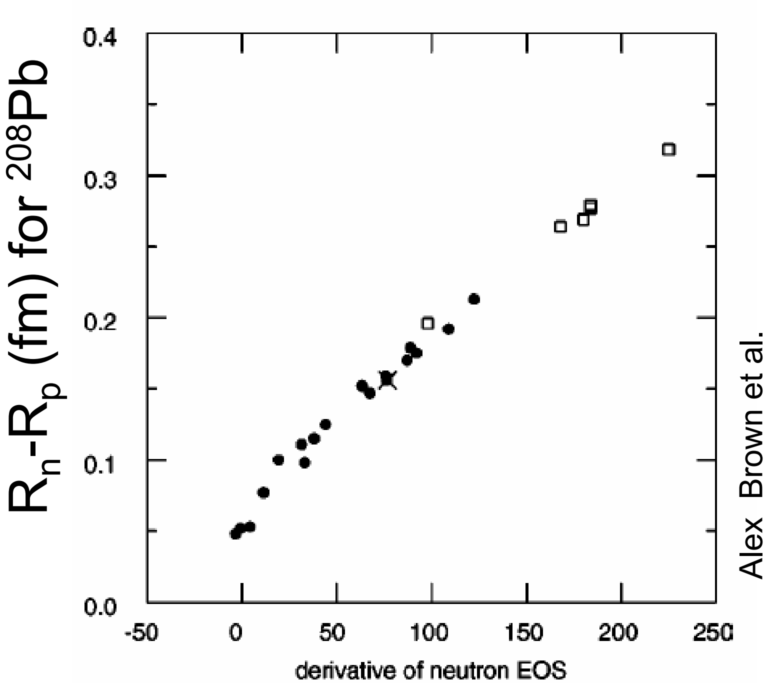
Pb Radius Measurement

- Pressure forces neutrons out against surface tension. Large pressure gives large neutron radius.
- Pressure depends on derivative of energy with respect to density.
- Energy of neutron matter is E of nuc. matter plus symmetry energy.

$$E_{neutron} = E_{nuclear} + S(\rho)$$

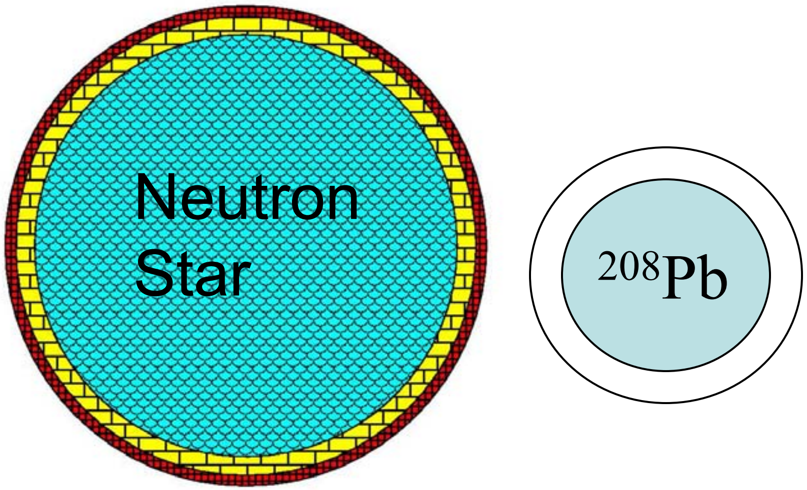
$$P \rightarrow dE/d\rho \rightarrow dS/d\rho$$

- **Neutron radius determines P of neutron matter at $\approx$ 0.1 fm^{-3} and the density dependence of the symmetry energy dS/dρ.**



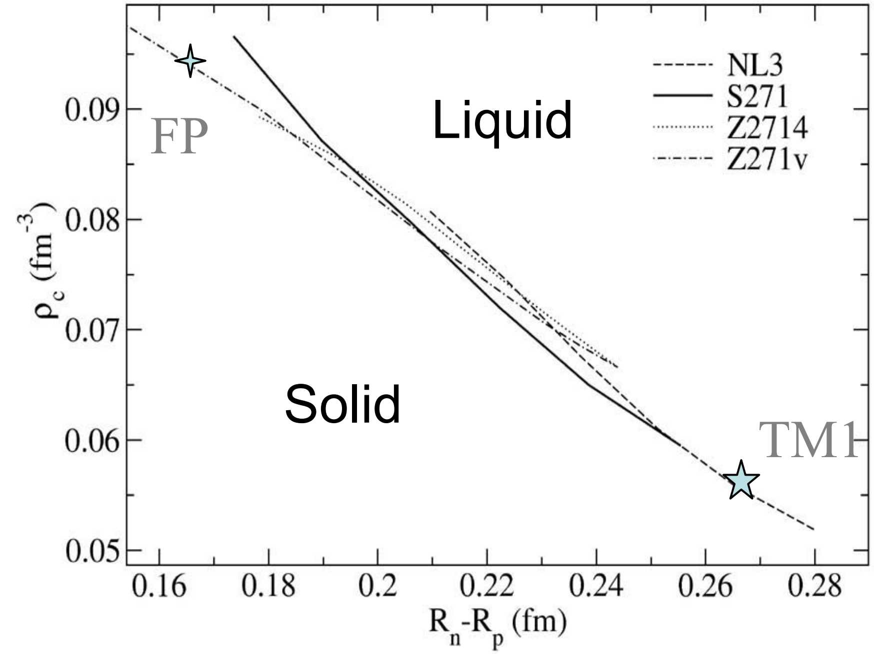
Neutron minus proton rms radius of Pb versus pressure of pure neutron matter at ρ=0.1 fm^{-3}.

Neutron Star Crust vs Pb Neutron Skin



• Neutron star has solid crust (yellow) over liquid core (blue).
• Nucleus has neutron skin.
• Both neutron skin and NS crust are made out of neutron rich matter at similar densities.
• Common unknown is EOS at subnuclear densities.

