Cooling Neutron Stars

What we actually see.

The Equilibrium

- We discussed the equilibrium in neutron star cores through this reaction (direct Urca). $n \rightleftharpoons p + e^- + \bar{\nu}_e$

$$\mu_n = \mu_p + \mu_e + \mu_{\bar{\nu}_e}$$

- Does the reaction actually occur?

$$E_n = E_p + E_e + E_{\bar{\nu}_e}$$

$$\mu_n + kT \approx \mu_p - kT + \mu_e - kT + E_{\bar{\nu}_e}$$

$$E_{\bar{\nu}_e} \sim kT$$

Why?

- People assumed that the first neutron stars to be discovered would be glowing embers of a supernova explosion, slowly releasing the heat generated by the cataclysm that created them.
- How does a neutron star cool?
- How bright is its surface?

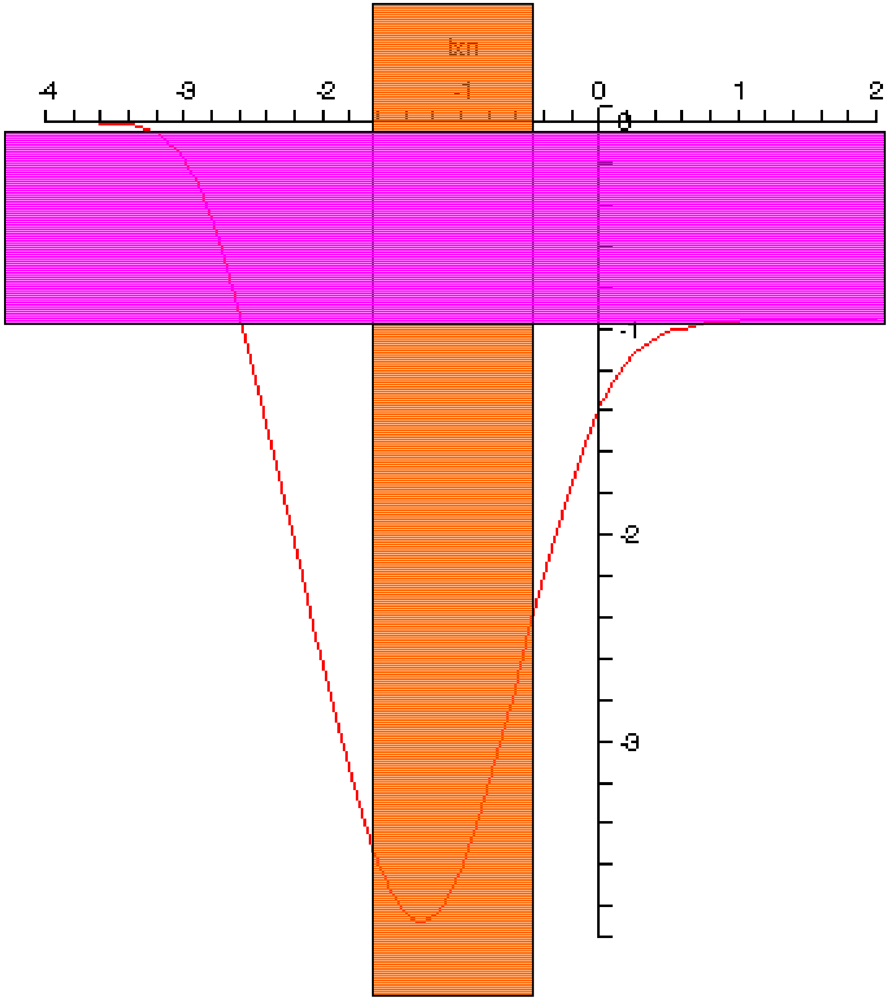
Conservation Laws

- Energy: $$E_n = E_p + E_e + E_{\bar{\nu}_e}$$
  $$\mu_n + kT \approx \mu_p - kT + \mu_e - kT + E_{\bar{\nu}_e}$$
  $$E_{\bar{\nu}_e} \sim kT$$
- Momentum: $$\mathbf{p}_n = \mathbf{p}_p + \mathbf{p}_e + \mathbf{p}_{\bar{\nu}_e}$$
  $$p_n \leq p_p + p_e + p_{\bar{\nu}_e}$$
  $$p_{F,n} \leq p_{F,p} + p_{F,e} + \frac{kT}{c}$$
- Remember that the neutron is a bit above the Fermi sea and the others are a bit below.

Can momentum be conserved?

- If the fraction of protons equals or exceeds 1/8 the number of neutrons, momentum can be conserved.
- Low densities and high densities, but not nuclear densities.

$$x_n \approx 0.4 \left(\frac{\rho}{\rho_n}\right)^{1/3}$$



Where is Urca?

What happens at nuclear densities?

- Modified Urca reaction:

$$n + n' \rightleftharpoons p + n' + e^- + \bar{\nu}_e$$

$$\mathbf{p}_n + \mathbf{p}_{n'} = \mathbf{p}_p + \mathbf{p}_{n'} + \mathbf{p}_e + \mathbf{p}_{\bar{\nu}_e}$$

$$p_n - p_{n'} \leq p_p + p_{n'} + p_e + p_{\bar{\nu}_e}$$

$$0 \leq p_{F,p} + p_{F,n} + p_{F,e} + \frac{kT}{c}$$

- This last inequality is indeed true! Modified Urca is a go!

What about quark matter?

- In quark matter, the quarks are relativistic (at least the ups and downs) so

$$d \leftrightharpoons u + e^- + \bar{\nu}_e$$

can occur and produce neutrinos.

Reaction rates

- Fermi's Golden Rule:

$$\lambda_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 \rho_f$$

Transition probability

Matrix element for the interaction

Density of final states

$$\epsilon_\nu \sim E_\nu \lambda_{if} f_i n$$

Energy loss rate

Energy of neutrino

Fraction of particles in initial state

Getting the scaling! (1)

- The density of states of the outgoing neutrino is $\propto (kT)^2$ and its energy is $\propto kT$.
- The number of states for the outgoing proton and electron are:

$$4\pi {p_F}^2 \frac{dp}{dE} kT \frac{g}{h^3} = 4\pi p_F \mu kT \frac{g}{h^3 c^2}$$

- The fraction of initial particles near the Fermi energy. $\dfrac{3\mu kT}{p_F c^2}$

Getting the scaling! (2)

- Let's drop all dimensionless numbers to make it easier.
- Direct Urca: $\epsilon_\nu \propto (kT)^3 p_{F,e}\mu_e (kT) p_{F,p}\mu_p (kT)\frac{\mu_n kT}{p_{F,n}^2 c^2}$

$$\propto (kT)^6 \frac{p_{F,e}p_{F,p}\mu_e\mu_p\mu_n}{p_{F,n}^2 c^2}$$

- Quark Urca: $\epsilon_\nu \propto (kT)^6 n$

Getting the scaling! (3)

- Modified Urca:

$$\epsilon_\nu \propto (kT)^3 p_{F,e}\mu_e(kT)p_{F,p}\mu_p(kT)p_{F,n}\mu_n(kT)\left(\frac{\mu_n kT}{p_{F,n}^2 c^2}\right)^2$$

$$\propto (kT)^8 \frac{p_{F,e}p_{F,p}p_{F,n}\mu_e\mu_p\mu_n^3}{p_{F,n}^4 c^4} \propto T^8$$

The Results

- Quark Urca:

$$L_\nu^{\mathrm{quark}} \approx (1.3 \times 10^{44} \mathrm{erg\ \ s}^{-1}) \frac{M}{M_\odot} T_9^6$$

- Modified Urca:

$$L_\nu^{\mathrm{Urca}} \approx (5.3 \times 10^{39} \mathrm{erg\ \ s}^{-1}) \frac{M}{M_\odot} \left(\frac{\rho_{\mathrm{nuc}}}{\rho} \right)^{1/3} T_9^8$$

Photon Cooling (1)

- The luminosity from the surface of a neutron star

$$L_\gamma = 4\pi R^2 \sigma T_e^4 = 7 \times 10^{36} \mathrm{erg\ \ s}^{-1} \left(\frac{R}{10\mathrm{km}}\right)^2 T_{e,7}^4$$

