Accreting Neutron Stars

Magnetic field, spin and disks

Spherical Accretion (1)

- The equations of fluid dynamics:
  - Equation of state

$$P = K\rho^\Gamma \quad K, \Gamma \ \ \text{constant}$$

  - Continuity (conservation of stuff)

$$\nabla \cdot (\rho \mathbf{u}) = \frac{1}{r^2}\frac{d}{dr}(r^2 \rho u) = 0$$

  - Euler (conservation of energy)

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2}$$

Spherical Accretion (2)

- Integrals of these equations:
  - Continuity (conservation of stuff)

$$4\pi r^2 \rho u = \dot{M} = \text{constant}$$

  - Euler (conservation of energy): Bernoulli

$$\frac{1}{2}u^2 + \frac{1}{\Gamma - 1}a^2 - \frac{GM}{r} = \text{constant}$$

$$= \frac{1}{\Gamma - 1}a_\infty^2$$

Spherical Accretion (3)

- Working with the continuity equation:

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho u) = \frac{1}{r^2}(2r\rho u + r^2\rho' u + r^2\rho u') = 0$$

$$\frac{\rho'}{\rho} + \frac{u'}{u} + \frac{2}{r} = 0$$

- Working with Euler equation:

$$uu' + a^2\frac{\rho'}{\rho} + \frac{GM}{r^2} = 0$$

$$\text{where } \; a \equiv \left(\frac{dP}{d\rho}\right)^{1/2} = \left(\frac{\Gamma P}{\rho}\right)^{1/2}$$

Spherical Accretion (4)

- We can solve for u' and ρ':

$$u' = \frac{2a^2/r - GM/r^2}{\rho^2 u(u^2 - a^2)} \qquad \rho' = \frac{2u^2/r - GM/r^2}{\rho u^2(u^2 - a^2)}$$

- We would like u to diverge as r goes to zero (like in free-fall), but u starts at zero at infinity, so at some point $u=a$. Unless the numerators vanish too, the derivatives will diverge at this "sonic point."

The Sonic Point (1)

- At the sonic point: $$u^2 = a^2 = \frac{1}{2}\,\frac{GM}{r_s}$$
- Using the Bernoulli equation:

$$\frac{1}{2}a_s^2 + \frac{1}{\Gamma - 1}a_s^2 - 2a_s^2 = \frac{1}{\Gamma - 1}a_\infty^2$$

$$\frac{5 - 3\Gamma}{2\Gamma - 2}a_s^2 = \frac{1}{\Gamma - 1}a_\infty^2$$

$$u_s^2 = a_s^2 = \left(\frac{2}{5 - 3\Gamma}\right) a_\infty^2 \qquad r_s = \left(\frac{5 - 3\Gamma}{4}\right)\frac{GM}{a_\infty^2}$$

The Sonic Point (2)

- We know the velocity and position of the sonic point. Let's calculate the density.

$$a = \left(\frac{\Gamma P}{\rho}\right)^{1/2} = \left(\frac{\Gamma K \rho^\Gamma}{\rho}\right)^{1/2} \qquad \frac{a}{a_\infty} = \left(\frac{\rho}{\rho_\infty}\right)^{\Gamma/2}$$

$$\rho_s = \rho_\infty \left(\frac{a_s}{a_\infty}\right)^{2/(\Gamma-1)} = \rho_\infty \left(\frac{2}{5-3\Gamma}\right)^{1/(\Gamma-1)}$$

$$\dot{M} = 4\pi \rho_s r_s^2 u_s = 4\pi \rho_\infty a_\infty \left(\frac{GM}{a_\infty}\right)^2 \frac{1}{4} \left(\frac{2}{5-3\Gamma}\right)^{(5-3\Gamma)/(2\Gamma-2)}$$