Liquid/Solid Transition Density



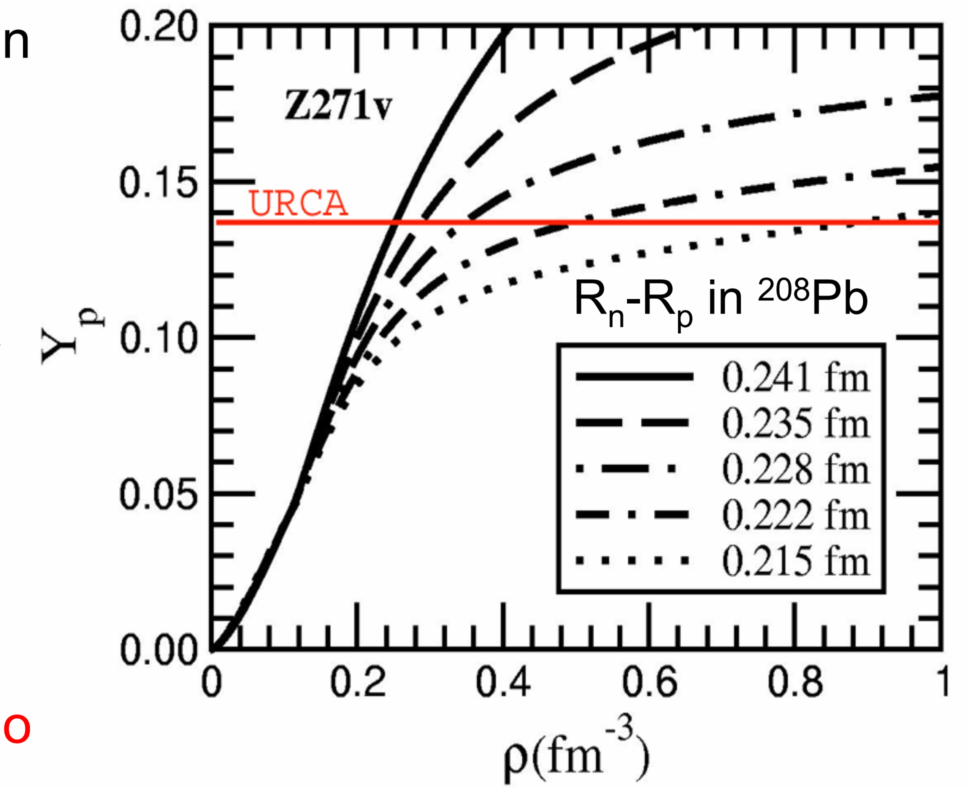
- Thicker neutron skin in Pb means energy rises rapidly with density→ Quickly favors uniform phase.
- Thick skin in Pb→low transition density in star.

Pb Radius vs Neutron Star Radius

- The ^{208}Pb radius constrains the pressure of neutron matter at subnuclear densities.
- The NS radius depends on the pressure at nuclear density and above.
- Most interested in density dependence of equation of state (EOS) from a possible phase transition.
- Important to have both low density and high density measurements to constrain density dependence of EOS.
  - If Pb radius is relatively large: EOS at low density is stiff with high P. If NS radius is small than high density EOS soft.
  - This softening of EOS with density could strongly suggest a transition to an exotic high density phase such as quark matter, strange matter, color superconductor, kaon condensate…

PREX Constrains Rapid Direct URCA Cooling of Neutron Stars

- Proton fraction Y_p for matter in beta equilibrium depends on symmetry energy S(n).
- R_n in Pb determines density dependence of S(n).
- The larger R_n in Pb the lower the threshold mass for direct URCA cooling.
- If R_n-R_p<0.2 fm all EOS models do not have direct URCA in 1.4 $M_\odot$ stars.
- If R_n-R_p>0.25 fm all models do have URCA in 1.4 $M_\odot$ stars.



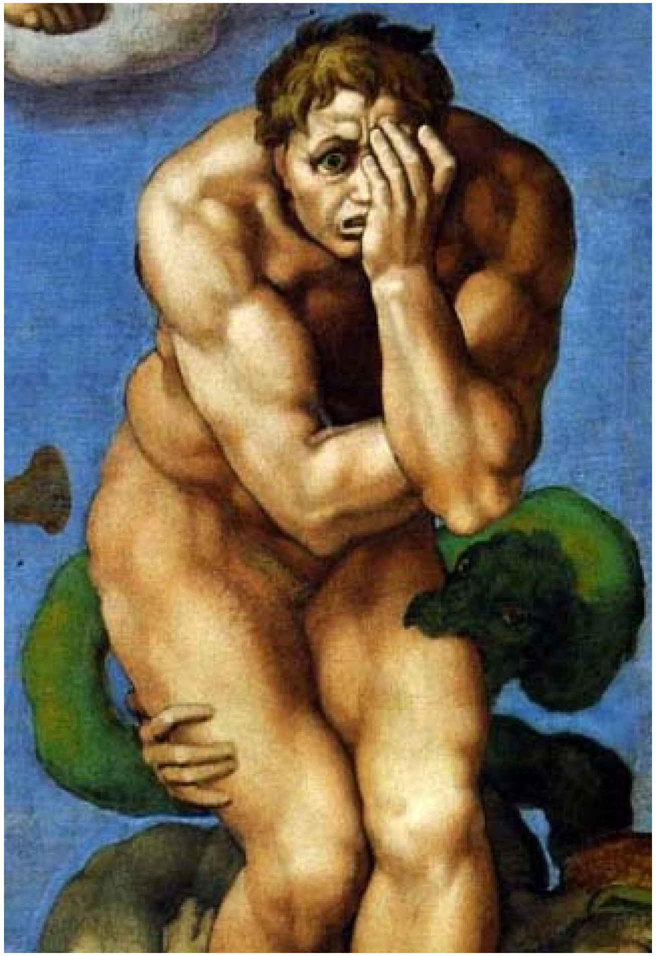
If Y_p > red line NS cools quickly via direct URCA $n \rightarrow p+e+\nu$

Nuclear Pasta

In the inner crust of neutron stars and supernovae

All conventional matter is frustrated

- It is correlated at short distances from attractive strong interactions.
- And anti-correlated at large distances from coulomb repulsion.
- Normally these length scales are well separated so nucleons bind into nuclei segregated on a crystal lattice.



Michelangelo

Nuclear Pasta

- At great densities, the attractive nuclear and repulsive atomic length scales are comparable.
  – Leads to a complex ground state.
  – Can involve sphere (meat ball), rod (spaghetti), plate (lasagna), or other shapes.
- This “nuclear pasta” is expected in neutron star crusts and supernovae.
- It should have unusual properties and dynamics because of the frustration.

Molecular Dynamics Simulations

- Charge neutral system of n, p and e. [e provide screening length λ for Coulomb.]
- Thermal wave length of heavy clusters is small compared to inter cluster spacing: semi-classical approx. should be good.
- n, p interact via classical 2-body pot.

