LETTERS TO THE EDITOR

PHYSICAL SCIENCES

Rotating Neutron Stars, Pulsars and Supernova Remnants

I SHALL discuss here some problems connected with theories linking the pulsars to the rotation of neutron stars (ref. 1 and a preprint by L. Woltjer). Because neutron stars can be formed during a supernova explosion, their rotation could be coupled with the surrounding gaseous remnant[2,3]: the following considerations will therefore also refer to the problem of the activity observed in the Crab Nebula and similar objects.

Gold (private communication) has noted that the direct detection of a pulsar in the Crab Nebula would be made very difficult by the dispersion inside the nebula itself. It is also obvious that a pulsed emission will not result if the basic cause of the pulses (for instance, the stellar rotation) has a time scale shorter than the width of the individual pulses. A rotating neutron star could then be present, say, in the Crab Nebula even if there is no evidence for a pulsating radio source in this part of the sky (Drake, private communication). Long term brightness variations similar to the ones found in the pulsars could still be detectable: a search for this kind of variability in the compact low frequency source in the Crab is needed.

Following my earlier paper[3] and more recent remarks by several authors (ref. 1 and preprints by L. Woltjer and by B. J. Eastlund), I shall consider a neutron star which has a dipole magnetic field and rotates about an axis different from that of the magnetic field. The oblique rotator configuration seems common among magnetic stars where the angle between the field and the rotation axes tends to be close to 90° (ref. 4). Even if the two axes coincided in the original star, the mass loss during the supernova explosion would certainly not be perfectly spherically symmetric, especially if large magnetic fields are present. As a consequence, the mutual inclination of the axes in a newly born neutron star is likely to be almost arbitrary. Spitzer[5] has noted that an oblique rotator configuration cannot persist indefinitely because the body is rotating about an axis which is not a principal axis of inertia. Dissipation mechanisms in the form of hydromagnetic waves can be important close to the surface of the star and eventually result in an acceleration of particles. Remarkable evidence for this kind of activity close to the magnetic poles of some A_p stars comes from the distribution of peculiar elements in these stars (ref. 4 and a preprint by L. Woltjer). The possibility of a connexion between this kind of hydromagnetic activity in an oblique rotator and the activity observed in some supernova remnants is worth mentioning.

I shall not attempt to evaluate a lifetime for the oblique rotator configuration, as the problem is extremely difficult. There is, however, a difficulty of a different kind associated with the existence of the magnetic field itself in an old neutron star. The decay time of the field is proportional to the product σR^2, where σ is the electrical conductivity and R is the radius of the star. An order of magnitude for σ can be roughly estimated by putting

$$\sigma = \frac{e^2}{m_e v S} \qquad (1)$$

where v is the electron velocity and S is the Coulomb cross-section. For a relativistic electron gas we take $v=c$ and S of the order of the classical value $\pi r_e^2 = 10^{-24}$ cm^2. The electrical conductivity turns out to be $\sigma \sim 10^{22}$ s^{-1}, that is, about 10^4–10^5 times larger than in the Sun. For a neutron star we have $R \simeq 10^6$ cm $\simeq 10^{-5}$ $R_\odot$, and for the Sun the decay time is about 10^{10} yr, so it is at least uncertain whether the fossil magnetic field of a neutron star can last longer than about 10^4–10^5 yr. It has to be emphasized that this estimate does not take into account the gas degeneracy and, even more important, the possibility of a superconductive behaviour of the neutron star matter in some density range. A complete investigation of the problem is certainly desirable.

I shall now describe the electromagnetic field around a neutron star rotating with an angular velocity ω in a quasi-vacuum. By quasi-vacuum we mean that the field of the star cannot be compensated by the currents induced in the surrounding medium. The corresponding density limit can be evaluated by noting that the maximum current density in the circumstellar gas is $j_{max} = n_e e c$. The Maxwell equation curl $\mathbf{H} = (4\pi/c)\,\mathbf{j}$ then gives the maximum induced field. If we take curl $H \sim 1/r$ (r is of the order of the size of the system) we obtain $H_{max} \simeq 4\pi n_e r$. The field of the star, or variations of the field of the order of the field itself, cannot be compensated by the induced currents if $n_e < H/(4\pi e r)$. Assuming $H \simeq 10^{10}$ gauss and $r \simeq R \simeq 10^6$ cm the critical electron density is about 10^{13} cm^{-3}. This limit is certainly not very stringent and is likely to be violated only soon after the birth of the neutron star. The assumption of a quasi-vacuum then seems legitimate near a neutron star having a very strong magnetic field.