- It is this radiation that we may observe.
- The photons diffuse through the envelope of the star.
- You can define a conductivity:

$$F = -\tilde{\kappa}\frac{dT}{dr}$$

Photon Cooling (2)

- In astrophysics is customary to use the opacity. $\frac{dT}{dr} = -\frac{3 \quad \kappa\rho \quad L}{4acT^3 4\pi r^2}$

- Let's combine this with the hydrostatic equilibrium equation:

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2}$$

$$\frac{dP}{dT} = \frac{4ac}{3}\frac{T^3}{\kappa}\frac{4\pi GM}{L} = \frac{4aT^3}{3}\frac{\sigma_T}{m_p\kappa}\frac{L_{\mathrm{Edd}}}{L}$$

Photon Cooling (3)

- Combine this with the EOS and opacity.

$$P = \frac{\rho}{\mu m_u} kT, \quad \text{and} \quad \kappa = \kappa_0 \rho T^{-7/2}$$

- Yields the following:

$$\frac{dP}{dT} = \frac{4a}{3} \frac{\sigma_T}{m_p \kappa_0} \frac{L_{\rm Edd}}{L} \frac{T^{13/2}}{\rho} = \frac{4a}{3} \frac{\sigma_T}{m_p \kappa_0} \frac{L_{\rm Edd}}{L} \frac{k}{\mu m_u} \frac{T^{15/2}}{P}$$

$$P^2 = \frac{4}{17} \frac{4a}{3} \frac{\sigma_T}{m_p \kappa_0} \frac{L_{\rm Edd}}{L} \frac{k}{\mu m_u} T^{17/2}$$

$$\left(\frac{k}{\mu m_u} \rho T \right)^2 = \frac{4}{17} \frac{4a}{3} \frac{\sigma_T}{m_p \kappa_0} \frac{L_{\rm Edd}}{L} \frac{k}{\mu m_u} T^{17/2}$$

Photon Cooling (4)

- Rearranging: $$\rho^2 = \frac{4}{17}\frac{4a}{3}\frac{\sigma_T}{m_p \kappa_0}\frac{L_{\rm Edd}}{L}\frac{\mu m_u}{k} T^{13/2}$$
- To keep things simple let's assume that the temperature where the electrons become degenerate is the core temperature. At this point, $T_F = T$. $$kT_F = \frac{p_F^2}{2m_e}, \rho = \frac{8\pi}{3h^3} p_F^3 (\mu_e m_u)$$
- I have assumed that the electrons are not relativistic: $k\,T \ll m_e\,c^2$, $T \ll 5.93 \times 10^9$ K.

Photon Cooling (5)

- Substituting the equation for ρ in terms of T_F:

$$\left(\frac{8\pi}{3h^3}\right)^2 (2m_e kT)^3 (\mu_e m_u)^2 = \frac{4}{17}\ \frac{4a}{3}\ \frac{\sigma_T}{m_p \kappa_0}\ \frac{L_{\rm Edd}}{L}\ \frac{\mu m_u}{k}\ T^{13/2}$$

$$\frac{L}{L_{Edd}} = 7 \times 10^{-32} \frac{1}{Z(1+X)}\ \frac{\mu}{\mu_e^2} \left(\frac{T}{1K}\right)^{7/2}$$

$$L_\gamma = 9.3 \times 10^6 \ \text{erg} \ \text{s}^{-1} \frac{1}{Z(1+X)}\ \frac{\mu}{\mu_e^2} \left(\frac{T}{1K}\right)^{7/2}$$

$$= 1.6 \times 10^5 \ \text{erg} \ \text{s}^{-1} \left(\frac{T}{1K}\right)^{7/2} \ \text{for iron}$$

$$T_{e,7} = T_9^{7/8} \left(\frac{R}{10 \ \text{km}}\right)^{-1/2}$$

Heat Capacity

- Let's estimate the heat capacity of a neutron star. $C_v = \frac{\pi^2(x^2+1)^{1/2}}{x^2} Nk \left(\frac{kT}{mc^2}\right)$

$$x_n \approx 0.4 \left(\frac{\rho}{\rho_n}\right)^{1/3}$$

- Integrating up from zero temperature

$$U_n = 31 \left(\frac{\rho}{\rho_{\rm nuc}}\right)^{-2/3} \frac{M}{m_n} \left(\frac{k^2 T^2}{m_n c^2}\right)$$

$$= 6 \times 10^{47} \mathrm{erg} \frac{M}{M_\odot} \left(\frac{\rho}{\rho_{\rm nuc}}\right)^{-2/3} T_9^2$$

Thermal Evolution (1)

- We can now determine how the inside of the star cools.

$$\frac{dU}{dt} = -\left(L_\nu + L_\gamma\right)$$

- These equations look a lot like the ones for the spin.
- If we assume that only one neutrino or photon process dominates, they are integrable.

Thermal Evolution (2)

- Modified Urca:

$$\Delta t \simeq 1 \ \ \mathrm{yr} \left(\frac{\rho}{\rho_{\mathrm{nuc}}}\right)^{-1/3} T_9^{-6} \left\{1 - \left[\frac{T_9}{T_{9,i}}\right]^6\right\}$$

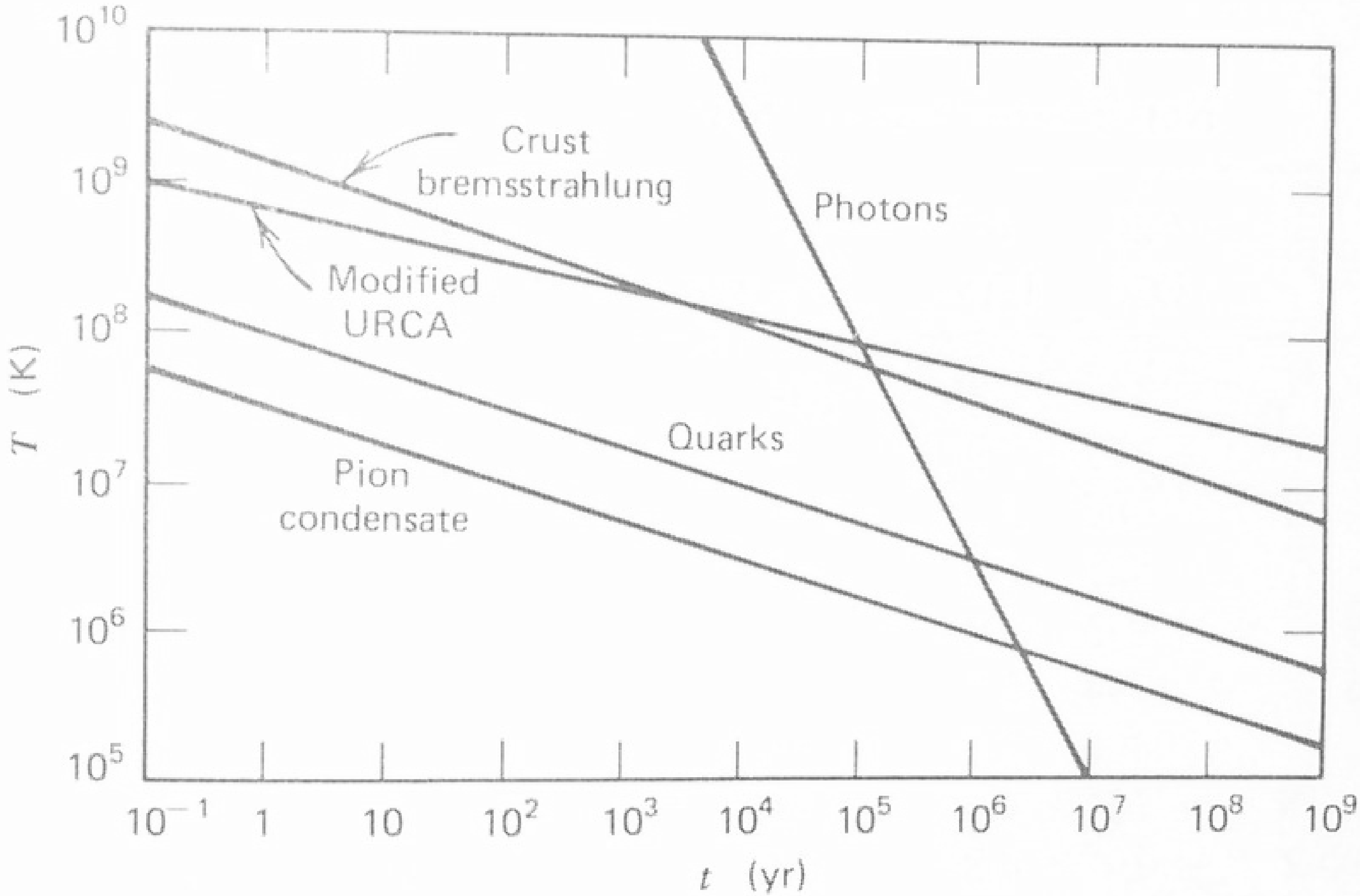
- Quark Urca:

$$\Delta t \simeq 1 \ \ \mathrm{hr} \left(\frac{\rho}{\rho_{\mathrm{nuc}}}\right)^{-1/3} T_9^{-4} \left\{1 - \left[\frac{T_9}{T_{9,i}}\right]^4\right\}$$

