Stellar Black Holes

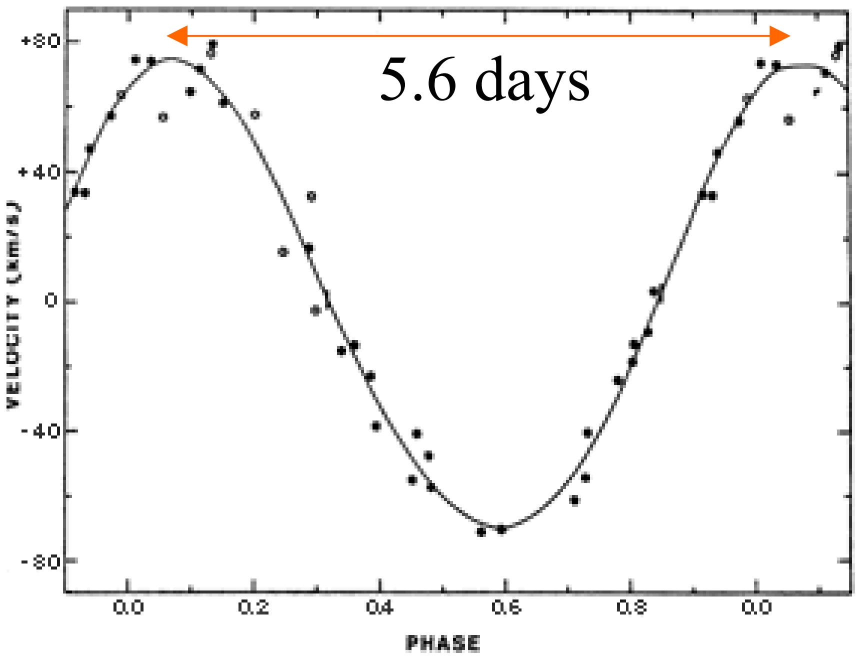
Accretion Disks

Why black holes? (1)

- The limiting mass for a neutron star under generous assumptions is around 3 solar masses.
- Supernova simulations find that stars with masses greater than about 20 solar masses leave remnants greater than 3 solar masses.

Why black holes? (2)

- Cygnus X-1 was one of the first X-ray sources to have been discovered.
- It is associated with a OB supergiant HDE226868 whose radial velocity varies with a 5.6-day period.



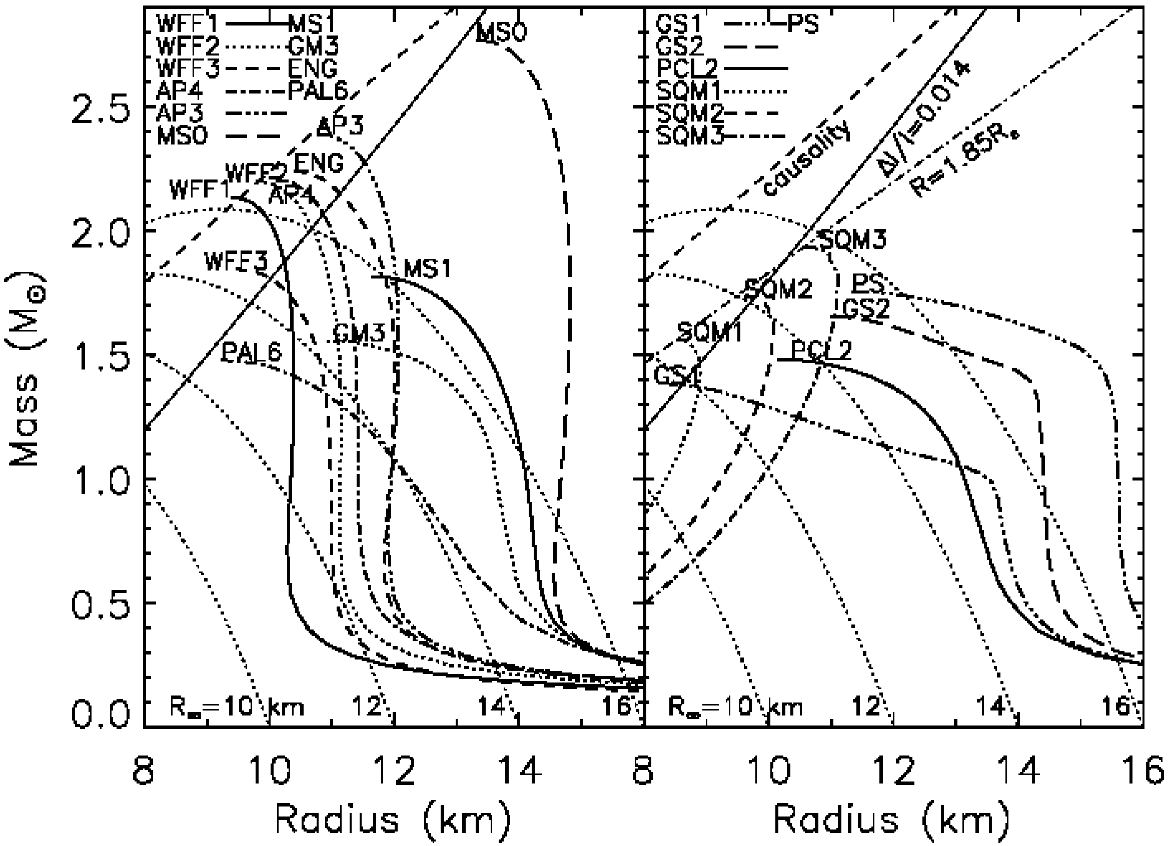
- The X-ray emission also varies with a 5.6-day period.