The Whole Bondi Solution

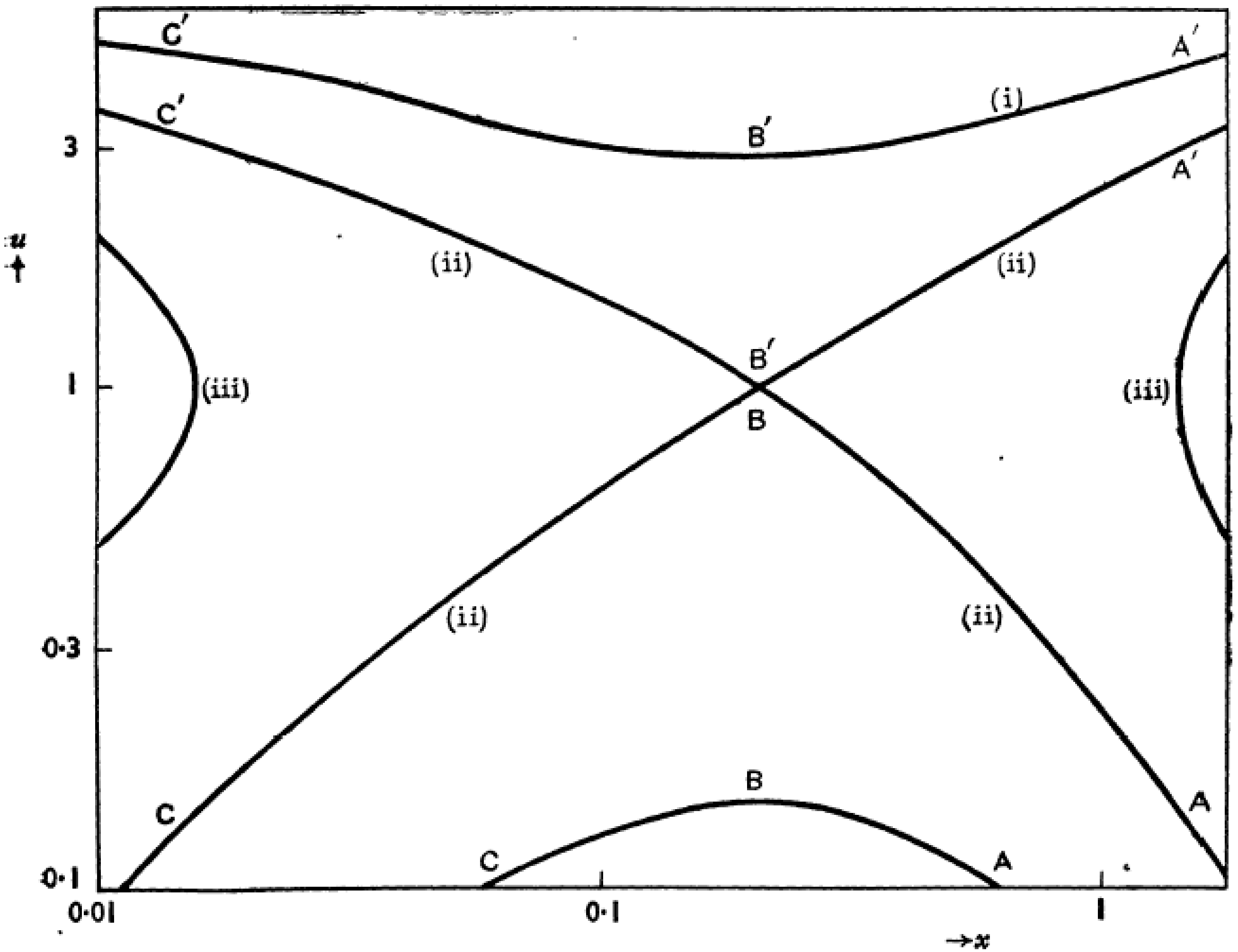


FIG. 2.—*u as function of x for* $\gamma=\frac{7}{5}$.
(i) $\lambda=\frac{1}{4}\lambda_c$;
(ii) $\lambda=\lambda_c$;
(iii) $\lambda=4\lambda_c$.

What is missing?

The work of the present paper, together with previous work, is likely to give a fair estimate of the order of magnitude of accretion in all cases of physical interest. Further progress in this field will probably require the consideration of non-steady states.

Trinity College,
Cambridge :
1951 *October* 2.

What is missing.

- Material falling onto a compact object generally has angular momentum
- A neutron star has two additional complications:
  - A surface
  - A magnetic field

Angular Momentum

- If the material initially has some tangential velocity, this velocity will increase as the material falls toward the star.

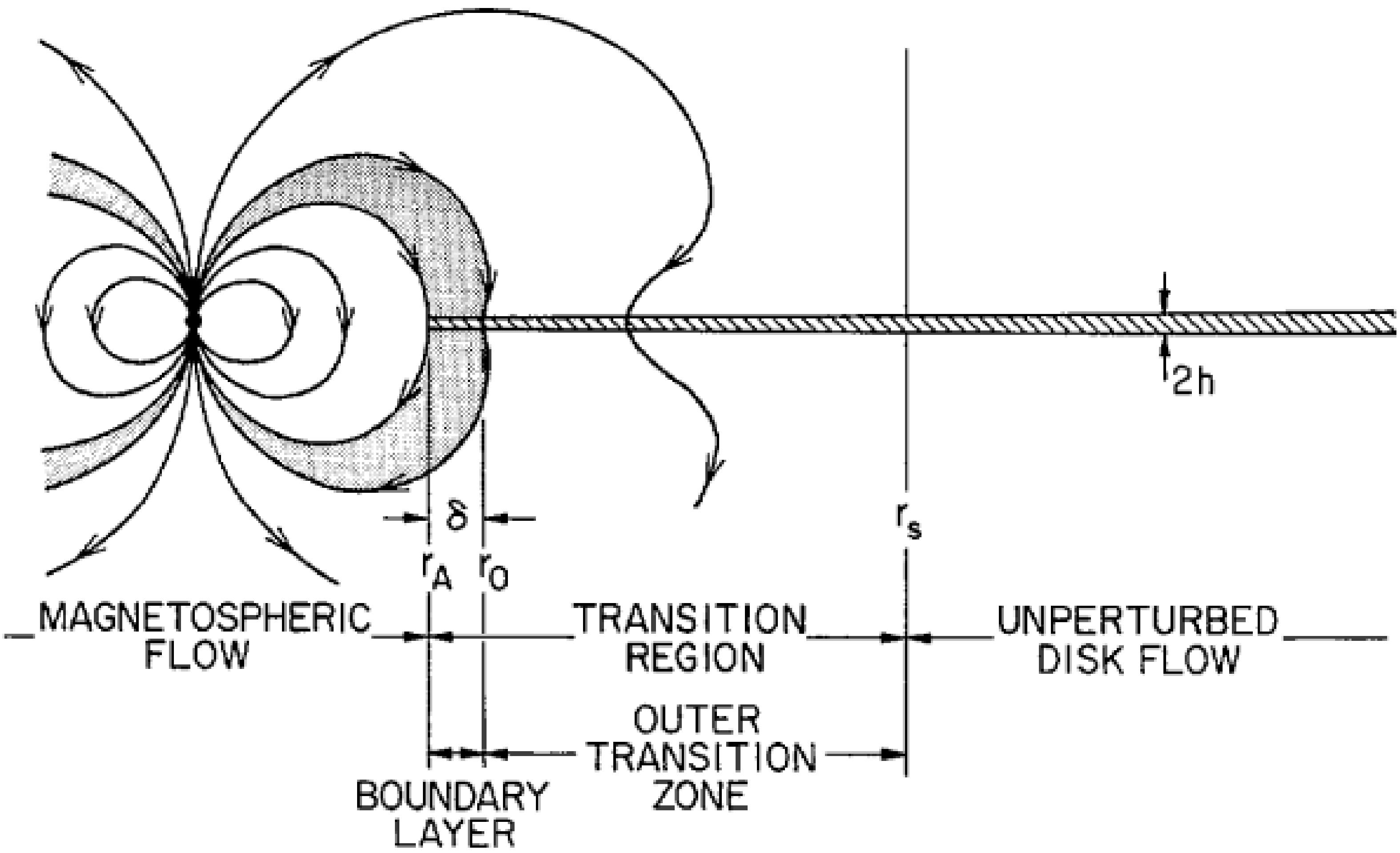
$$L/m = v_\phi r \quad \longrightarrow \quad v_\phi \propto 1/r$$

- The radial velocity if there were no pressure goes as

$$\frac{1}{2} v_r^2 = \frac{GM}{r} \quad \longrightarrow \quad v_r \propto 1/r^{1/2}$$

- So at some radius the swirling motion will become important.

Magnetized Disk



The Regions

- Outside of r_s, there is no magnetic field in the disk.
- Outside of r_0, the disk motion is Keplerian; that is, $v_r = v_z = 0, v_\phi^2 = GM/r.$
- Inside of r_A, there is no disk.