$$H = K + \sum_{i<j} v(r_{ij})$$

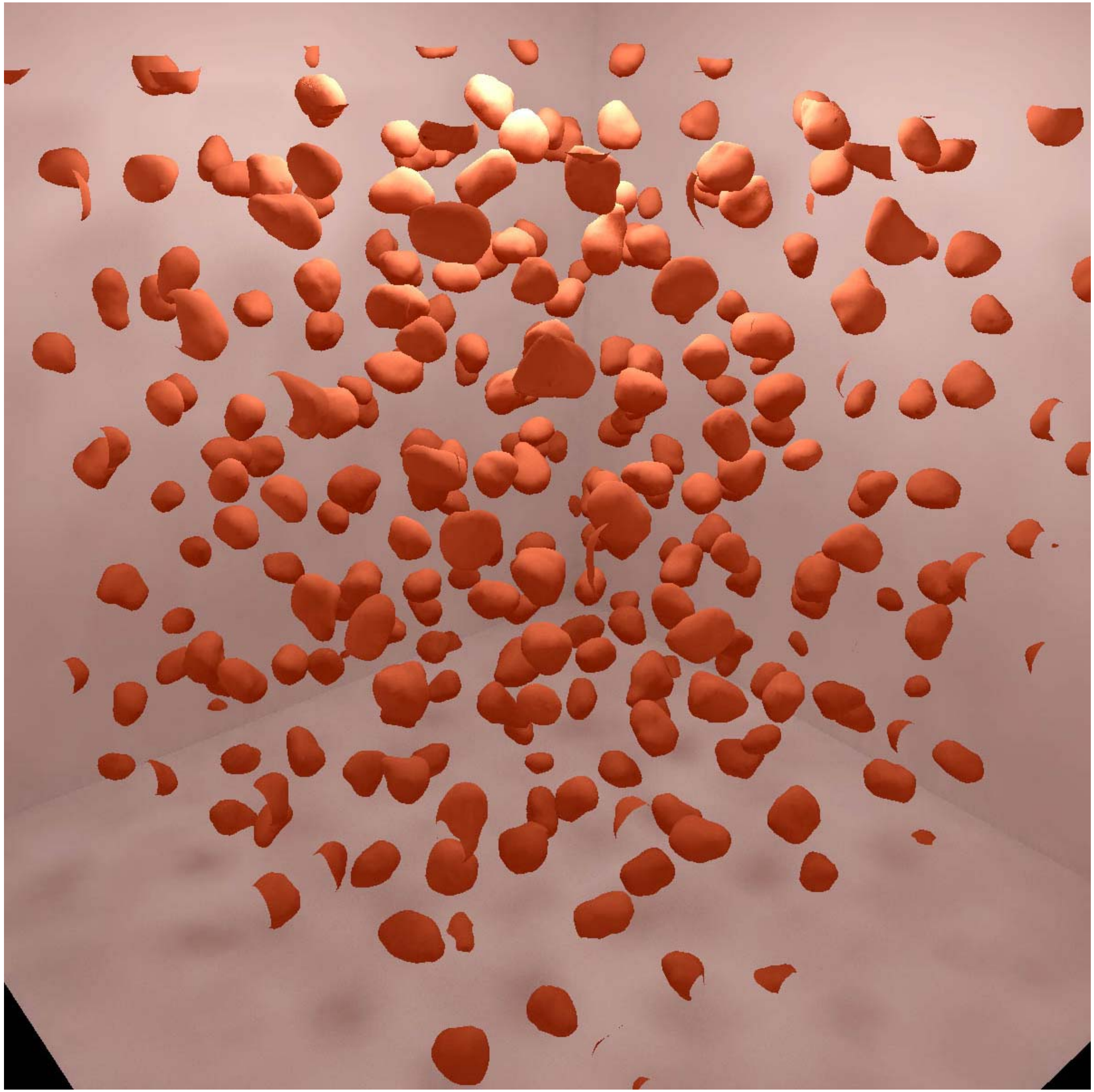
$$v(r)=a\,\mathrm{Exp}[-r^2/\Lambda]+ b_{ij}\,\mathrm{Exp}[-r^2/2\Lambda] + e_i e_j\,\mathrm{Exp}[-r/\lambda]/r$$

- Parameters a, b_{ij}, Λ fit to binding E and saturation density of nuclear matter and a reasonable symmetry energy.
- Watanabe et al have done similar QMD simulations for smaller systems.

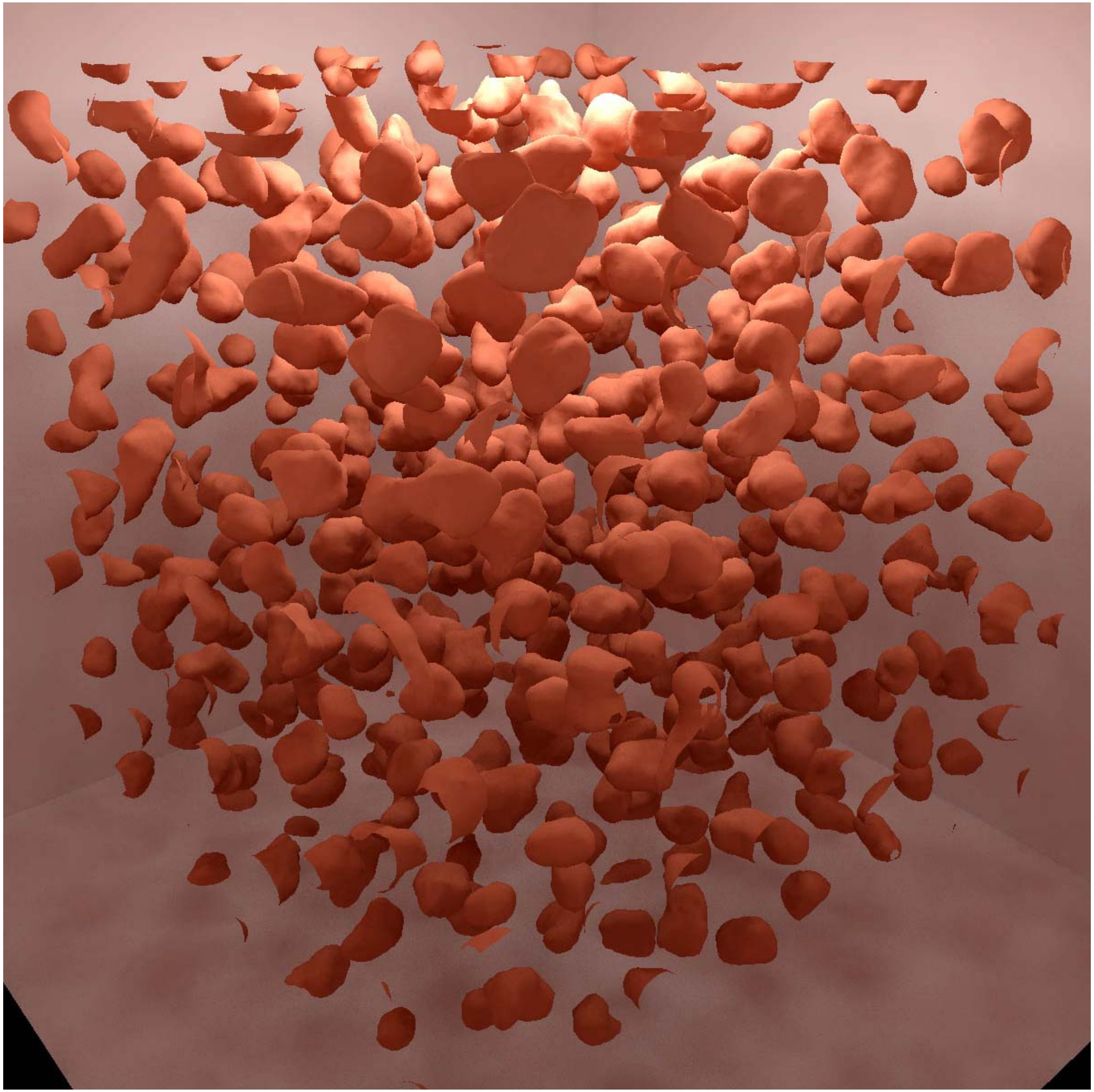
Molecular Dynamics Simulation with 40,000 nucleons at T=1 MeV, ρ=0.01 fm^{-3}, Y_p=0.2

Isosurface of proton density is shown.