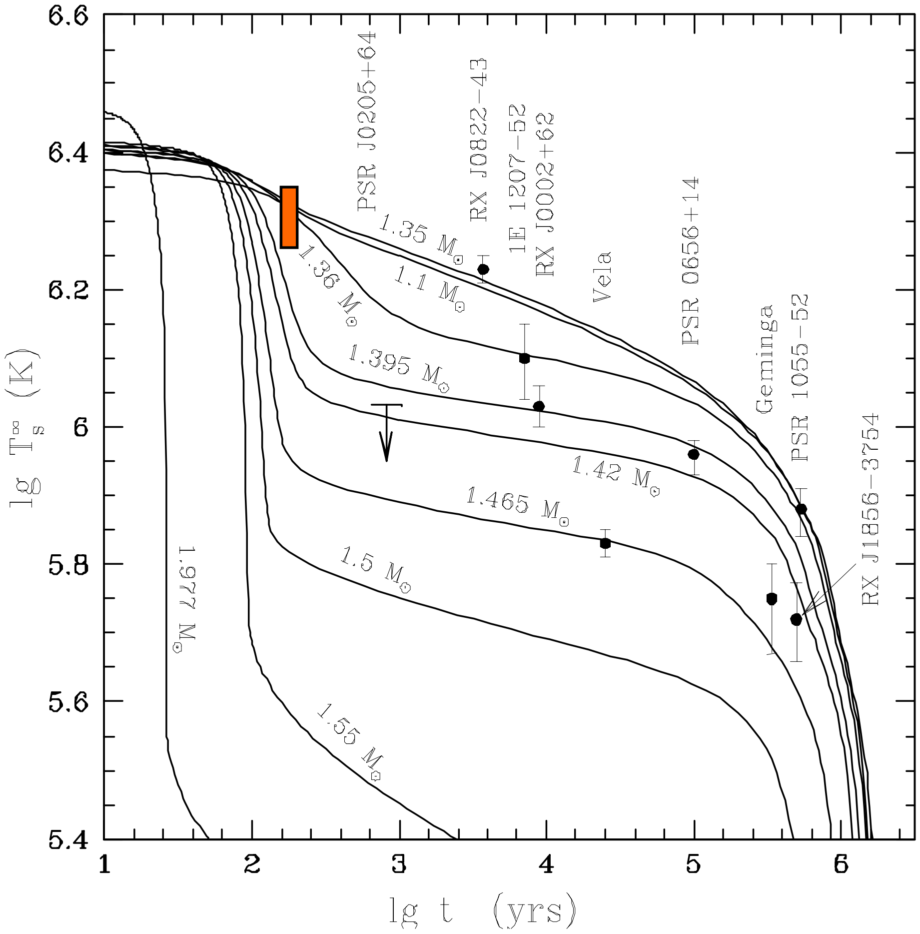
- Photons:

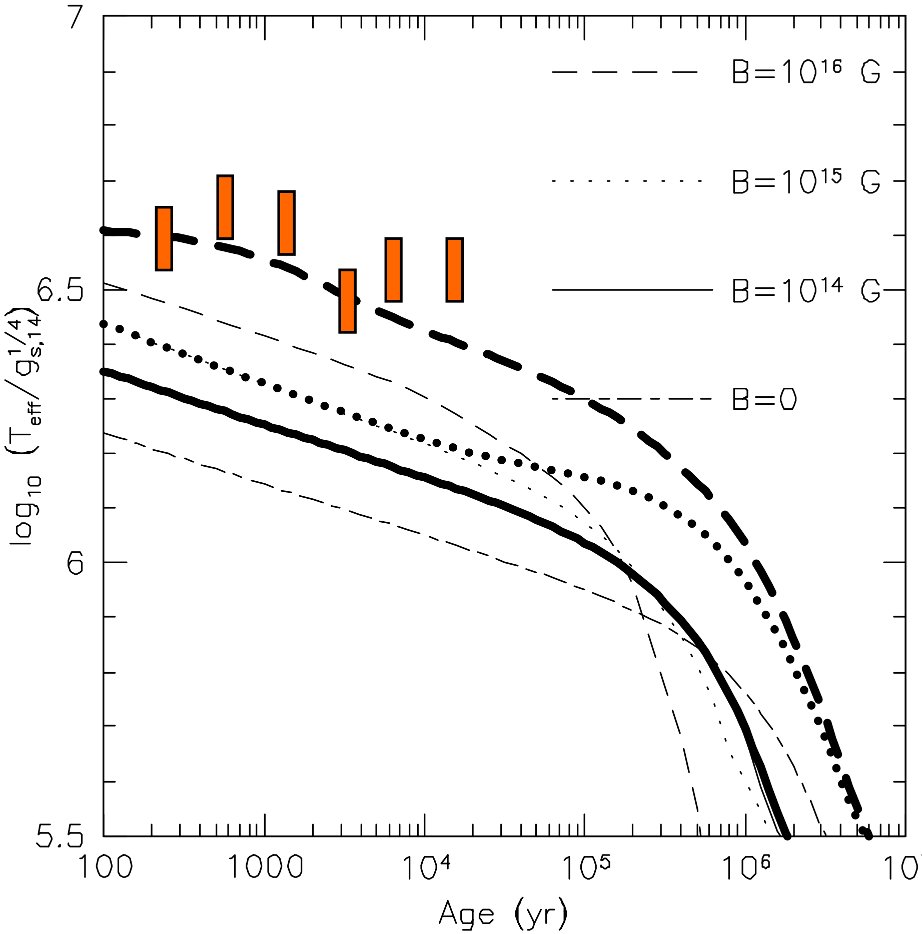
$$\Delta t \simeq 5000 \ \ \mathrm{yr} \left(\frac{\rho}{\rho_{\mathrm{nuc}}}\right)^{-2/3} T_9^{-3/2} \left\{1 - \left[\frac{T_9}{T_{9,i}}\right]^{3/2}\right\}$$

Thermal Evolution (3)



Thermal Evolution (4)