The Alfvén Radius (1)

- Let's assume for now that the flow is spherical and in the transonic regime. The fluid is more or less freely falling:

$$v_r \approx v_{ff} = \left(\frac{2GM}{r}\right)^{1/2} \quad \text{and} \quad \rho = \frac{\dot{M}}{4\pi v r^2}$$

- The magnetic field is given by

$$B \approx \frac{\mu}{r^3} = (10^{12}\mathrm{G})\mu_{30}R_6^{-3}\left(\frac{R}{r}\right)^3$$

The Alfvén Radius (2)

- Let's compare the magnetic energy density to the kinetic energy density of the flow:

$$\frac{B^2(r_A)}{8\pi} = \frac{1}{2}\rho(r_A)v^2(r_A) \longrightarrow \frac{\mu^2}{8\pi r_A^6} = \frac{1}{2}\frac{\dot{M}}{4\pi r_A^2}\left(\frac{2GM}{r_A}\right)^{1/2}$$

$$r_A = \left(\frac{\mu^4}{2GM\dot{M}^2}\right)^{1/7} = \left(\frac{\mu^4 GM}{2L^2R^2}\right)^{1/7}$$

$$= 3.5\times 10^8 L_{37}^{-2/7}\mu_{30}^{4/7}\left(\frac{M}{M_\odot}\right)^{1/7} R_6^{2/7} \text{ cm}$$

Transition Radius (1)

- The angular momentum that remains in the disk at r_0 ends up on the star.

$$\frac{d}{dt}(I\Omega_s) = \dot{M}\left(\frac{L}{m}\right)_0 + N$$

- I is the moment of inertia of the star and corotating magnetosphere. N is any torque that is exerted at r_0.

$$\frac{\dot{P}}{P} = \frac{\dot{M}}{M}\left[\frac{M}{I}\ \frac{dI}{dM} - \left(\frac{L}{m}\right)_0 \frac{M}{I\Omega_s}\right] - \frac{N}{I\Omega_s}$$

Transition Radius (2)

- What is the value of r_0?
  - The disk is 2-D, so the mass flow density is concentrated by a factor of $\sim r/h$, so r_0 should be less than r_A for the spherical case.
  - However, the magnetic field is somewhat concentrated in the disk which mitigates this effect, so $\left(\frac{h}{r}\right)_0^{2/7} r_A \cdot \quad r_0 \cdot \quad r_A$
  - The detailed calculation gives $r_0 \approx 0.5 r_A$

Spin Evolution (1)

- Let's estimate the various terms:

$$I = \frac{0.21 MR^2}{1 - 2GM/Rc^2}, \quad \frac{d \ln I}{d \ln M} = \frac{1}{1 - 2GM/Rc^2}$$

$$\left(\frac{L}{m}\right)_0 = \frac{\Omega r_0^2}{1 - 2GM/r_0 c^2} = \frac{\sqrt{GMr_0}}{1 - 2GM/r_0 c^2}$$

$$\frac{I\Omega_s}{M} = \frac{0.21 \Omega_s R^2}{1 - 2GM/Rc^2}$$

- So the second term in the brackets is larger than the first.

Spin Evolution (2)

- Let's ignore the relativity bits, the torque and the first term to get:

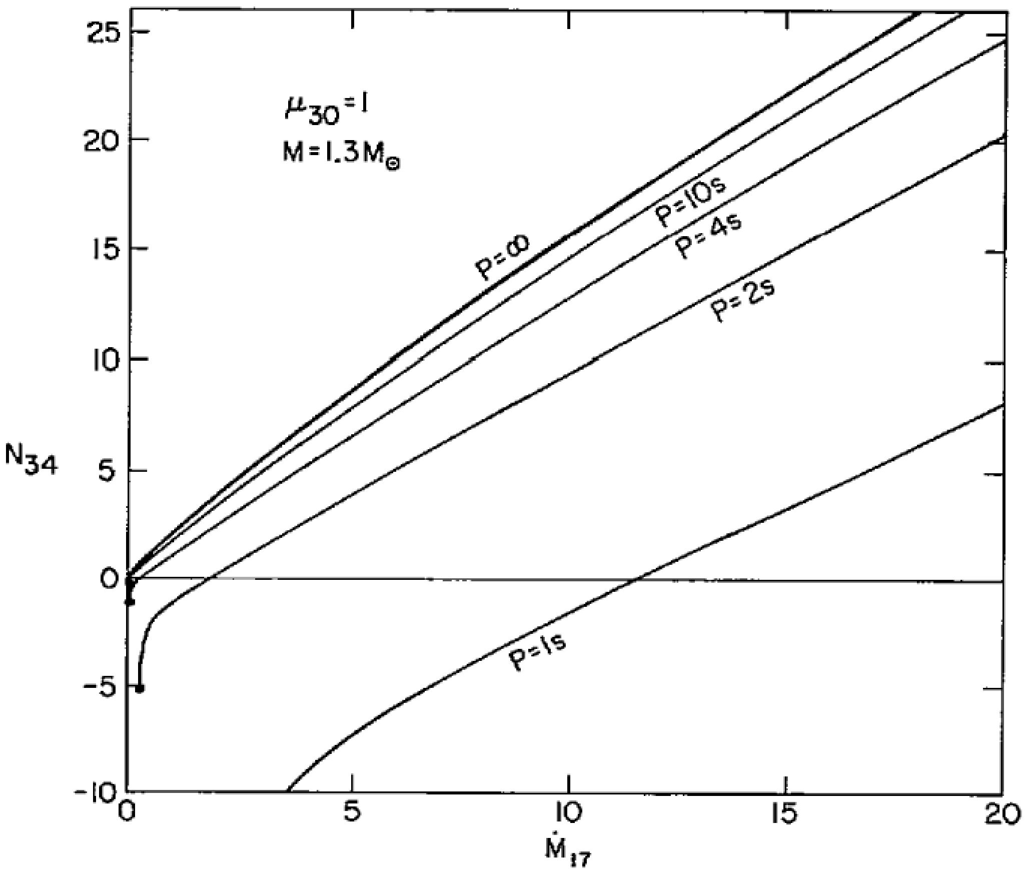
$$\frac{\dot{P}}{P} = -\frac{(GMr_0)^{1/2}\,\dot{M}}{I\Omega_s} = -\left(\frac{\mu^4}{2GM\dot{M}^2}\right)^{1/14}\frac{(0.5GM)^{1/2}\,\dot{M}}{I\Omega_s}$$