Why black holes? (3)

- What is the mass of the X-ray source?

Why not a funny neutron star?

- The maximum mass of a neutron star is less than 3 solar masses.
- It could be another type of object but it mustn't have a hard surface otherwise you would get lots more energy.



A Keplerian Disk

- What is the specific angular momentum of material in the disk?

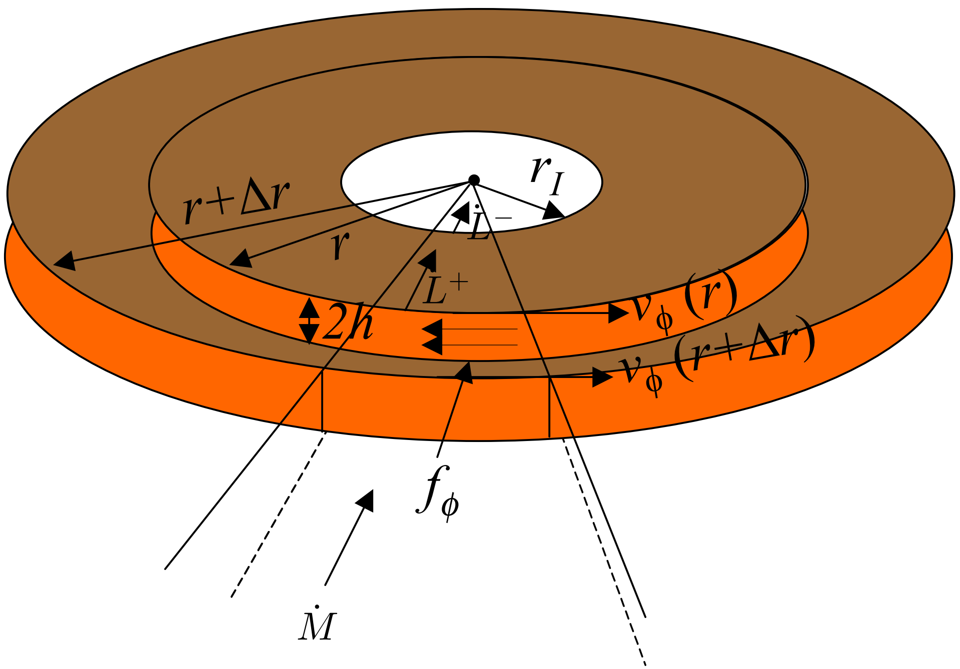
$$\Omega^2 = \frac{GM}{r^3}$$

$$L = mr^2\Omega = m\,(GMr)^{1/2}$$

- Since the mass moves in, so does angular momentum at a rate.

$$\dot{L}^+ = \dot{M}\,(GMr)^{1/2}$$

- Some ends up on the black hole, $\dot{L}^- = \beta\dot{M}\,(GMr_I)^{1/2}$



Angular Momentum (1)

- There is a torque on the material in the accretion disk,

$$\text{torque} = (\text{force along } \hat{\phi}/\text{area}) \times (\text{area}) \times r = \dot{L}^{+} - \dot{L}^{-}$$
$$= f_\phi (2\pi r \cdot 2h)(r) = \dot{M}\left[(GMr)^{1/2} - \beta\,(GMr_I)^{1/2}\right]$$

- f_ϕ is the viscous stress. For rotating disk of fluid we also know that

$$f_\phi = -\eta \frac{d\Omega}{d\ln r} = -\eta r \frac{d}{dr}\left(\sqrt{GM}\, r^{-3/2}\right) = \frac{3}{2}\eta\Omega$$

- Heat is generated by viscosity at the rate (per unit volume) $\dot{Q} = \rho T \dot{s} \approx \frac{\left(f_\phi\right)^2}{\eta}$

Angular Momentum (2)

- Argh!! What is η? It is the coefficient of dynamic viscosity (g cm^{-1} s^{-1}). And we can only guess what it is. But we can make progress anyhow by using both expressions for the stress.

$$2h\dot{Q} = \frac{1}{\eta}\left(\frac{3}{2}\eta\Omega\right)\left\{\frac{1}{2\pi r^2}\dot{M}\left[(GMr)^{1/2} - \beta\,(GMr_I)^{1/2}\right]\right\}$$

$$= \frac{3\dot{M}}{4\pi r^2}\,\frac{GM}{r}\left[1 - \beta\left(\frac{r_I}{r}\right)^{1/2}\right]$$

- This is the heat generated per unit surface area of the disk.