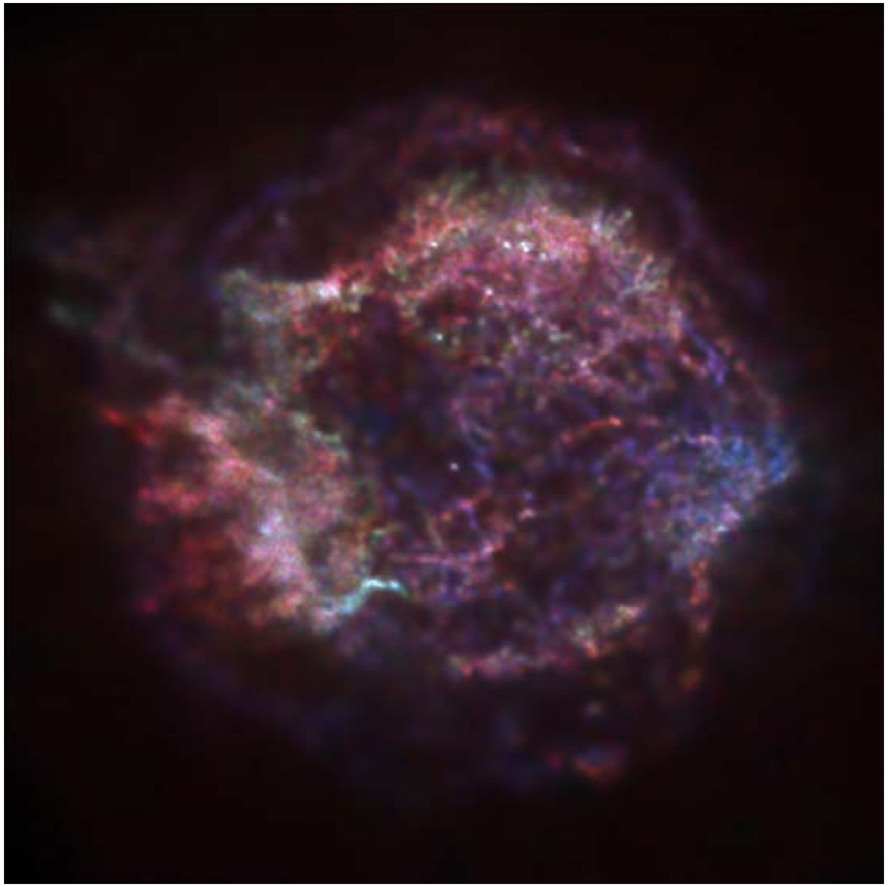


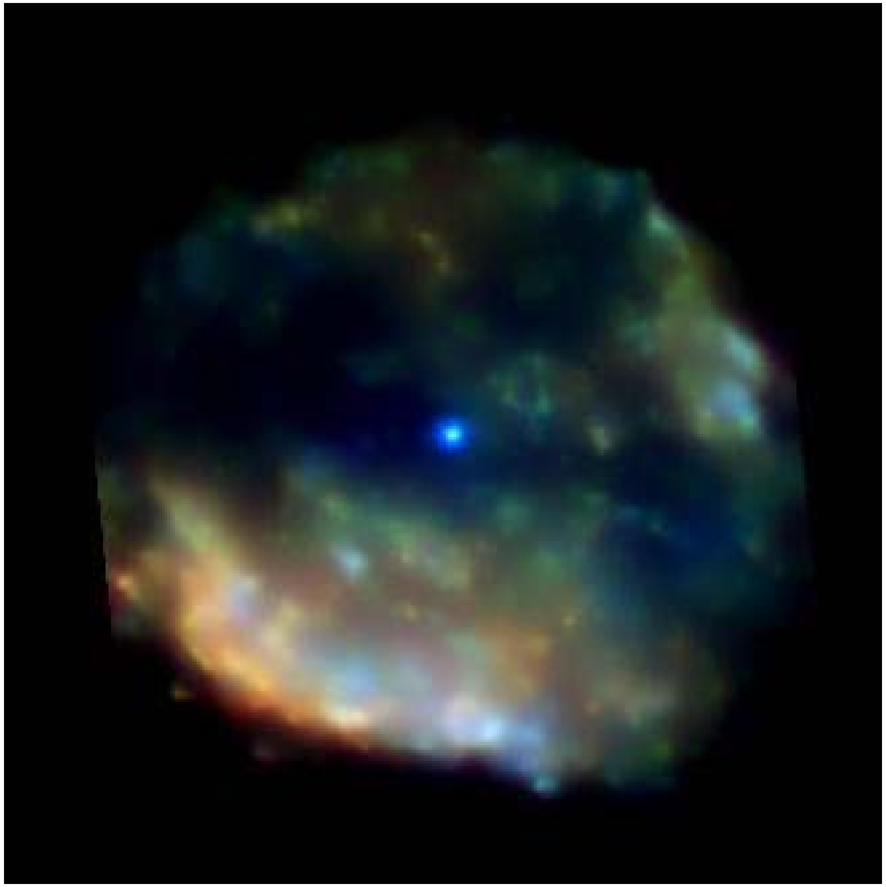


Young Neutron Stars (1)

Evidence	Plerionic Remnant	Composite Remnant	Pure Shell Remnant
Pulsar + Supernova Remnant (15+2)	G106.6+2.9 [PSR J2229+6114] G130.7+3.1 (3C 58) [PSR J0205+64] G184.6−5.8 (Crab) [PSR B0531+21] N157B (in LMC) [PSR J0537−6917]	G5.4−1.2 (Duck) [PSR B1757−24] G11.2−0.3 [PSR J1811−1925] G29.7−0.3 (Kes 75) [PSR J1846−0258] G34.7−0.4 (W44) [PSR B1853+01] G69.0+2.7 (CTB 80) [PSR B1951+32] G114.3+0.3 [PSR B2334+61] G263.9−3.3 (Vela) [PSR B0833−45] G292.0+1.8 [PSR J1124−5916] G308.8−0.1 [PSR J1341−6220] G320.4−1.2 (MSH 15−52) [PSR B1509−58] N158A (in LMC) [PSR B0540−69]	G180.0−1.7 (S147) [PSR J0538+2817] G292.2−0.5 [PSR J1119−6127]
Exotic/Possible NS + Supernova Remnant (16+1)	G54.1+0.3 [CXOU J193030.1+185214]	G0.9+0.1 [SAX J1747−2809] G119.5+10.2 (CTA 1) [RX J000702+7302.9] G189.1+3.0 (IC 443) [CXOU J061705.3+222127] G291.0−0.1 (MSH 11−62) [AX J1111−6040]	G27.4+0.0 (Kes 73) [AX J1841−045] (AXP) G29.6+0.1 [AX J1845−0258] (AXP?) G39.7−2.0 [SS 433] (binary) G78.2+2.1 (gamma Cygni) [RX J2020.2+4026] (NS?) G109.1−1.0 (CTB 109) [1E 2259+586] (AXP) G111.7−2.1 (Cas A) [CXO J232327.9+584842] (NS?) G260.4−3.4 (Puppis A) [RX J0822−4300] (NS?) G266.2−1.2 (RX J0852.0−4622) [SAX J0852.0−4615] (NS?) G296.5+10.0 (PKS 1209−51/52) [1E 1207.4−5209] G321.9−0.3 [Cir X-1] (binary) G332.4−0.4 [RCW 103] (1E 161348−5055) (NS?) N49 (in LMC) [SGR 0526−66] (SGR)
X-ray and Radio nebula (9)	G20.0−0.2 G21.5−0.9 G74.9+1.2 G328.4+0.2	G16.7+0.1 G39.2−0.3 G326.3−1.8 (MSH 15−56) G327.1−1.1 G344.7−0.1	
Radio nebula only (8)	G6.1+1.2 G27.8+0.6 G63.7+1.1	G24.7+0.6 G293.8+0.6 G318.9+0.4 G322.5−0.1 G351.2+0.1	