$$\dot{P} = -\frac{P^2\mu^{2/7}}{4\pi I}\left(\frac{2L^2R^2}{GM}\right)^{3/7}$$

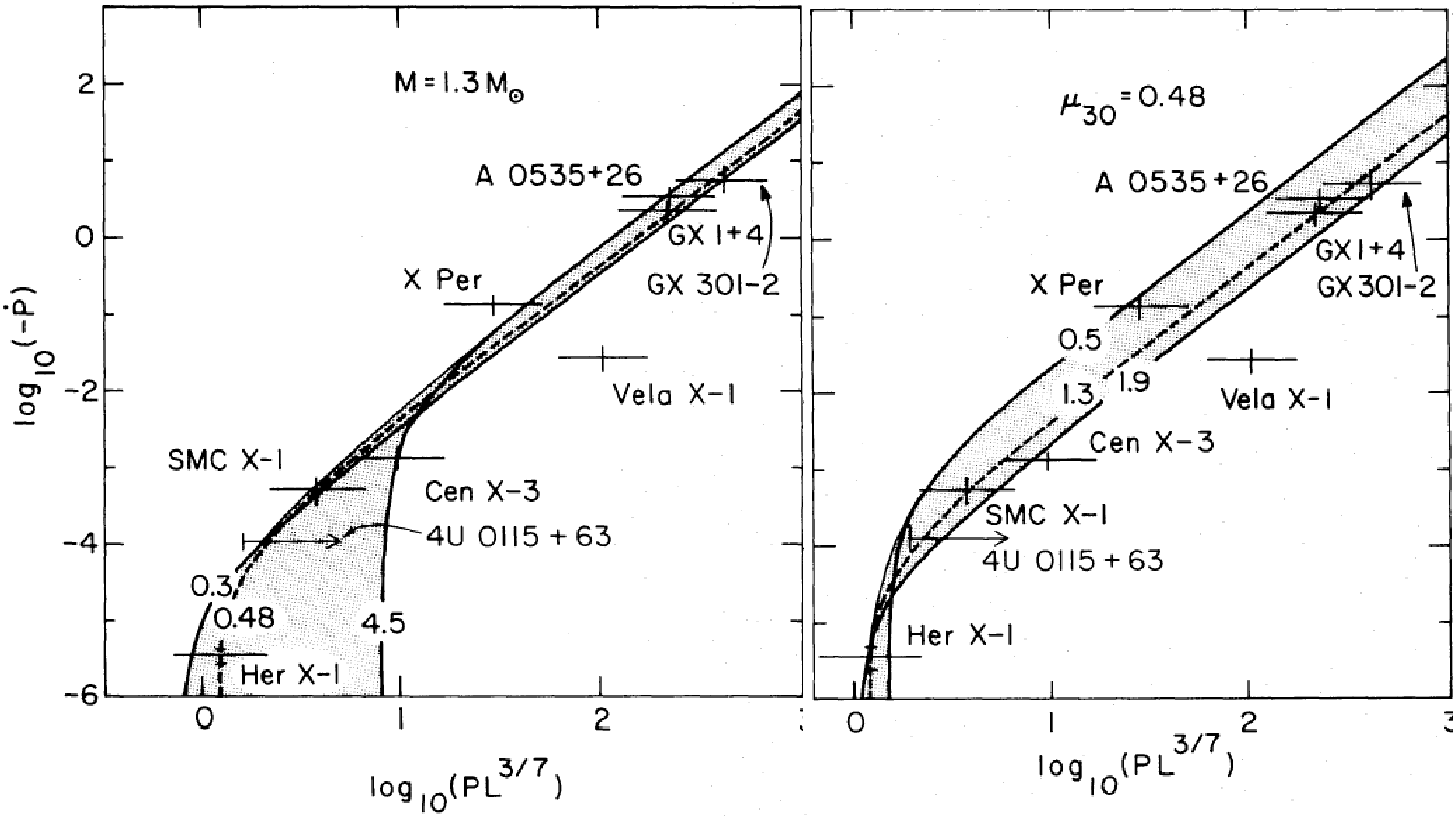
$$= 5.8\times 10^{-5}\left[\mu_{30}^{2/7}R_6^{6/7}\left(\frac{M}{M_\odot}\right)^{-3/7}I_{45}^{-1}\right]\left(PL_{37}^{3/7}\right)^2\ \mathrm{s}\ \mathrm{yr}^{-1}$$

Spin Evolution (3)

- What about that torque *N* that we neglected?
- If Ω_s & Ω_K , then the star is a "fast rotator" and the torque is large.
- If $\Omega_s \gg \Omega_K$, then it is a propeller.



Observations of X-ray PSRs



Radiation from Accreting Neutron Stars (1)

- The disk itself radiates over a wide range of energies; we will see this in a few weeks.
- The highest energy emission comes from where the gas hits the surface of the neutron star (it also accounts for more than one half of the luminosity if $r_A > R$, why?)

Radiation from Accreting Neutron Stars (2)

- Let's assume that the matter is channeled onto a polar cap so,
$$L = \Omega R^2 \sigma T^4 \rightarrow T = 1 \ \mathrm{keV} R_6^{-1/2} L_{37}^{1/4} \left(\frac{\Omega}{4\pi}\right)^{-1/4}$$
- Let's estimate Ω. The opening angle of the cap corresponds to field lines that have $r_{\max} \sim r_0$.
- Remember that $\frac{\sin^2\theta}{r}$ is constant along a field line.

Radiation from Accreting Neutron Stars (3)

- Let us assume that $r_0 \gg R$, so

$$\sin^2\theta = \frac{R}{r_0}, \quad \theta \approx \left(\frac{R}{r_0}\right)^{1/2}$$

- The solid angle is

$$\Omega = 2\int_0^\theta 2\pi \sin\theta d\theta = 4\pi(1-\cos\theta) \approx 2\pi\theta^2$$

- Therefore, $T = 1 \ \mathrm{keV} R_6^{-1/2} L_{37}^{1/4} \left(\frac{r_A}{R}\right)^{1/4}$

$$= 4.3 \ \mathrm{keV} L_{37}^{5/28} \mu_{30}^{1/7} \left(\frac{M}{M_\odot}\right)^{1/28} R_6^{19/28}$$

Radiation from Accreting Neutron Stars (4)

- The collimation of the flow increase the energy of the emission significantly, but more importantly, it makes the emission pulsed with the rotation period of the neutron star.   We have an X-ray pulsar!

What happens now?

- Accretion deposits lots of hydrogen and helium (**fuel**) onto the hot surface of a neutron star.
- Shell burning on stars is general unstable, so the nuclear energy emerges in bursts.
- What would happen if the energy emerged continually?