Emission (1)

- Let's assume that the heat is not trapped in the disk but radiated where it is produced

$$F(r) = \frac{1}{2} \times 2h\dot{Q} = \frac{3\dot{M}}{8\pi r^2}\frac{GM}{r}\left[1 - \beta\left(\frac{r_I}{r}\right)^{1/2}\right]$$

- The total luminosity of the disk is

$$L = \int_{r_I}^{\infty} 2F \times 2\pi r dr = \left(\frac{3}{2} - \beta\right)\frac{GM\dot{M}}{r_I}$$

Vertical Structure (1)

- The disk is in hydrostatic equilibrium in the vertical direction against the vertical component of gravity.

$$\frac{1}{\rho}\frac{dP}{dz} = -\frac{GM}{r^2}\frac{z}{r}, \qquad \frac{P_c}{\rho_c h} \approx \frac{GM}{r^3}h$$

$$h \approx \left(\frac{P}{\rho}\right)^{1/2}\left(\frac{r^3}{GM}\right)^{1/2} \approx \frac{c_s}{\Omega}$$

Vertical Structure (2)

- We have assumed that the disk stays thin what does this say about the gas in the disk.

$$\frac{h}{r} \ll 1, \qquad \left(\frac{P}{\rho}\right)^{1/2} \frac{1}{\Omega r} \ll 1$$

$$\left(\frac{2kT}{m_p}\right)^{1/2} \left(\frac{r}{GM}\right)^{1/2} \ll 1$$

$$kT \ll \frac{1}{2} \frac{GMm_p}{r}$$

Here we have assumed that gas pressure dominates but the argument works for a combination of gas and radiation pressure.

- So the disk stays thin if the energy is radiated away. The alternative is like the Bondi solution (a.k.a ADAF - advection dominated accretion flow)

Viscosity (1)

- What is the viscosity? Let’s do some dimensional analysis. η has units of g cm^{-1} s^{-1}, density times velocity time length. Shakura & Sunyaev assumed that viscosity is produced by turbulence in the flow so $\eta \approx \rho v_{\rm turb} l_{\rm turb}$.
- The size of the eddy is limited by the thickness of the disk and the velocity is limited by the sound speed so….

Viscosity (2)

- The viscosity is limited to $\eta < \rho c_s h.$
- Remember that the stress is given by
  $f_\phi = \frac{3}{2}\eta\Omega < \frac{3}{2}\rho c_s h\Omega \approx \rho c_s^2 \approx P$
- In general we have $f_\phi = \alpha P \text{ where } \alpha \leq 1.$
- Models constructed using this model are called "α-disks." Observations of accretion disks give α in the range 0.1-1.

Putting things together

- If we combine the rule for α-disks with the stress in the disk we get

$$\alpha P(2\pi r \cdot 2h)(r) = \dot{M}\left[(GMr)^{1/2} - \beta\,(GMr_I)^{1/2}\right]$$