Let α be the angle between the magnetic and the rotation axes. If $\alpha \neq 0$ retardation effects become important for the field at a distance r such that $\omega r \sim c$: for $\omega r \gg c$ the field becomes a wave progressing outwards. As noted[3] the oblique rotator model leads to the release of the rotational energy of the star by the radiation of electromagnetic waves having the same frequency as the star rotation.

Deutsch has given a complete description of the field of a rotating sphere[6]. Following this work we shall introduce an inertial reference system (r, θ, φ) having its origin in the centre of the star and axis ω. If we assume that the internal field is frozen into the star, its components can be written

$$H(r,\theta,\varphi) = \mathbf{e}_r H_r(r,\theta,\lambda) + \mathbf{e}_\theta H_\theta(r,\theta,\lambda) + \mathbf{e}_\varphi H_\varphi(r,\theta,\lambda) \qquad (2)$$

where λ is an azimuthal coordinate measured from a meridian fixed in the star. In the same reference system the internal electric field is given by

$$\mathbf{E} = -\mu_0 (\boldsymbol{\omega} \times \mathbf{r}) \times \mathbf{H} \qquad (3)$$

If we impose the continuity conditions at the surface of the star for the field components, the external field is fully determined by the Maxwell equations.

Let H_0 be the surface value of the magnetic field and a be the radius of the star: the general expression for the external electromagnetic field turns out to be very complicated[6] and I shall only give its components in the near region $r \ll (c/\omega)$ and in the far-wave region $r \gg (c/\omega)$.

(a) In the near region the field rotates with the star and depends on the azimuthal coordinate λ which is measured from a meridian fixed in the star. The components are

$$H_r = H_0 \left(\frac{a}{r}\right)^3 [\cos\alpha\cos\theta + \sin\alpha\sin\theta\cos\lambda]$$

$$H_\theta = \frac{1}{2} H_0 \left(\frac{a}{r}\right)^3 [\cos\alpha\sin\theta - \sin\alpha\cos\theta\cos\lambda]$$

$$H_\varphi = \frac{1}{2} H_0 \left(\frac{a}{r}\right)^3 \sin\alpha\sin\lambda \qquad (4)$$

$$E_r = -\frac{1}{4}\,\omega\, a\, \mu_0 H_0 \left(\frac{a}{r}\right)^4 [\cos\alpha\,(3\cos 2\theta + 1) + 3\sin\alpha \sin 2\theta \cos\lambda]$$

$$E_\theta = -\frac{1}{2}\,\omega\, a\, \mu_0 H_0 \left(\frac{a}{r}\right)^2 \left[\frac{a^2}{r^2}\cos\alpha \sin 2\theta + \sin\alpha\left(1 - \frac{a^2}{r^2}\cos 2\theta\right)\cos\lambda\right] \qquad (5)$$

$$E_\varphi = \frac{1}{2}\,\omega\, a\, \mu_0 H_0 \left(\frac{a}{r}\right)^2 \left(1 - \frac{a^2}{r^2}\right)\sin\alpha\cos\theta\sin\lambda$$

(*b*) Far-wave zone $r \gg (c/\omega)$; in this case the retardation effects dominate and we obtain

$$H_r = \frac{\omega}{c} H_0\, a^3 \frac{1}{r^2}\sin\alpha\sin\theta\sin\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right]$$

$$H_\theta = \frac{1}{2}\frac{\omega^2}{c^2}\, a^3 H_0 \frac{1}{r}\sin\alpha\cos\theta\cos\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right] \qquad (6)$$

$$H_\varphi = -\frac{1}{2}\frac{\omega^2}{c^2}\, a^3 H_0 \frac{1}{r}\sin\alpha\sin\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right]$$

$$E_r = 0$$

$$E_\theta = -\frac{1}{2}\frac{\omega^2\mu_0}{c}\, a^3 H_0 \frac{1}{r}\sin\alpha\sin\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right] \qquad (7)$$

$$E_\varphi = -\frac{1}{2}\frac{\omega^2\mu_0}{c}\, a^3 H_0 \frac{1}{r}\sin\alpha\cos\theta\cos\left[\omega\left(\frac{r}{c} - t\right) + \varphi\right]$$

If the magnetic and rotation axes coincide, there is no radiation field: the electromagnetic field external to the star can be obtained simply by putting $\alpha = 0$ in the set of equations (4) and (5).