High-T Hydrogen Burning

- At high temperatures, hydrogen burning proceeds by the CNO-cycle:

$$\begin{tikzcd}
 & {}^{14}\mathrm{O} \arrow[ddr, bend left, "{101\mathrm{s}\ (\beta^+)}"] & \\
{}^{12}\mathrm{C} \arrow[r, "{(p,\gamma)}"] & {}^{13}\mathrm{N} \arrow[u] \arrow[r, "{(\beta^+)\ 863\mathrm{s}}"] & {}^{13}\mathrm{C} \arrow[d, "{(p,\gamma)}"] \\
{}^{15}\mathrm{N} \arrow[u, "{(p,\alpha)}"] & {}^{15}\mathrm{O} \arrow[l, "{(\beta^+)\ 176\mathrm{s}}"'] & {}^{14}\mathrm{N} \arrow[l, "{(p,\gamma)}"']
\end{tikzcd}$$

Beta-limited H-burning

- At sufficiently high temperatures, the proton captures take little time compared to the beta decays, so it takes a set time for a single CNO-nucleus to process 4 protons into 1 alpha particle
  - 277 s above $10^{7.8}$ K (hot-CNO)
  - 1039 s is the minimum time for regular CNO
- In a solar mix, you have 350 protons per CNO nucleus - at least 6 (25) hours to burn all of the hydrogen.

rp-process

- If it is really hot, the material "breaks out" of the *CNO*-cycle into the *rp*-process in which nuclei up to and beyond iron capture protons (potential presentation topic).
- This doesn't occur during the pile-up but occurs often during the bursts.

Type-I X-ray Bursts

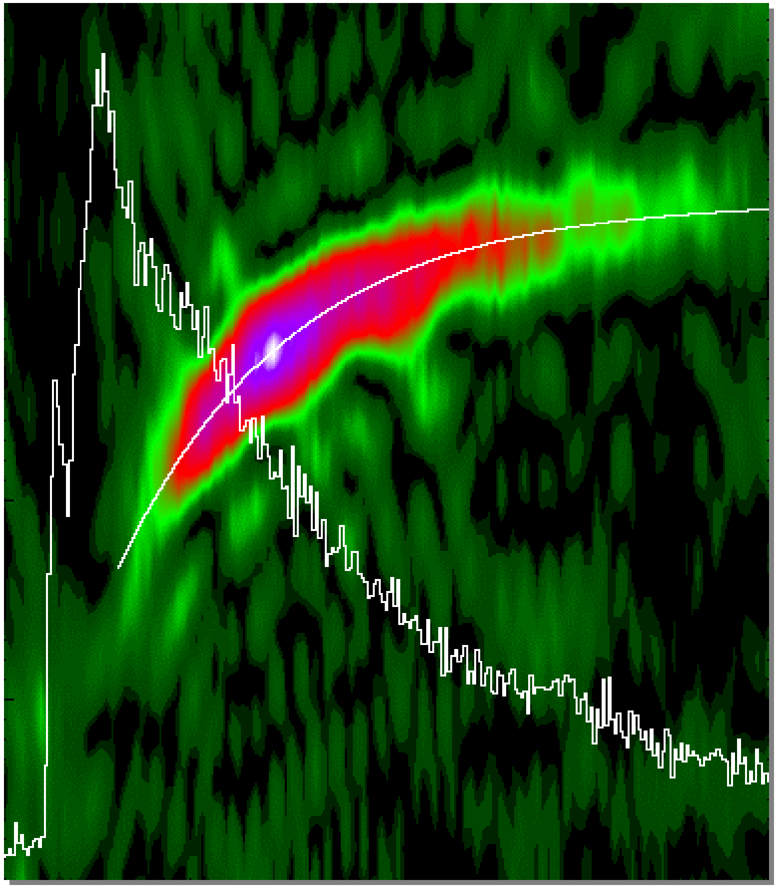
- The bursts of unstable hydrogen and helium burning are called Type-I X-ray Bursts.
  - Type-II X-ray bursts occur when a big glob of material falls suddenly onto the surface of a neutron star and forms and pair-plasma fireball (like an SGR burst)

Different Types of Bursts

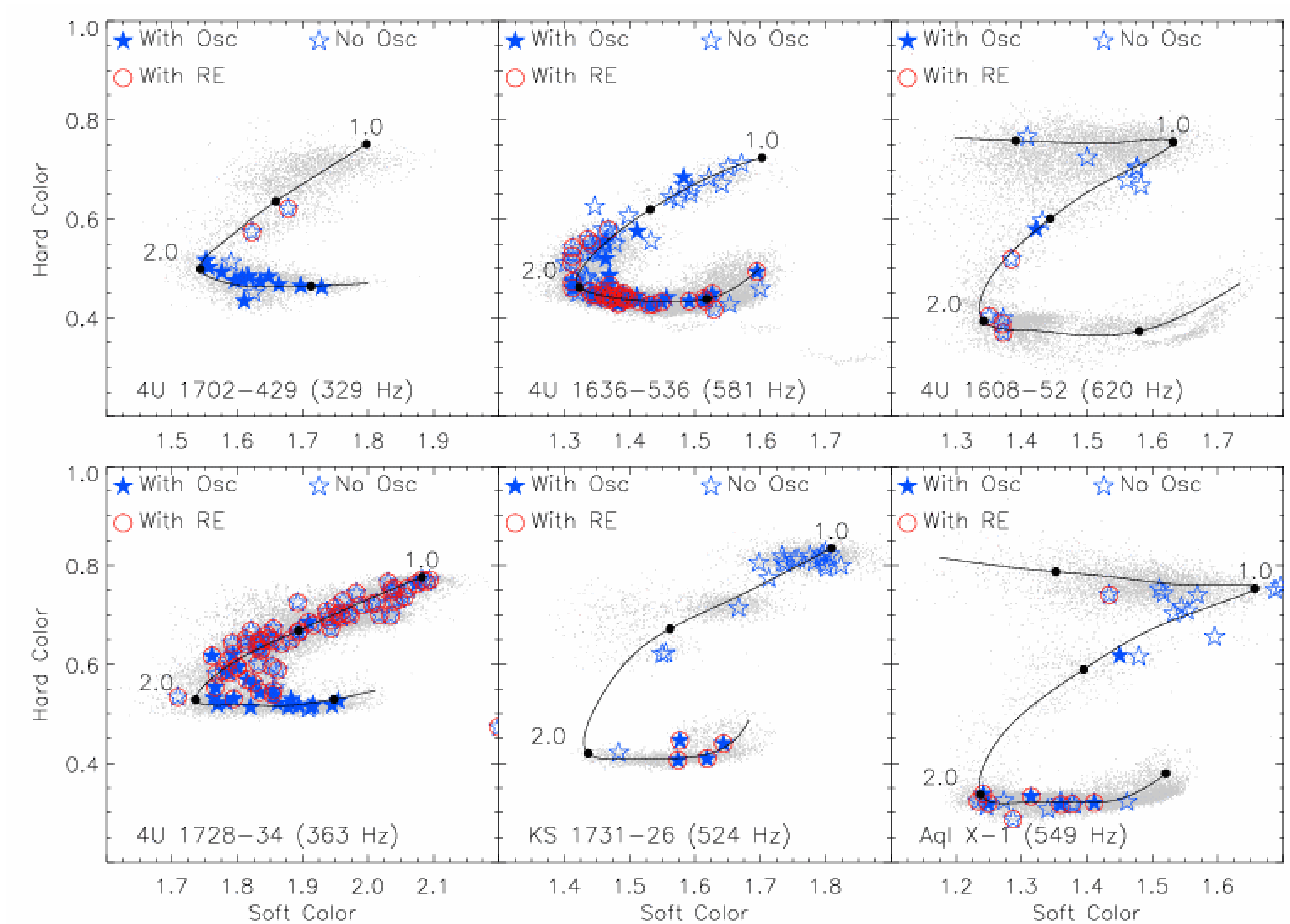
- At really low accretion rates, hydrogen burning is unstable.
- At low accretion rates, it takes a long time to accumulate an unstable layer so the hydrogen is exhausted: helium bursts.
- At higher accretion rates, there is hydrogen left: mixed bursts.
- At accretion rates approaching the Eddington rate, the burning becomes stable.