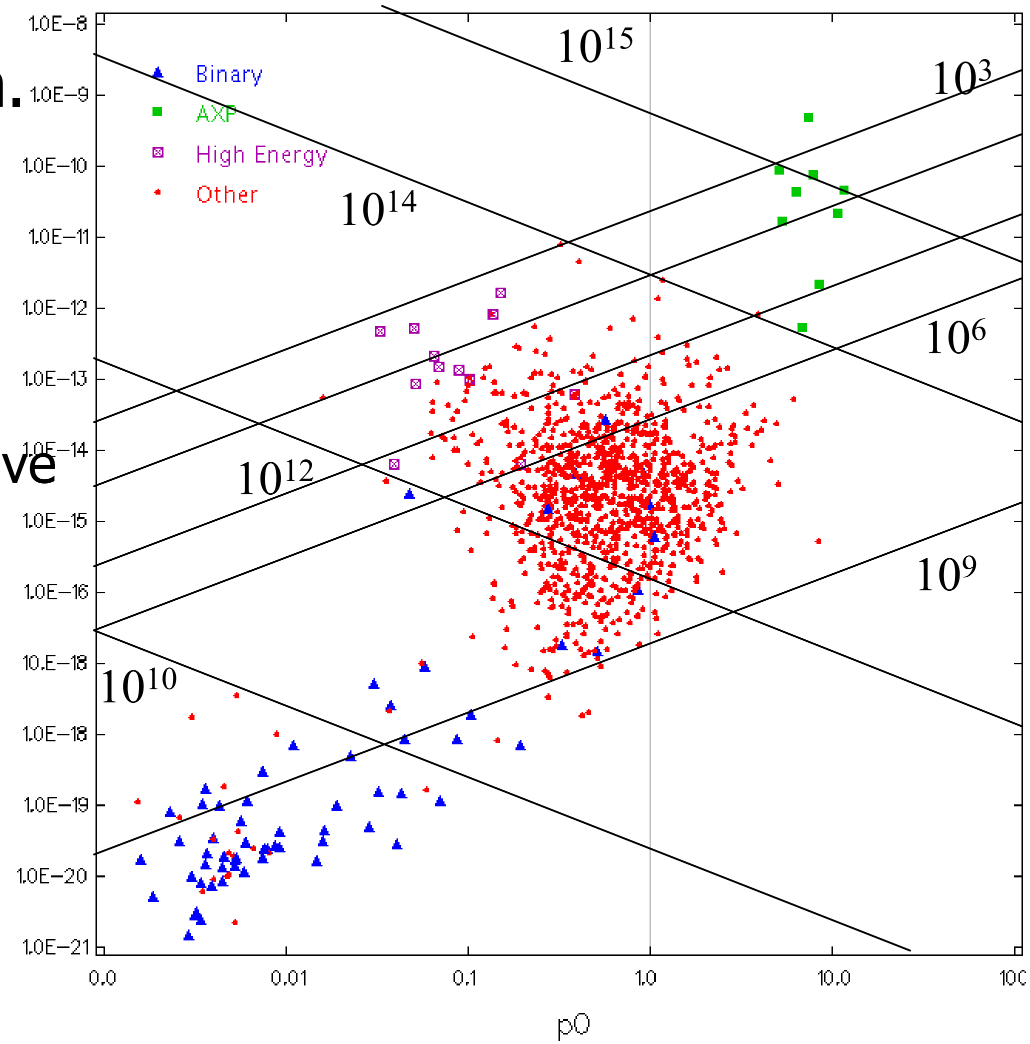
Young Neutron Stars (2)





Young Neutron Stars (3)

- On the P-P-dot diagram.
- Things to notice:
  - most red dots (isolated radio pulsars): $10^{11\text{-}13}$ G, $10^{5\text{-}8}$ yr
  - most PSRs in binaries have short periods
  - Many young PSRs have high-energy emission or are AXPs (no radio and thermal x-ray)

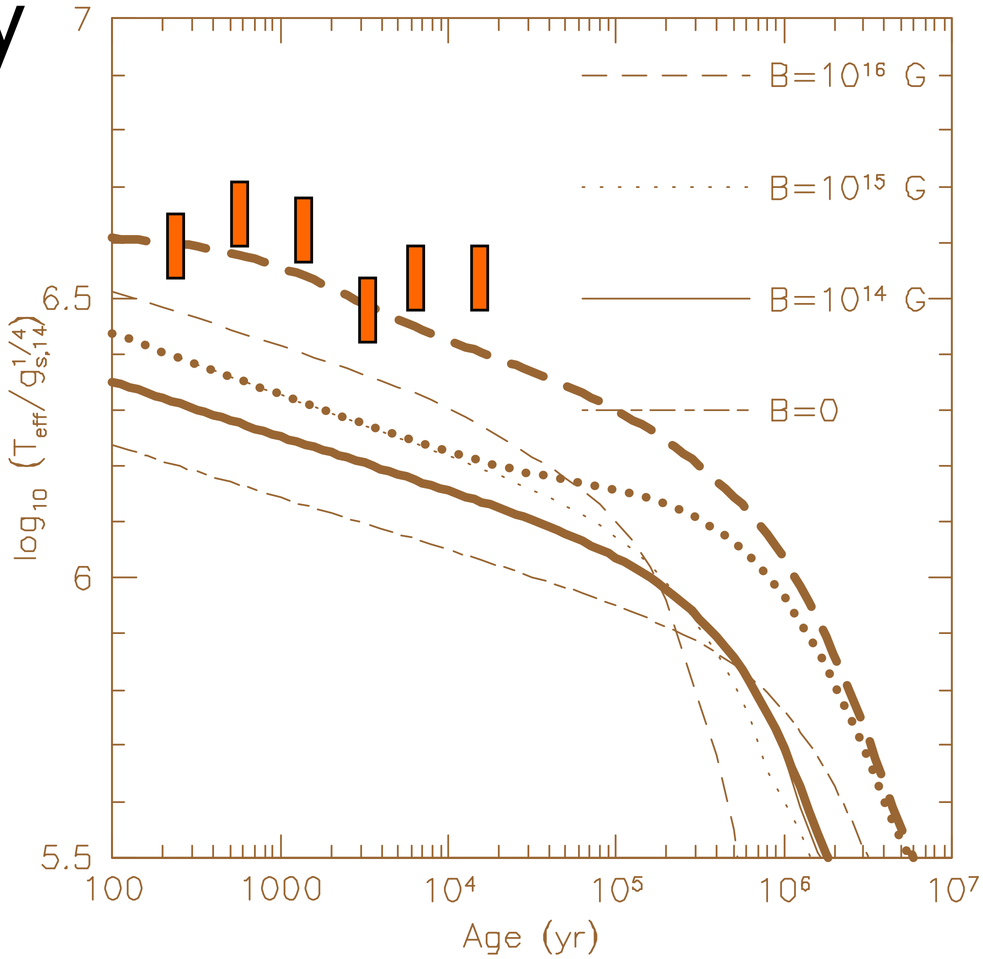


Anomalous X-ray Pulsars

- Young isolated neutron stars (often in SNRs):
  - consistent spin down with glitches,
  - periods of several seconds,
  - thermal spectra in X-rays, $L \sim 10^{34}$ erg/s,
  - really faint in optical
  - inferred $B \sim 10^{15}$ G
  - too bright to be standard cooling, too faint for standard accretion
- Accretion from tiny disk?

Powering the AXPs

- Magnetic fields play a dominant and dynamic role.
  - Electron conduction,
  - Field decay,
- Early on neutrinos dominate the cooling; later photons do.