Not shown, low density neutron gas between clusters.

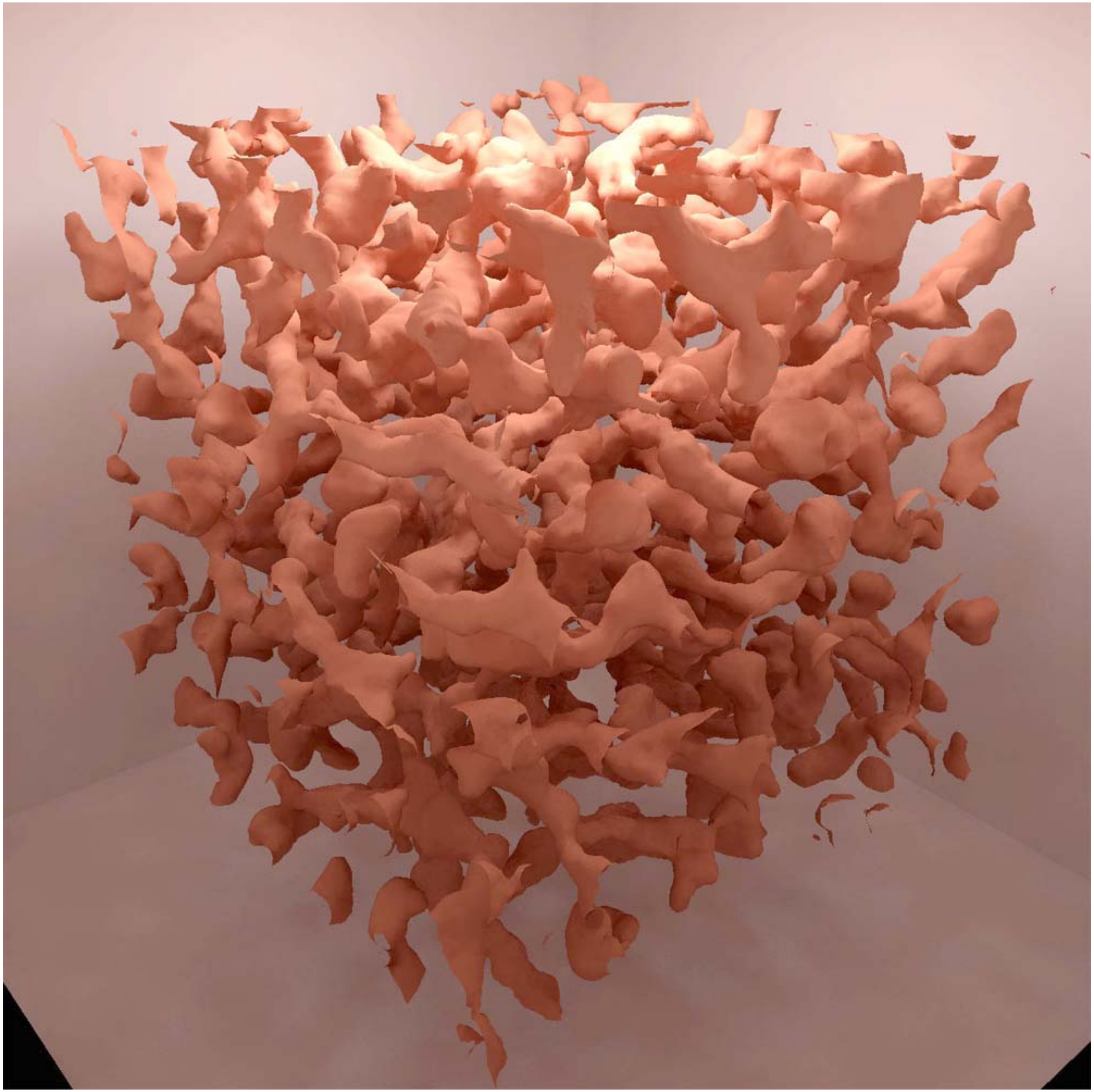


Graphics by Brad Futch FSU



Proton density at ρ=0.025 fm^{-3}

Graphics by Brad Futch FSU



Simulation with 100,000 nucleons at ρ=0.05 fm^{-3} showing pasta phase.

Graphics by Brad Futch FSU

Static Structure Factor S_q

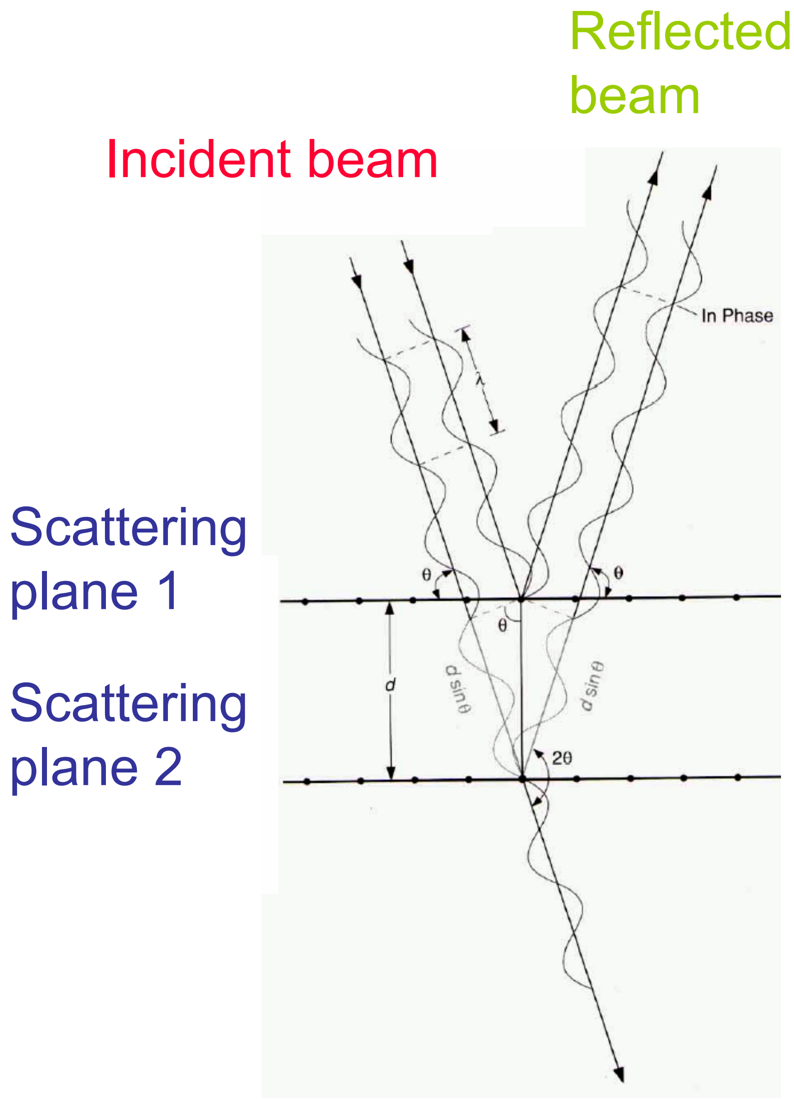
- Neutrino pasta scattering in SN
- Effective cross section per nucleon is