How does a burst appear?

- The burst begins at a point and spreads over the surface in about one second.
- The flux during the burst varies periodically.
- During the cooling portion of the burst, the observed frequency increases by about one Hertz.

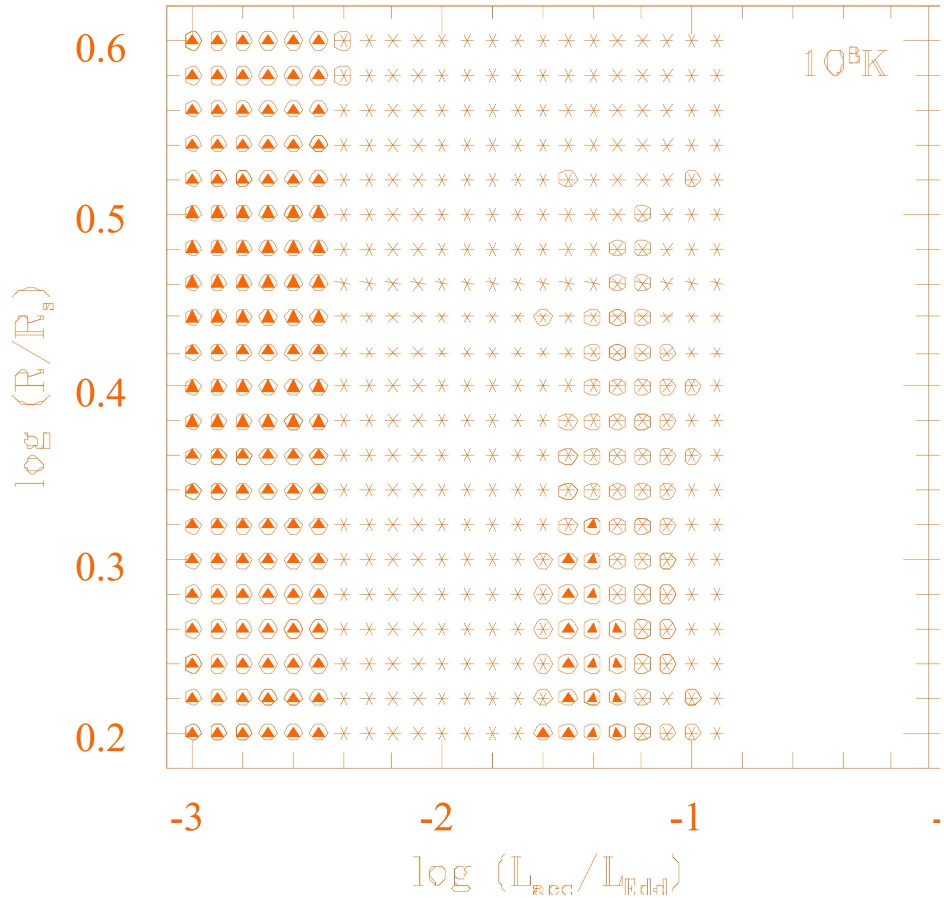


Radius-Expansion Bursts



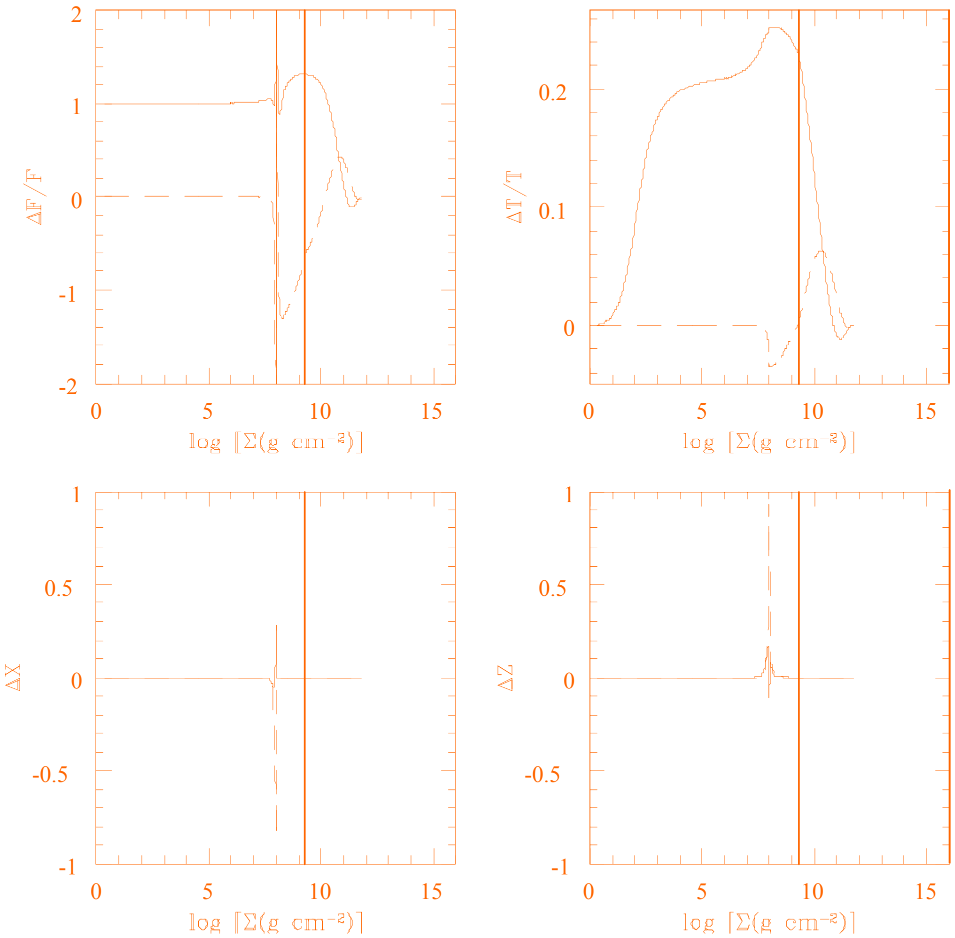
Burst Regimes

- We find the same major burst regimes:
  - H bursts
  - He bursts
  - Mixed bursts
- Prompt v. delayed bursts
- Unstable v. overstable triggers

