Astro 304 Homework set # 2. Due Friday 9 April

1: Because it came both for the CMB and for looking at pulsars, lets work out how light goes through plasma. We will find an expression for ϵ and you can always find $(c/n)^2 = \epsilon\mu_o$ on your own.

A: An electric field

$$\vec{E}(t) = \hat{x}\ exp^{i\omega t} \tag{1}$$

acts on an electron (charge e, mass m) which is in a neutral low density plasma. Assume that the electron moves in synchrony with $\vec{E}$ as

$$\vec{x}(t) = \vec{x}_o\ exp^{i\omega t} \tag{2}$$

Find the amplitude of motion, x_o by matching F and $ma = m\ddot{x}$.

B: The protons in this plasma are, by comparison, big and heavy, and hardly move. Find the dipole moment per electron, $\vec{p}$, caused by one electron moving.

C: If there are n_e electrons per cubic metre, the polarization density is

$$\vec{P} = n_e\vec{p} \tag{3}$$

and

$$\vec{D} = \epsilon_o\vec{E} + \vec{P} = \epsilon\vec{E}. \tag{4}$$

Show that ϵ is frequency dependent and can be written as

$$\epsilon(\omega) = \epsilon_o(1 + \omega_p^2/\omega^2). \tag{5}$$

D: Does Eq. 5 capture the features in the pulsar talk, that different frequency components of a pulse arrive at different times and that the effect is largest at low frequencies? Look up the density of a typical interstellar plasma and calculate ω_p.

2: Last week you found the critical density. Let's continue and find how the expansion rate evolves. You should have found an expression relating the scale factor, $a(t)$, its time-derivative,$\dot{a}(t)$ and the density, ρ. Assume that the universe has exactly the critical density, *ie* kinetic and binding energy are equal. Also, assume that density scales as a to some power. Specifically, for a universe filled only with radiation, the energy density is

$$\rho_\gamma \propto T^4 \propto a(t)^{-4}. \tag{6}$$

The critical-density universe grows forever. Substitute $a(t) = a_o t^p$ into your energy balance equation and find the exponent, p for a radiation-dominated universe.

How does $a(t)$ very with time in a matter-dominated universe, where instead of Eq. 6 we have $\rho_M \propto a(t)^{-3}$?