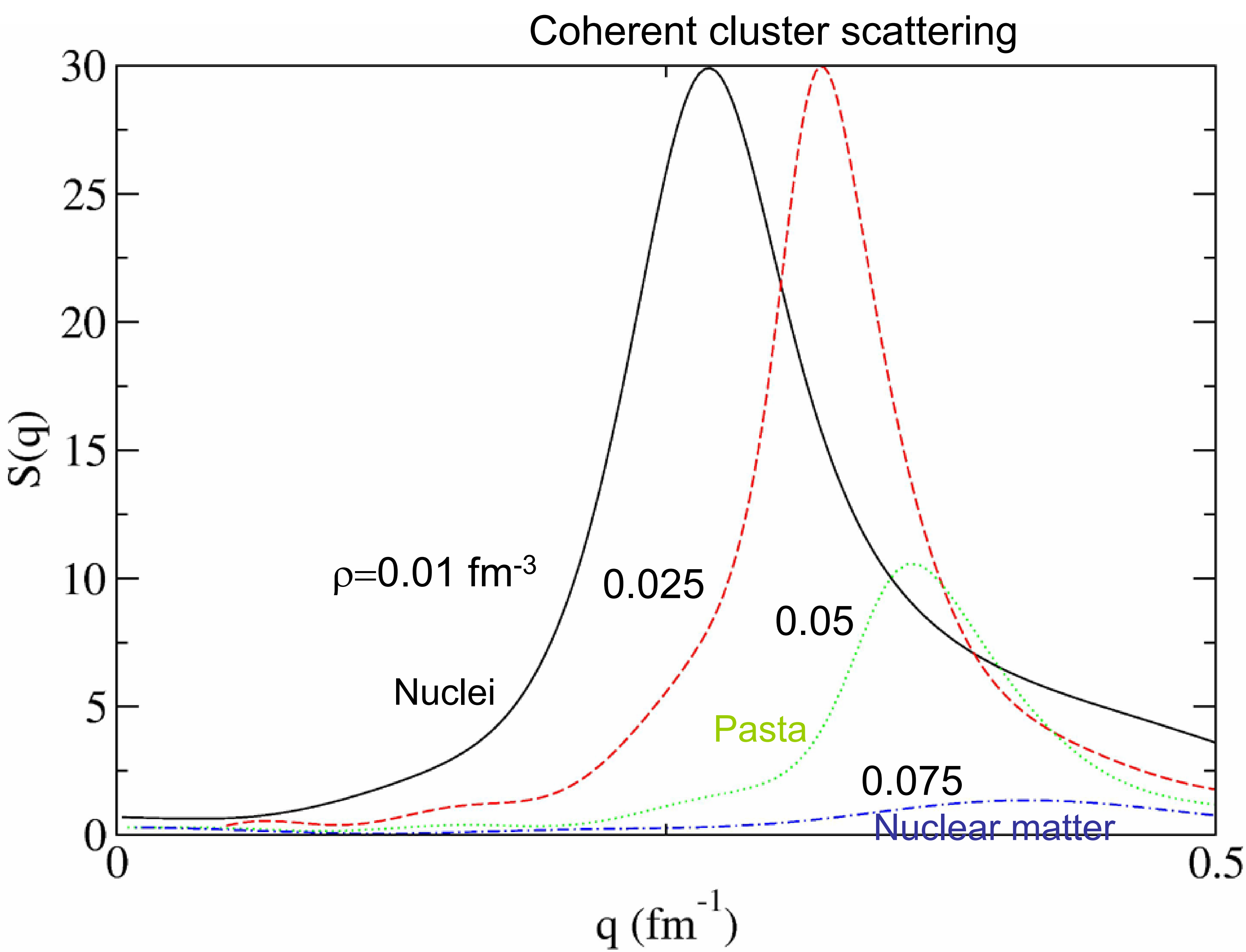
  $d\sigma/d\Omega = S(q)\, d\sigma/d\Omega|_{free}$

- Sum over all possible reflections.

  $S(q) \propto \sum_{i,j} \exp[iq \cdot r_{ij}]$

- S(q) gives the degree of coherence.





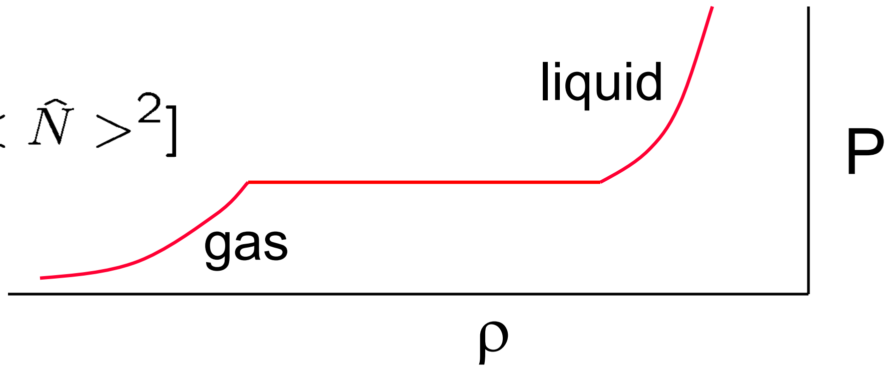
Coherent cluster scattering
S(q)
q (fm^{-1})
30
25
20
15
10
5
0
0
0.5
ρ=0.01 fm^{-3}
Nuclei
0.025
0.05
Pasta
0.075
Nuclear matter

Liquid Vapor Phase Transition

- In a first order phase transition, low density vapor is in equilibrium with high density liquid.
- Large density fluctuations arise as liquid is converted to/ from vapor.
- Static structure factor at q=0 related to density fluctuations.

$$S(q=0) = \frac{1}{N}[< \hat{N}^2 > - < \hat{N} >^2]$$

$$S(0) = kT/(dP/d\rho)$$



- Expect very large S(0) in two-phase coexistence region of simple first order phase transition $\rightarrow$ very short neutrino mean free paths in a supernova [J. Margueron, PRC**70**(2004)028801].
- We find no large enhancement of S(0) $\rightarrow$ system is ***not*** described by simple first order phase transition because ratio of liquid to vapor fixed by charge density of electrons. $\rightarrow$ mixed phase

There is no large first order liquid-vapor phase transition during a supernova.