- Now add the vertical structure

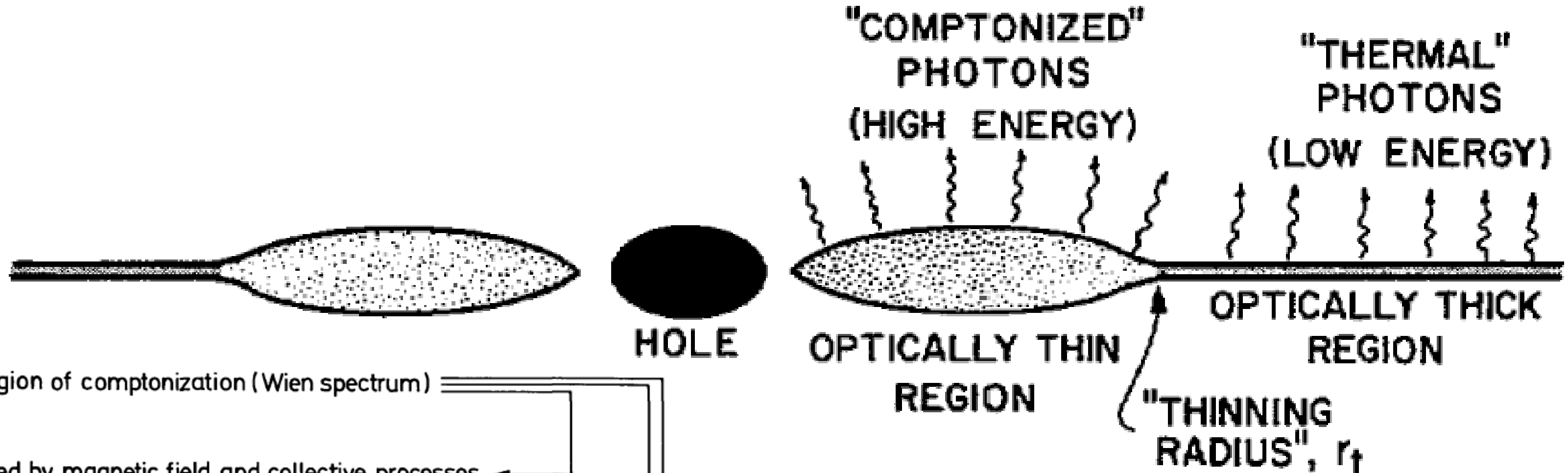
$$\alpha h^2\Omega^2\rho(2\pi r \cdot 2h)(r) = \dot{M}\left[(GMr)^{1/2} - \beta\,(GMr_I)^{1/2}\right]$$

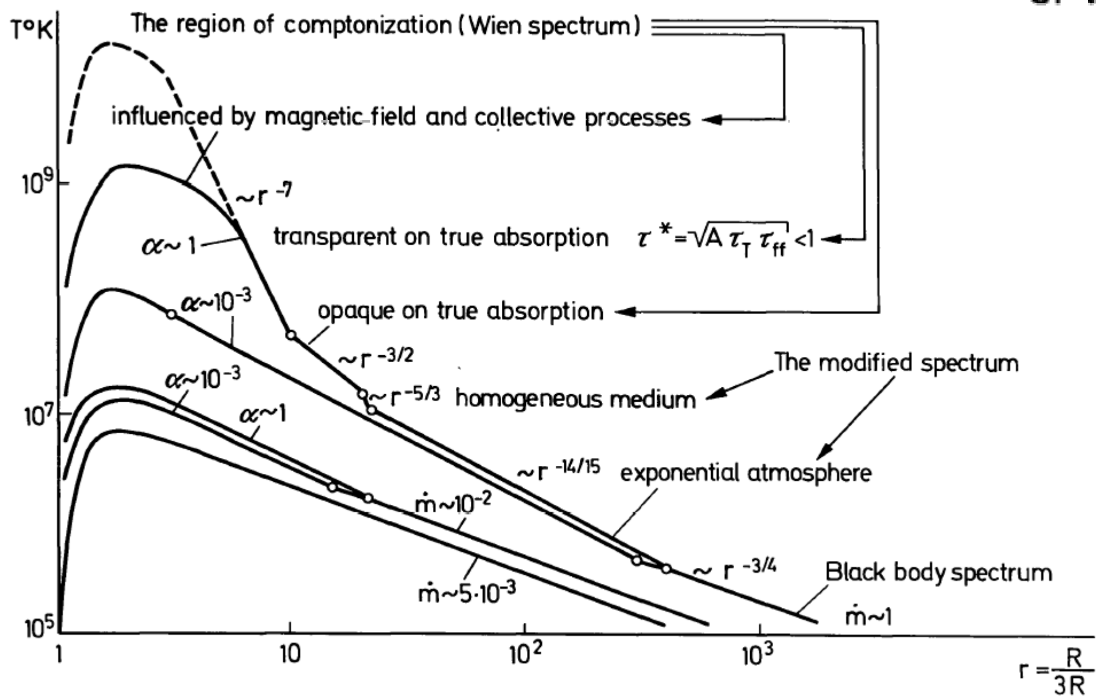
$$h^3 = \frac{\dot{M}}{4\pi r^2\rho}\,\frac{1}{\alpha\Omega^2}\left[(GMr)^{1/2} - \beta\,(GMr_I)^{1/2}\right]$$

$$h^3 = \frac{1}{\alpha}\,\frac{\dot{M}}{4\pi\rho\Omega}\left[1 - \beta\left(\frac{r_I}{r}\right)^{1/2}\right] = \frac{1}{2\alpha}\,\frac{v_r}{r\Omega}r^2h\left[1 - \beta\left(\frac{r_I}{r}\right)^{1/2}\right]$$

- So if α is big the disk is thin. Also if the horizontal velocity gets big, the disk gets flat.

Accretion Disk Structure





Accretion Disk Spectrum