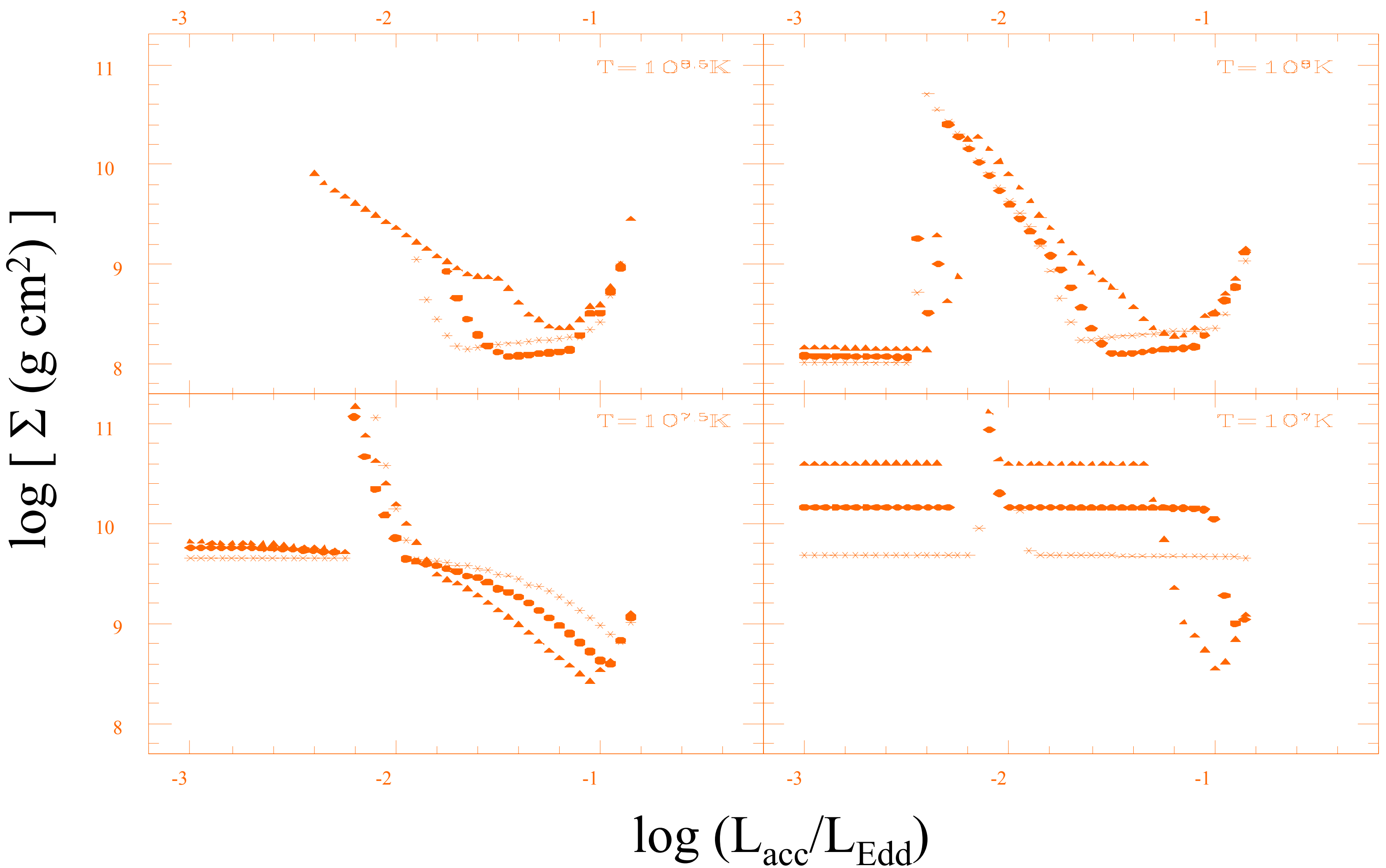


How do we figure it out?