Neutron Rich Matter in the Neutrino Atmosphere of a Proto-Neutron Star

Model Independent Virial Expansion for Neutron Rich Matter

- Neutrinos in a supernova decouple from the neutrino-sphere or neutrino atmosphere.
- Properties of this atmosphere determine the neutrino spectra and can be very important for SN dynamics.
- Conditions at neutrino-sphere:
  - Temperature ~ 4 MeV crudely observed with 20 SN1987a events.
  - $\rho \sim 10^{11}$ to 10^{12} g/cm^3 [$\sim 10^{-4}$ fm^{-3}] follows from known ν cross sections at these energies.
  - Proton fraction starts near ½ and drops to small values.
- What is the composition, equation of state, and neutrino response of nuclear matter under these conditions?
- *Virial expansion gives model independent answers!*

Universal Behavior of Neutron Matter

- Consider a low density fermi system with large scattering length $a \rightarrow \infty$, and effective range $r \rightarrow 0$ much less then inter-particle spacing.
- There are no length scales associated with interaction. Therefore system will exhibit universal behavior independent of details.
- To what extent does real neutron matter at low density approach this "unitary limit"? Real a=-19 fm, r=2.7 fm.
- Use Virial expansion to simply relate energy of neutron matter to nn scattering properties.
- A number of cold atom experiments to test universal behavior of fermions in this unitary limit.

Virial Expansion 101

- Assume (1) system in gas phase and has not undergone a phase transition with increasing density or decreasing temp. (2) fugacity $z=e^{\mu/T}$ with μ the chemical pot is small.
- Expand grand canon. partition function Q in powers of z: $P=T\ \ln Q/V$, $n=z\ d/dz\ \ln Q/V$

  $P=2T/\lambda^3[z+b_2z^2+b_3z^3+\ldots]$, $n=2/\lambda^3[z+2b_2z^2+3b_3z^3+\ldots]$

  Here λ=thermal wavelength=$(2\pi/mT)^{1/2}$
- 2nd virial coef. $b_2(T)$ calculated from 2 particle partition function: $Q_2=\sum_{states} \mathrm{Exp}[-E_2/T]$

  E_2 is energy of 2 particle state. Thus b_2 depends on density of states.

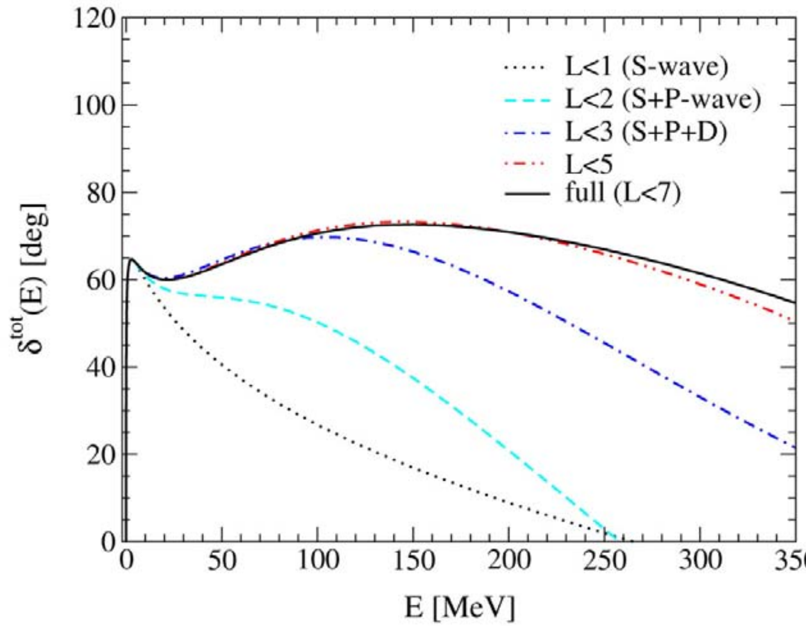
Density of states

- Put system in big spherical box of radius R
- Relative mom. k from $E_2=k^2/2m_{reduced}$.
- $\psi(r_1-r_2=R\to\infty)=0=\sin[kR+l\pi/2+\delta_l(k)]$ or
  $kR+l\pi/2+\delta_l(k)=n\pi$.
- Distance between states $\Delta k=\pi/(R+d\delta/dk)$ so $dn/dE \propto 1/\Delta k \propto R + d\delta/dk$
- $b_2=2^{1/2}\sum_B e^{E_B/T} +2^{1/2}/\pi \int_0^\infty dk\, e^{-E_k/2T} \sum_l{}' (2l+1)\, d\delta_l(k)/dk \pm 2^{-5/2}$
  with + for bose and – for fermions.
- b_2 Includes both bound states and scattering resonances on equal footing.

Neutron Matter

- Integrate by parts and include spin,
- $b_n = 1/(2^{1/2}\pi T)\int dE\, e^{-E/2T}\, \delta_{tot}(E) - 2^{-5/2}$
- $b_n(T)$=0.301, 0.306, 0.309 at T=2, 4, and 8 MeV
- b_n almost T independent, as s-wave phase falls with increasing energy, higher l contributions rise to almost cancel.

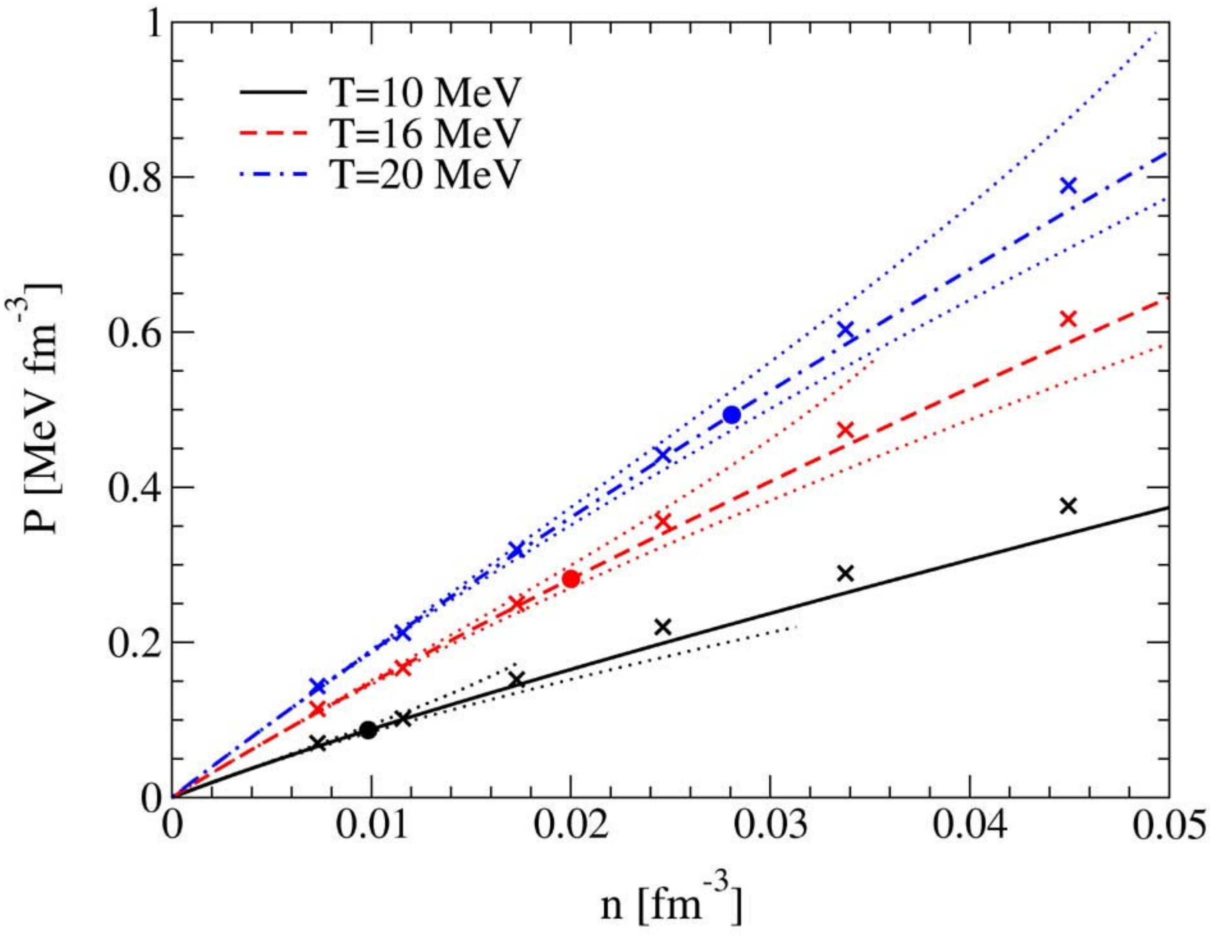
$$\delta^n_{tot}(E) = \delta_{^1S_0} + \delta_{^3P_0} + 3\delta_{^3P1} + 5\delta_{^3P_2} + 5\delta_{^1D_2} + \ldots$$