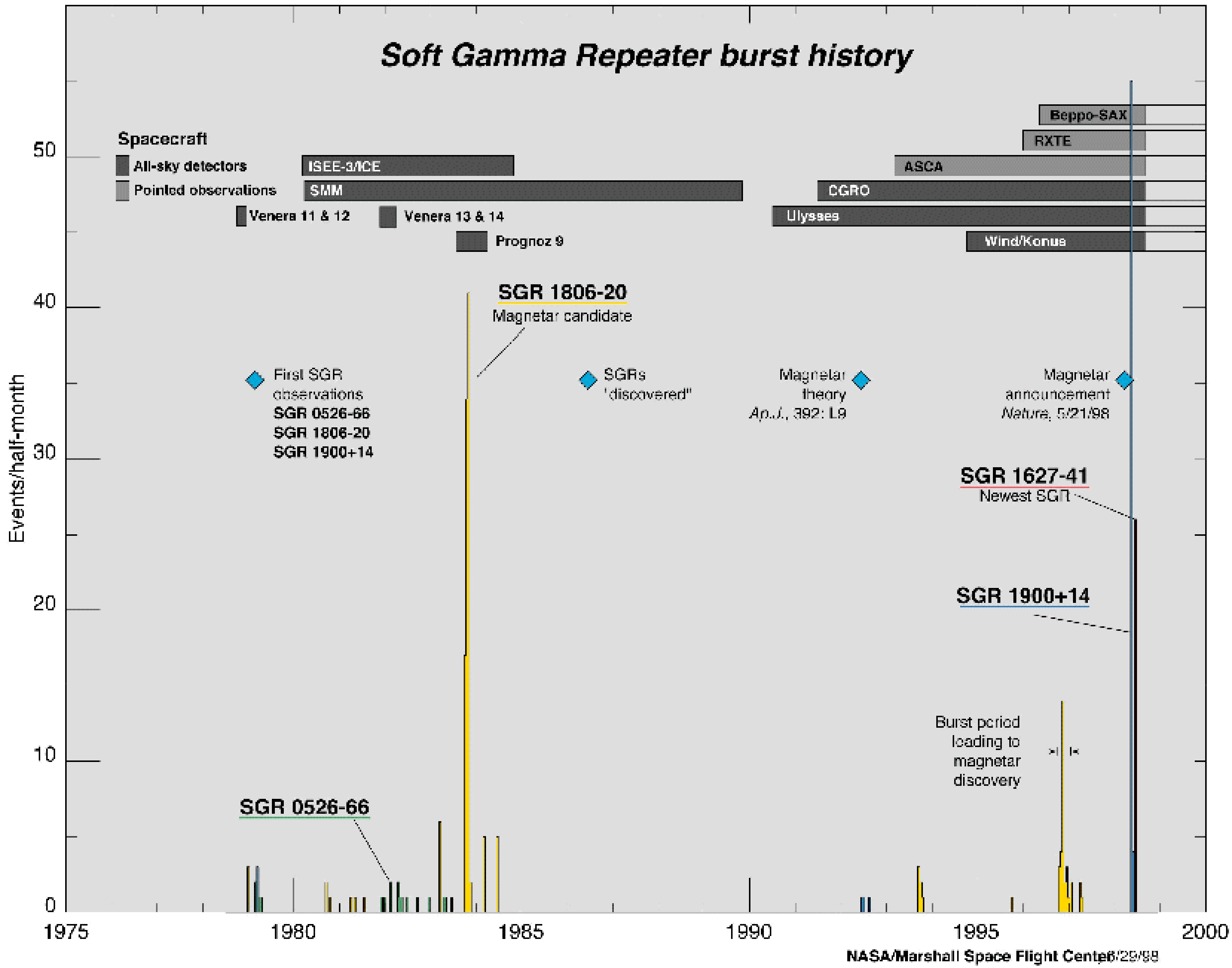


Soft Gamma Repeaters

- Young isolated neutron stars (sometimes in SNRs):
  - consistent spin down,
  - periods of several seconds,
  - thermal spectra in X-rays, $L \sim 10^{34}$ erg/s,
  - really faint in optical
  - inferred $B \sim 10^{15}$ G
  - too bright to be standard cooling, too faint for standard accretion

Plus they burst!

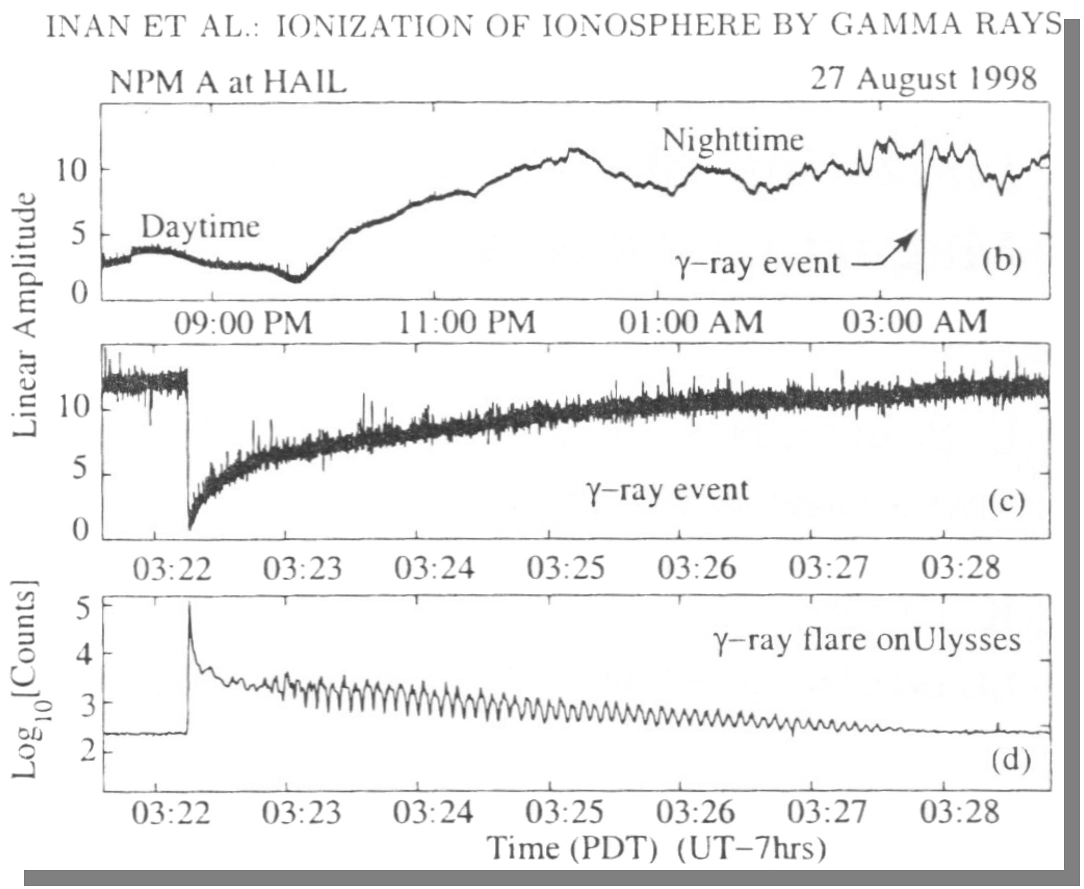
- Bursts last a few tenths of a second and radiate as much energy as the sun does in a year.   Soft compared to GRBs.
- Biggest explosions that don’t destroy the source.
- Magnetic stress builds in the crust until it fractures and the field rearranges itself locally leading to hard X-ray burst.



For additional information and illustrations: *http://www.magnetars.com*
NASA/Marshall Space Flight Center, 9/29/98

Some bursts are really big!

- March 5, 1979: SGR 0526-66
- August 27, 1998: SGR 1900+14
- The entire crust is disrupted leading to large-scale reconnection like a solar flare.



What is special about these objects?

- The energy of an electron in a magnetic field is quantized:

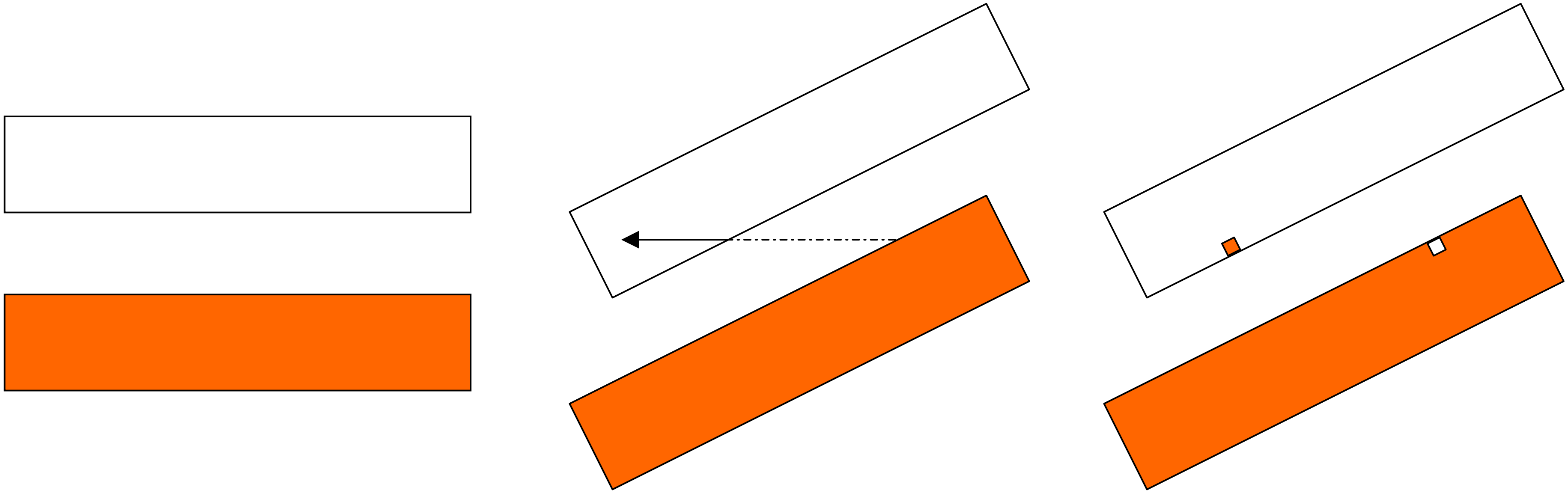
$$E = \left(n + \frac{1}{2}\right) \hbar\omega_g \text{ where } \omega_g = \frac{eB}{m_e c}$$