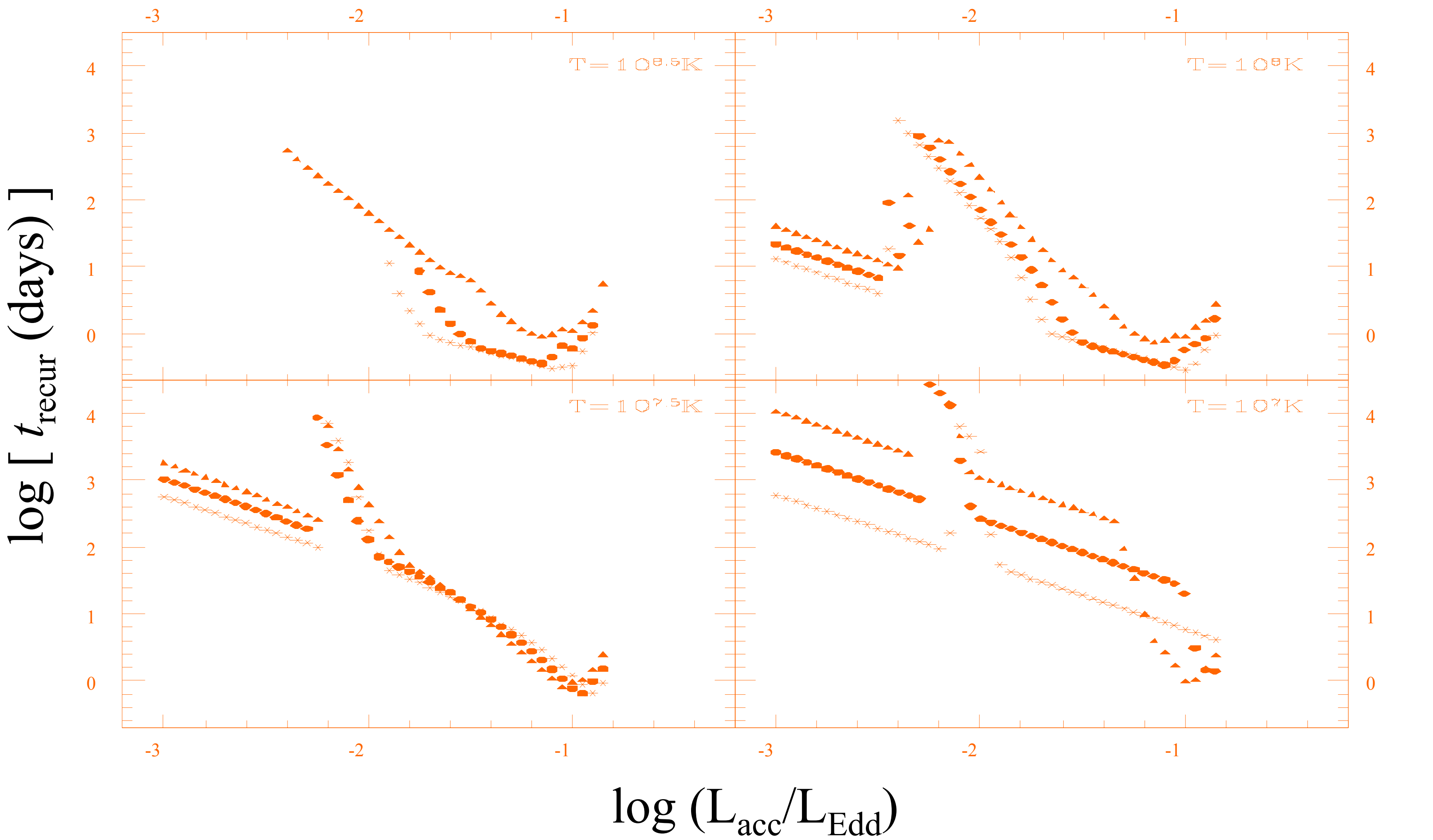
- Perturb the outgoing surface flux,
- Calculate the new solution [with $f_1 e^{\gamma t}$],
- Vary γ until $T_1=0$ at a depth where the diffusion time is $1/|\gamma|$.
- Bonus: $F_1=0$ too



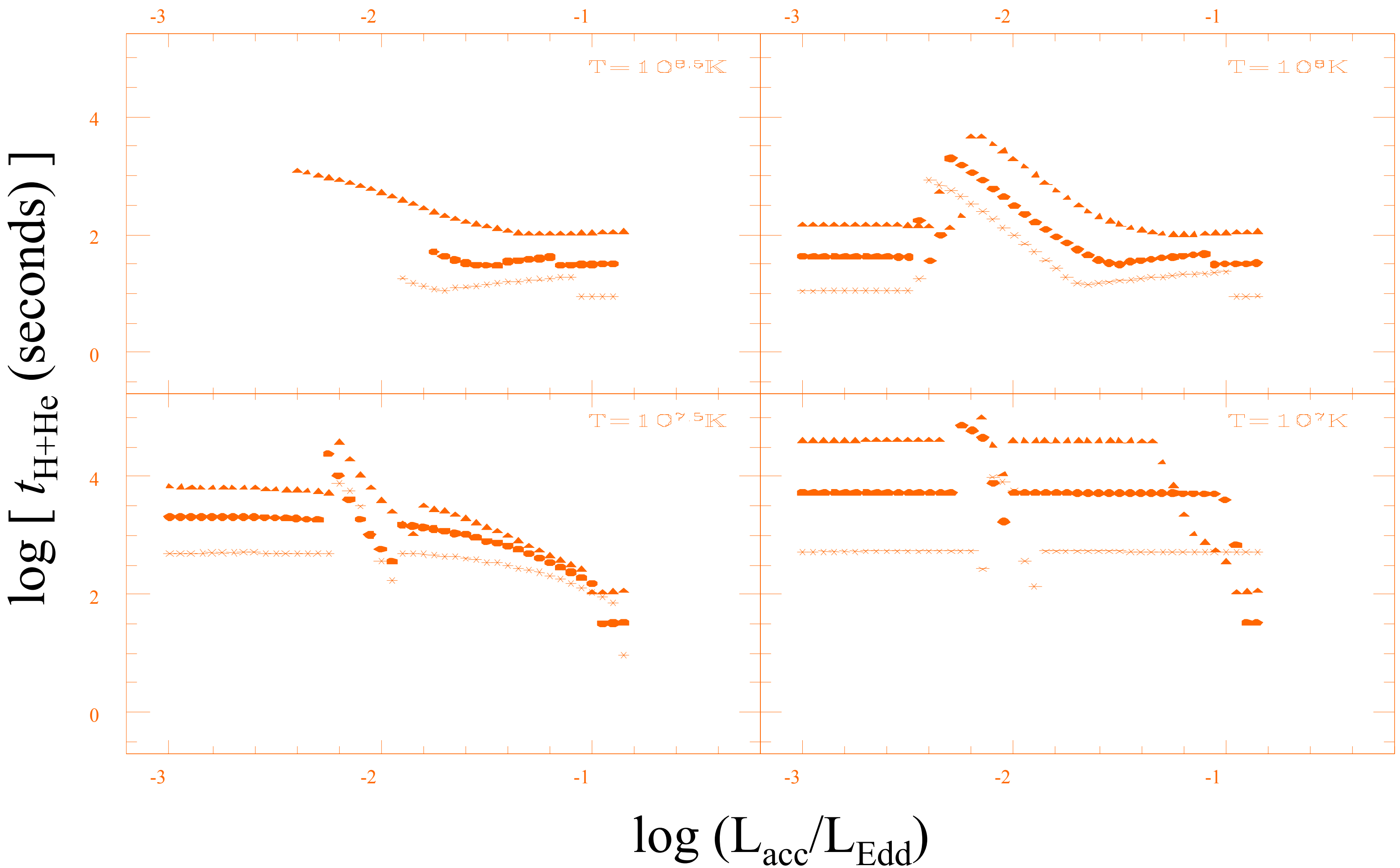
Critical Column Densities



Recurrence Time



Burst Duration



Observational Comparison (α)

- α is the ratio of the persistent flux to the flux during the burst
- Above 3% L_{Edd} we find the same trends in α as observed.
- Below 3% L_{Edd} we can't account for the low values of α.