- Use b_3 for error estimate. 3 n can't be in s state so expect b_3 to be small. Use $|b_3| \leq b_2/2$.

Neutron matter Equation of State

- Neutron matter virial nearly independent of temperature. $b_n(T)$=0.303, 0.309, 0.320 at T=2, 10, and 20 MeV.



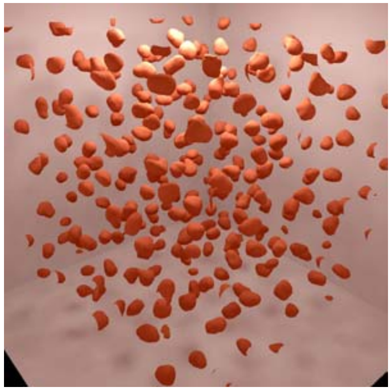
Error bars (dotted) from estimate of b_3

Crosses from microscopic FHNC calc. by Friedman + Pandharipande

Scaling of Neutron Matter EOS

- If $b_i(T)$ are independent of T the EOS will scale
  $P/T^{5/2} = f(n/T^{3/2})$.
- Neutron matter P is only a function of $n/T^{3/2}$ instead of a function of n and T separately
- From $P/T=g/\lambda^3[z+b_n z^2+\ldots]$ and $n=g/\lambda^3[z+2b_n z^2+\ldots]$ with $\lambda \propto T^{-1/2}$.
- Unitary Limit: calculate b_n with only s-wave and $a=-\infty$, $r=0$. $\delta(^1S_0)=\pi/2$
  $b_n(T)=3/2^{5/2}=0.5303$ independent of T.
- In unitary limit system clearly scales.
- Real neutron matter scales, to a very good approx., but with a $b_n \approx 0.3$ that is 40% smaller then unitary limit.
- In scaling limit energy density $\varepsilon=3/2P$ [Thomas et al have tested this for a universal system of cold ^{6}Li atoms.]

Nuclear Matter

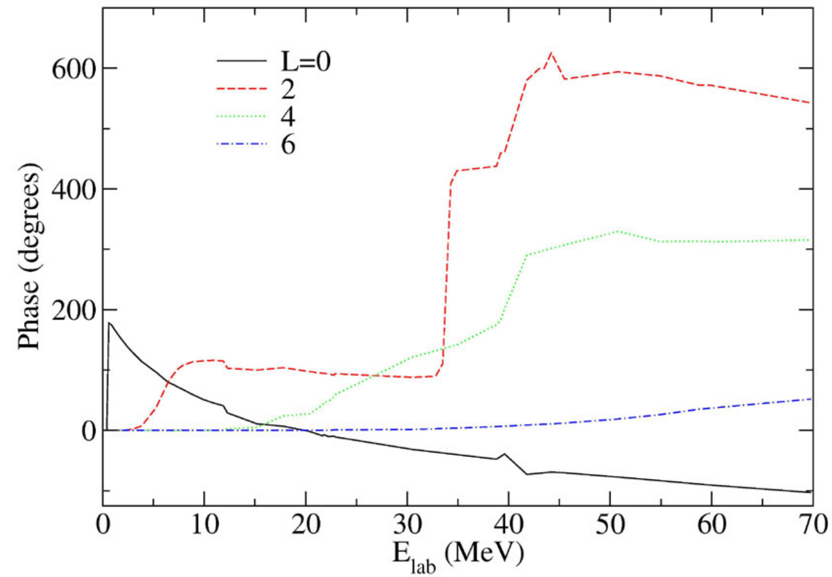


- Is very different from neutron matter because of cluster formation.
- Deuterons appear as bound state in b_2.
- α particles will appear as bound state in b_4.
- Large α binding E_α=28.3 MeV gives large $e^{+E\alpha/T}$ contribution to b_4.
- Nucleon only virial expansion may be accurate only over a very reduced density range because of the abnormally large b_4.
- Solution: include α explicitly and work with system of p, n, and α s. Chemical equilibrium $2\mu_p+2\mu_n=\mu_\alpha$ gives $z_\alpha=z_p^2 z_n^2\, e^{E\alpha/T}$.
- Work to 2nd order in z_p, z_n, z_α. Can include heavier nuclei at even higher densities.