- We can define a characteristic magnetic field for an electron.

$$\hbar\omega_g = \hbar\frac{eB}{m_e c} = m_e c^2 \text{ so } B = \frac{m_e^2 c^3}{e\hbar} \approx 4.4 \times 10^{13} \text{ G}$$

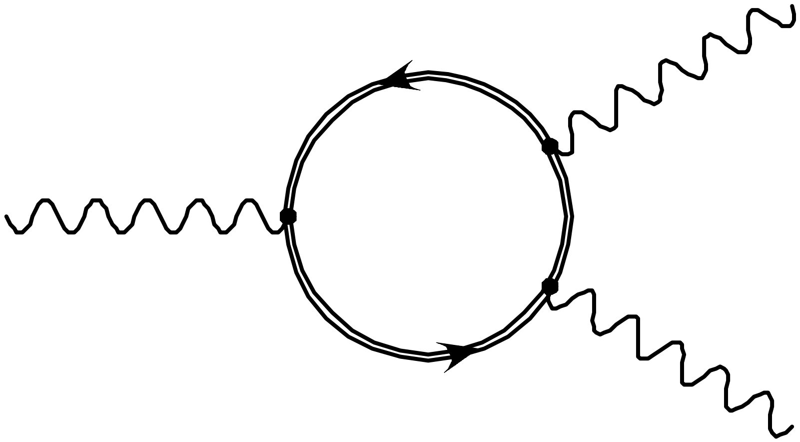
What happens? (1)

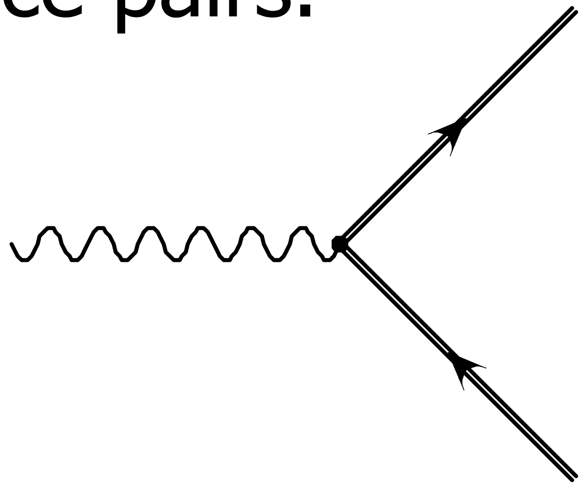
- First, what doesn't happen:
  - If you had an electric field this strong (about 10^{18} V/m), you would pull electron-positron pairs out of the vacuum (Klein paradox).



What happens? (2)

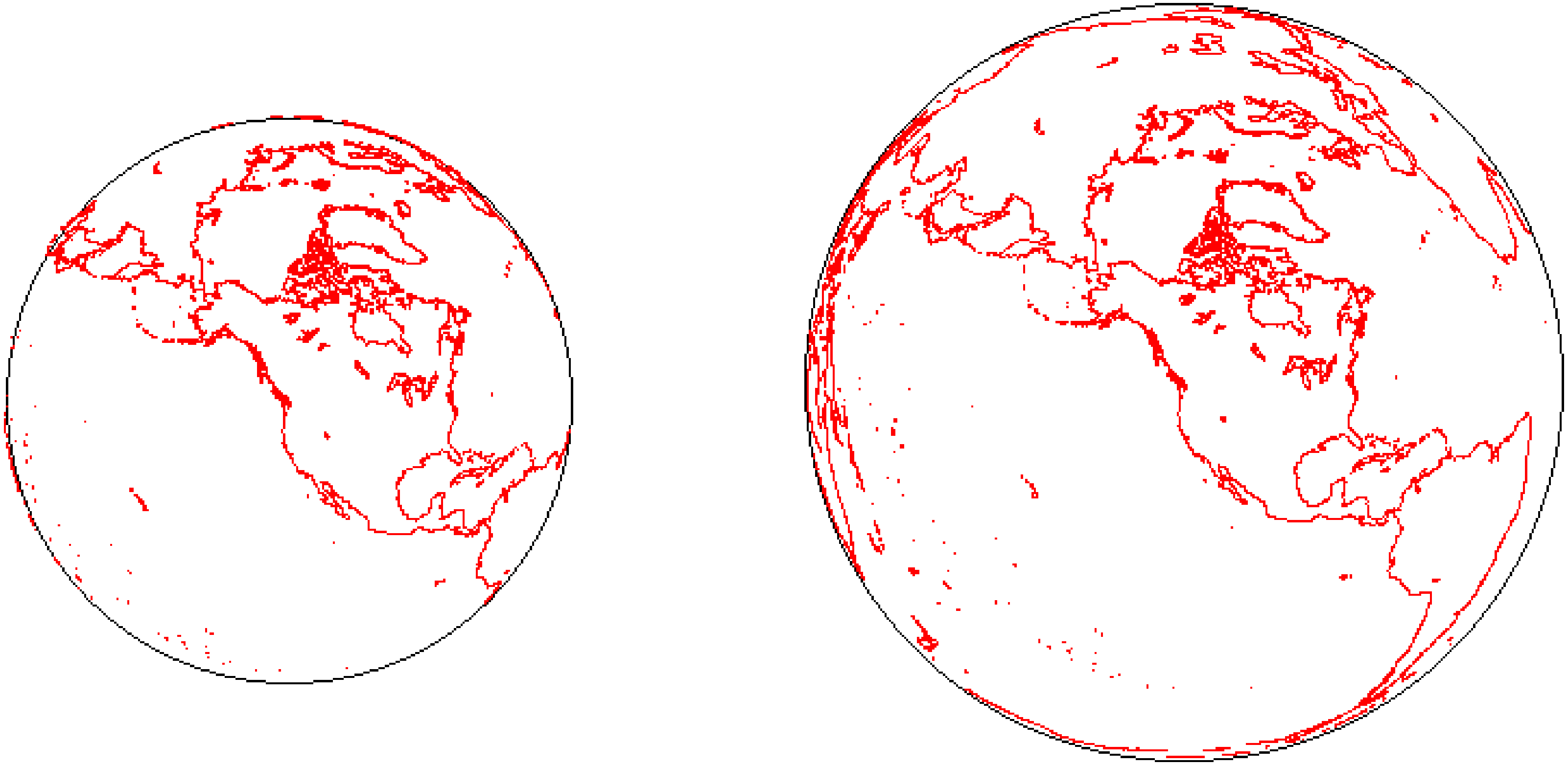
- Such a strong magnetic field is not unstable, but it does funny things to light.
- The index of refraction depends on the polarization of the light and its intensity.
- Photons can split or produce pairs.





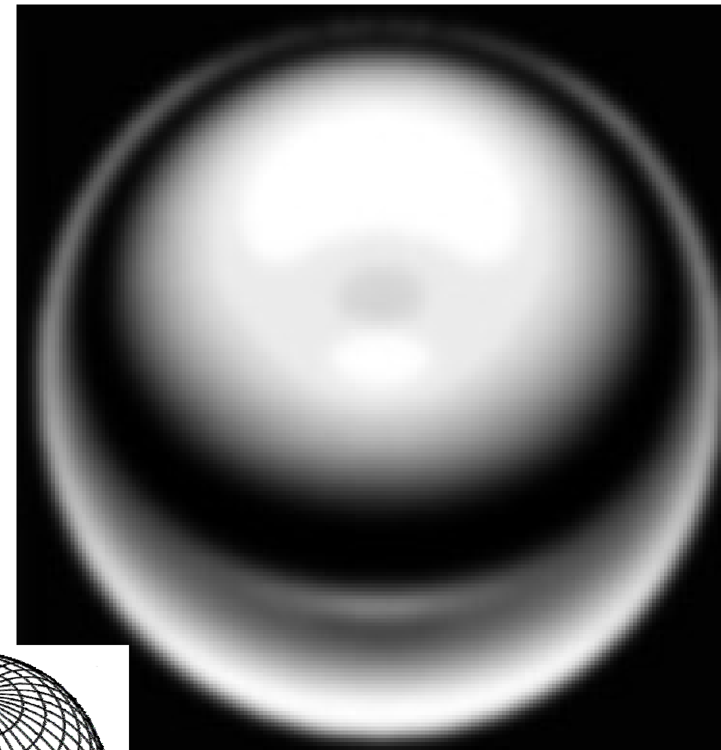
Looking at the surface (1)

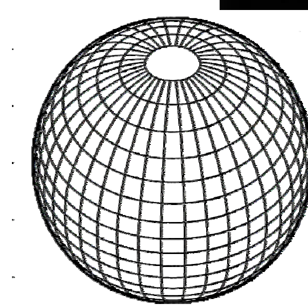
- Gravity distorts our view of neutron stars.