By using equations (6) and (7) we can obtain the Poynting vector and evaluate the rate at which the rotational energy W and the angular momentum L are radiated away from the star. The result is

$$\frac{dW}{dt} = -\frac{2\pi\,\omega^4\mu_0}{3c^3}\, a^6 H_0^2 \sin^2\alpha \qquad (8)$$

and

$$\frac{dL}{dt} = \frac{1}{\omega}\frac{dW}{dt} \qquad (9)$$

Once the electromagnetic waves are emitted and start to propagate, they will be reflected by the circumstellar gas if the plasma frequency exceeds the radiation frequency[3]. In our case this certainly occurs for every conceivable value of ω and gas density, and the electromagnetic waves will be unable to reach us. Generation of high energy particles can, however, be expected in the region where the waves are reflected and cause a rapid compression of the nebular gas. Because the radiation pattern of the star is directional, it becomes possible to speculate whether a pulsed radio source arises because of this periodical gas compression and consequent acceleration of particles. From an energy point of view no difficulty arises: the period of pulsar *CP* 1919 corresponds to $\omega = 4{\cdot}7$ s^{-1} and if we assume $H \sin\alpha \simeq 10^{10-11}$ gauss the energy output is 10^{27}–10^{29} ergs s^{-1} in agreement with the observational requirements. On the other hand, for a newly formed neutron star, rotational frequencies 100 or even 1,000 times larger are perfectly possible and the energy output corresponding to fields of 10^{11} gauss can be as high as 10^{41} erg s^{-1}. The high rotational velocity itself could wash out the pulse structure and lead to a continuum emission: this situation possibly corresponds to what is observed in the Crab Nebula.

Finally, in this model a pulsating radio source would show a relatively slow change in period. From equation (8) we can obtain an expression for the rotational energy $W = (1/2)k\,M\,a^2\,\omega^2$ left at a certain time t as a function of the original rotational energy. We have

$$\frac{W_0}{W(t)} = 1 + \frac{4\pi\,a^4}{3c^3\,k\,M}\,\omega_0^2\,(H_0\sin\alpha)^2\, t \qquad (10)$$

For a neutron star we can assume $k = 0{\cdot}2$, $a = 10^6$ cm and $M = 2\times10^{33}$ g. As $W \propto T^{-2}$ we have for the periods

$$T^2 = T_0^2\,(1 + 1{\cdot}2\times10^{-32}\,\omega_0^2\,H_0^2\sin^2\alpha\; t_{yr}) \qquad (11)$$

where T_0 is the initial rotation period. We can then put a limit to the product $H \sin\alpha$ for the pulsar *CP* 1919 because we know that the fractional change in period $\Delta T/T_0$ over 1 yr cannot exceed 10^{-7}. This gives $H_0 \sin\alpha < 10^{12}$ gauss.

It was my chief concern to discuss some possibilities which arise when the oblique configuration for the magnetic field of a rotating neutron star is assumed. In particular, I wanted to show that the model leads to a release of rotational energy from the star at a rate which can agree quantitatively with the requirements either of the pulsars or of objects such as the Crab Nebula.

I emphasize, however, that the consideration of the circumstellar plasma is going to be of prime importance in several respects. This plasma is the place where the radiation we actually observe originates: the rotating neutron star only provides the energy source and the timing of the basic excitation (periodic compression of the plasma by the low frequency electromagnetic waves). Hoyle, Narlikar and Wheeler[7] have noted that the scale height for a neutron star is very small and therefore the density of the gas that may exist outside the star is also small. Furthermore, the pressure exerted by the radiation is enormous and any residuum of gas near the star would be swept outwards. The neutron star itself will therefore be surrounded by the vacuum (at least as long as radiation pressure dominates over gravitational accretion) but, farther away, the electromagnetic waves will be reflected by the interstellar gas. The equations (8) to (11) assume radiation in a vacuum: whether or not the same loss of rotational energy occurs in real life seems to hinge on what happens to the energy and angular momentum of the electromagnetic waves when they interact with the plasma.

Finally, we note that the same model could also apply to a very rapidly rotating white dwarf. In this case, however, magnetic fields of 10^{10} gauss seem very unlikely and it would be more difficult to achieve the right conditions in the circumstellar gas.

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