Phys 350 - Problem Set #3

(Schaums 5.1)

#1 a force conservative iff $\nabla \times \vec{F} \equiv 0$

$\Rightarrow \exists\ V(\vec{x})$ such that $\vec{F} = -\nabla V$ where $\vec{x}, \nabla$ are with respect to generalized coordinates

a) $\boxed{V(\vec{x}) = mgy} \Rightarrow \vec{F} = -mg$

b) $\boxed{V(\vec{x}) = \frac{1}{2}kx^2 + \frac{1}{2}ky^2} \Rightarrow \vec{F} = -kx\hat{x} - ky\hat{y}$

c) $\nabla \times \vec{F} = 2k \not\equiv 0$ $\boxed{\text{not conservative}}$

d) $\nabla \times \vec{F} = By - Ax \not\equiv 0$ $\boxed{\text{not conservative}}$

e) $\boxed{V(\vec{x}) = -Axyz} \Rightarrow \vec{F} = Ayz\hat{x} + Axz\hat{y} + Axy\hat{z}$

f) $\boxed{V(r,\theta,\phi) = -Br^3\sin\theta\cos\phi}$ so that the generalized force is the one given.

g) $\boxed{V(r,\theta,\phi) = -\int_{r_0}^{r} dr' f_1(r') - \int_{\theta_0}^{\theta} d\theta' f_2(\theta) - \int_{\phi_0}^{\phi} d\phi' f_3(\phi')}$
gives the generalized force given

h) $\boxed{V(\vec{x}) = \frac{1}{2}kx^2\sin\omega t - \frac{1}{2}ky^2 t} \Rightarrow \vec{F} = -kx\sin\omega t\,\hat{x} + ky t$

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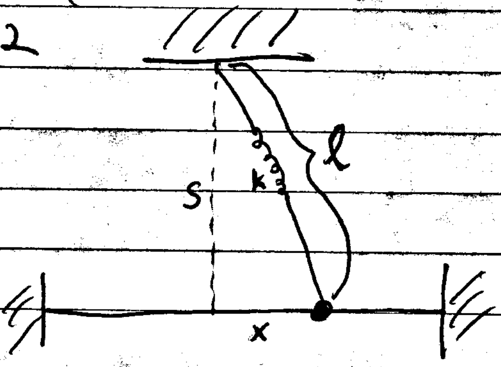
(Schaums #5.5)

#2



$$l^2 = s^2 + x^2$$

$$V = \frac{1}{2}k(l - l_0)^2$$
$$= \frac{1}{2}k\left(\sqrt{s^2+x^2} - l_0\right)^2$$
$$= \frac{1}{2}k\left(s^2 + x^2 - 2l_0\sqrt{s^2+x^2} + l_0^2\right)$$
$$= \frac{1}{2}k\left(x^2 - 2l_0\sqrt{s^2+x^2}\right)$$

where I have dropped constants from V.

For very small x

$$\sqrt{s^2+x^2} \simeq s + \frac{x^2}{2s} - \frac{x^4}{8s^3} + O(x^6)$$

so

$$V \simeq \frac{1}{2}k\left[x^2 - 2l_0\left(s + \frac{x^2}{2s} - \frac{x^4}{8s^3}\right)\right]$$

$$\boxed{V = \frac{k(s-l_0)}{2s}x^2 + \frac{kl_0}{8s^3}x^4 + \text{const.}}$$

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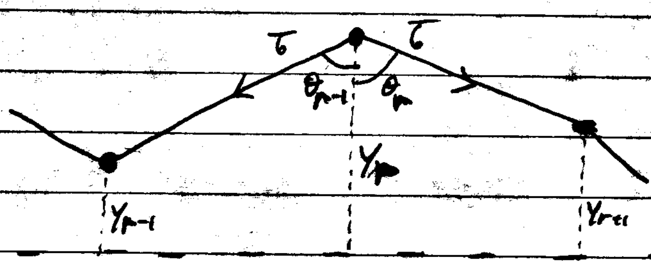
#3 (Schaums 5.12)

Consider the r^{th} bead

τ - tension force in string (assume constant



Assuming the x-component of the force cancels, the force on the r^{th} bead is

$$\vec{F}_r = \left(-\tau\cos\theta_{r-1} - \tau\cos\theta_r\right)\hat{y}$$

$$\simeq -\frac{\tau}{a}\Big[(y_r - y_{r-1}) + (y_r - y_{r+1})\Big]\hat{y}$$

$$= \left[-2\frac{\tau}{a}y_r + \frac{\tau}{a}y_{r-1} + \frac{\tau}{a}y_{r+1}\right]\hat{y} \qquad (3.1)$$

For n beads, the potential that gives rise to this force is

$$V = \frac{1}{2}\frac{\tau}{a}\sum_{r'=0}^{n}\left(y_{r'+1} - y_{r'}\right)^2$$ with $y_0 = y_{n+1} = 0$ being the endpoints

Check: $\vec{F}_r = -\dfrac{\partial V}{\partial y_r}\hat{y}$

$$= -\frac{1}{2}\frac{\tau}{a}\sum_{r'=0}^{n}\Big[2(y_{r'+1} - y_{r'})(\delta_{r,r'+1} - \delta_{r',r})\Big]\hat{y}$$

Kronecker delta

$$= -\frac{\tau}{a}\Big[y_r - y_{r-1} - y_{r+1} + y_r\Big]\hat{y}$$

$$= \left[-2\frac{\tau}{a}y_r + \frac{\tau}{a}y_{r-1} + \frac{\tau}{a}y_{r+1}\right]\hat{y}$$

which agrees with (3.1).

re-write potential

$$V = \frac{1}{2}\tau \sum_{r'=0}^{n} \left(\frac{y_{r'+1} - y_{r'}}{a}\right)^2 a$$

and if $y(x)$ is the position of string

$$V = \frac{1}{2}\tau \sum_{r'=0}^{n} \left(\frac{y(ar'+a) - y(ar')}{a}\right)^2 a.$$

In the continuum limit for a string of ~~...~~ l length

$$ar' \to x$$
$$a \to dx \qquad \frac{y(ar'+a) - y(ar')}{a} \to \frac{dy}{dx}$$
$$an \to l$$
$$\Sigma \to \int$$

so

$$\boxed{V = \frac{1}{2}\tau \int_0^l dx \left(\frac{dy}{dx}\right)^2}$$

For $y(x) = A\sin\frac{\pi x}{l} \Rightarrow \frac{dy}{dx} = \frac{A\pi}{l}\cos\frac{\pi x}{l}$

$$V = \frac{A^2\tau\pi^2}{2l^2}\int_0^l dx \cos^2\frac{\pi x}{l} \qquad \text{substitute } u = \frac{\pi x}{l} \Rightarrow dx = \frac{l}{\pi}du$$

$$= \frac{A^2\tau\pi}{4l}\int_0^{\pi} du\,(1 + \cos 2u)$$

$$= \frac{A^2\tau\pi^2}{4l}$$

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