Looking at the surface (2)

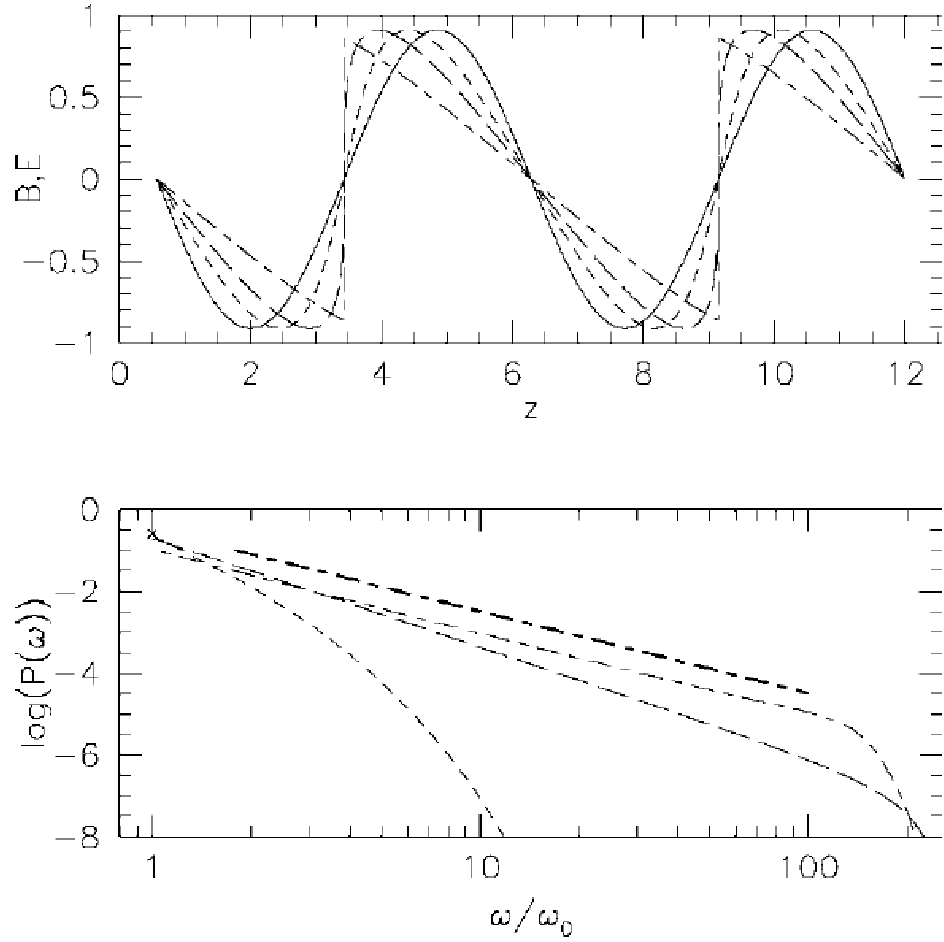
- The magnetic field magnifies one polarization but not the other. The poles are magnified more than the equatorial regions.



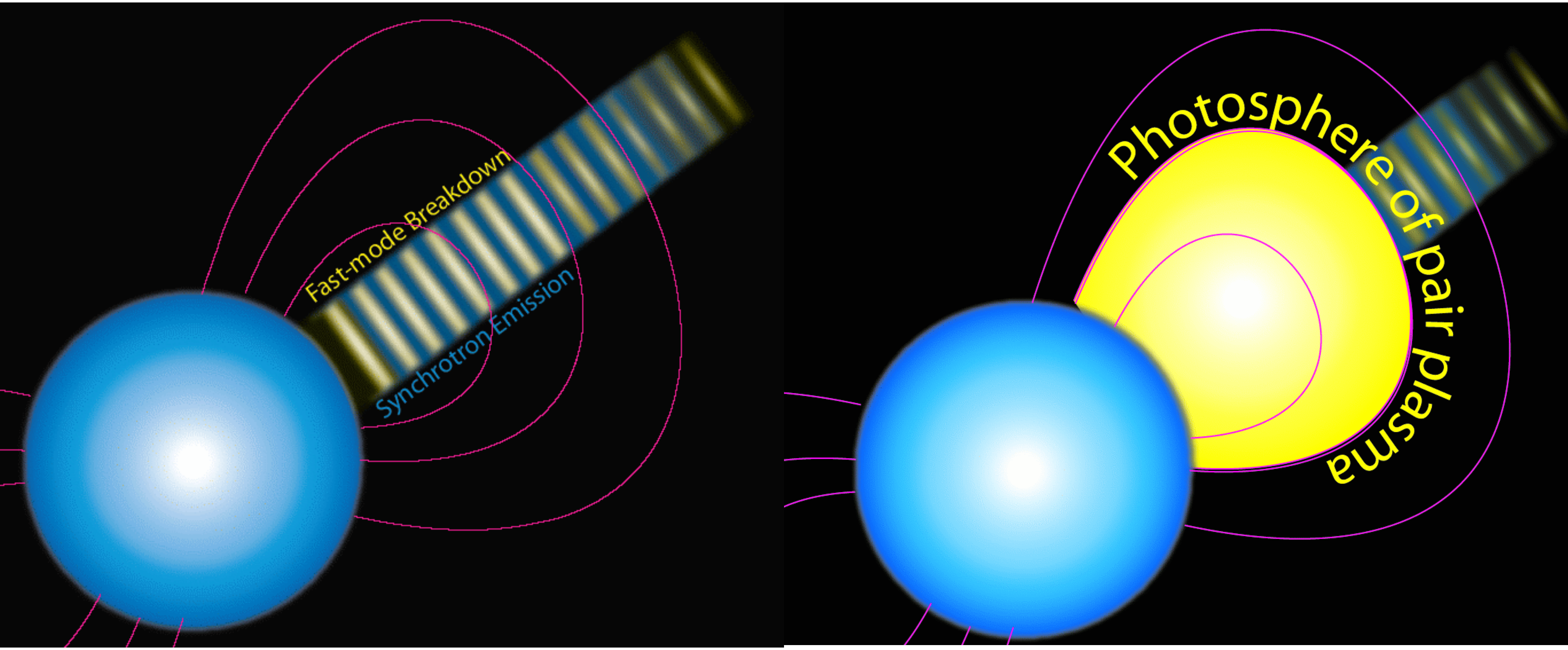


Electromagnetic Shocks

- The phase velocity of light depends on the strength of the wave.
- The peaks go slower than the troughs, so the wave steepens as it travels near the neutron stars.



Formation of a Fireball



Neutron Star Energetics

- Some typical energies for isolated neutron stars:

Rest-Mass Energy	$Mc^2 = 1.8 \times 10^{54} \frac{M}{M_\odot}$ erg
Gravitational Energy	$0.6 \frac{GM^2}{R} = 1.6 \times 10^{53} \left(\frac{M}{M_\odot}\right)^2 R_6^{-1}$ erg
Spin Energy	$\frac{1}{2} I \Omega^2 = 2 \times 10^{46} P_0^{-2}$ erg
Thermal Energy	$U_n = 6 \times 10^{47} \frac{M}{M_\odot} T_9^2$ erg
Magnetic Energy	$\frac{1}{12} R^3 B^2 = 8 \times 10^{46} R_6^3 B_{15}^2$ erg