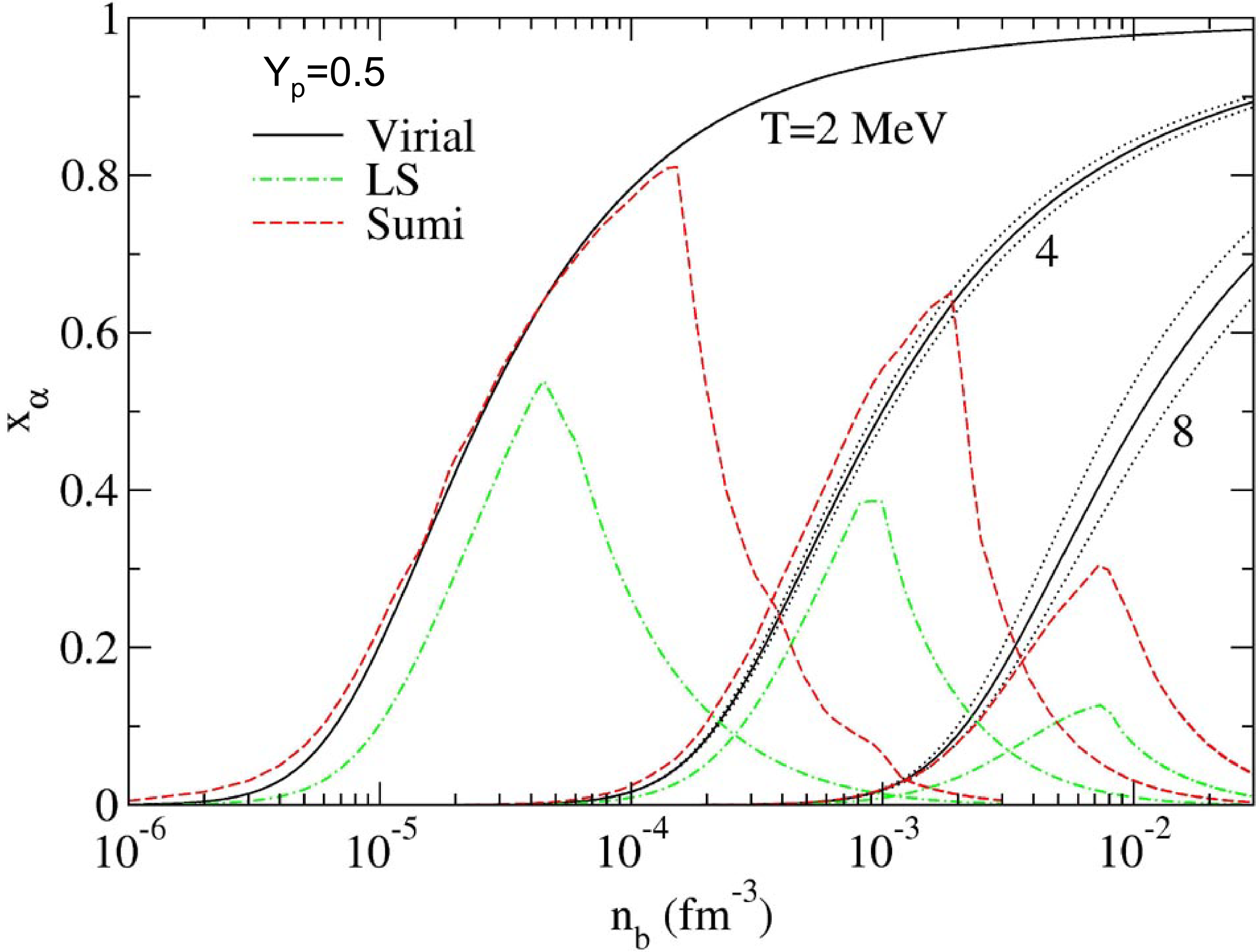
n, *p*, α system

$$\frac{P}{T} = \frac{2}{\lambda^3}[z_p + z_n + (z_n^2 + z_p^2) b_n + 2 z_n z_p (b_{nuc} - b_n)] + \frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + z_\alpha (z_p + z_n) b_{\alpha n}]$$

- Need four virial coefficients:
  - b_n for neutron matter,
  - b_{nuc} for symmetric nuclear matter,
  - b_α for alpha system,
  - $b_{\alpha n}$ for interaction between an α and N.
- Virials from NN, Nα and $\alpha\alpha$ elastic scattering phase shifts.



α–α Elastic Phase Shifts



Alpha mass fraction vs density for T=2, 4 and 8 MeV. Also shown are predictions for Lattime Swesty and Shen (Sumioshi) EOS models. These have x_α that drop at high density because of the formation of heavy nuclei.

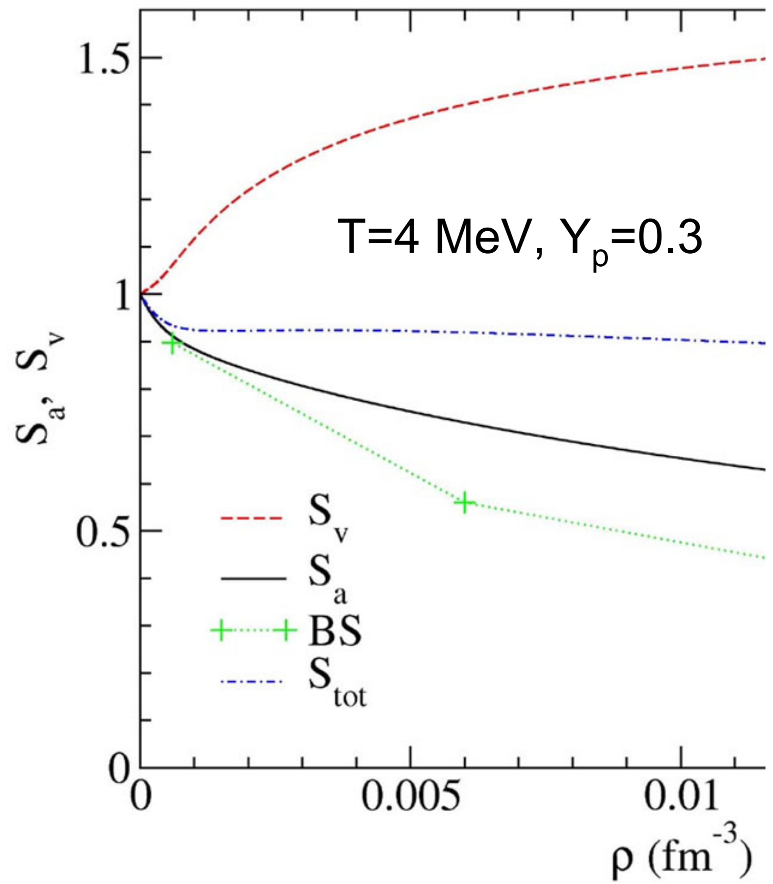
Neutrino Response in Virial Expansion

- ν cross section per volume for scattering from n, p and alphas

  $\sigma = G^2 E_\nu^{\,2}/4\pi\ [\ 3g_a^{\,2}(n_n+n_p)S_a + (n_n + 4n_\alpha)S_v\ ]$

- Here S_a is axial or spin response from virial expansion of spin polarized matter.
- S_v is vector or density response = static structure factor S_q in $q \to 0$ limit

  $S_v = S_{q=0} = T/(dP/dn)$



Future Work

- Calculate nucleon 3rd virial b_3 for neutron and nuclear matter. Example Paulo Bedaque + G. Rupak cond-mat/0206527
- Include heavy nuclei in addition to n, p, and α
  – As a single heavy nucleus with ave. <Z> and <A>.
  – As a distribution of many heavy nuclei (perhaps with simplified N-nucleus scattering).
- Include coulomb interactions.
- Study role of inelastic scattering.
- …

Conclusions

- Virial expansion provides *model independent* equation of state, composition, entropy, energy, and long wave length responses for neutron rich matter at low densities.
- Neutron matter EOS scales: $P=T^{5/2}f(n/T^{3/2})$, $\varepsilon=3/2P$
- Low density nuclear matter forms clusters and does not scale.
- We describe nuclear matter in n, p, and α coordinates with virial coefficients from NN, Nα, and $\alpha\alpha$ scattering.
- Incorporate d and α bound states and scattering resonances including ^{2}He, N-α p-waves, and ^{8}Be.
- Model independent results for α mass fraction disagree with all existing phenomenological EOS models.
- All existing microscopic calculations of low density nuclear matter EOS fail because of neglect of cluster formation.

J. Piekarewicz, M. A. Perez-Garcia, D. Berry, A. Schwenk, C. Horowitz