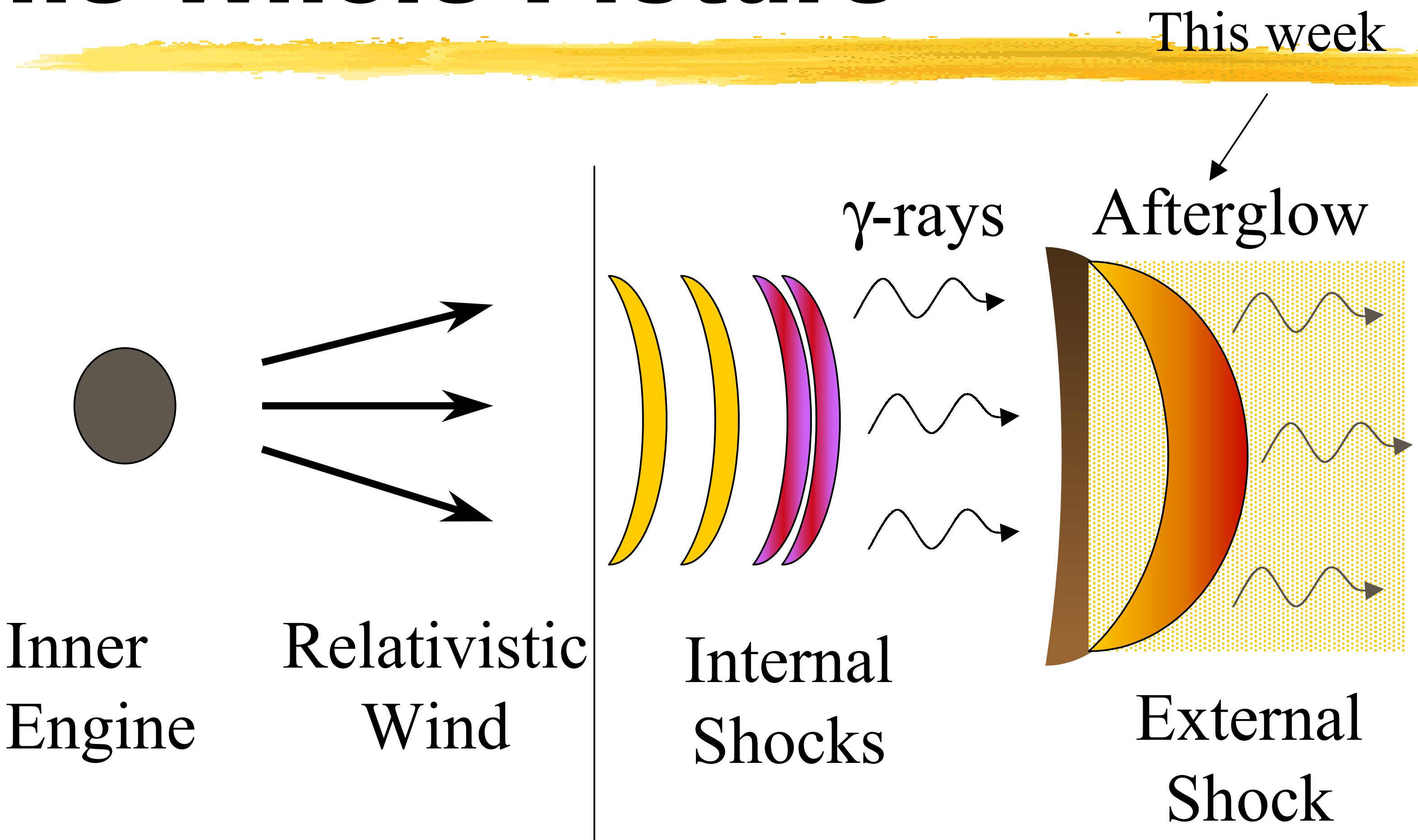
Gamma-Ray Burst Afterglows

The External Shock, Beaming and GRB Remnants

The Whole Picture

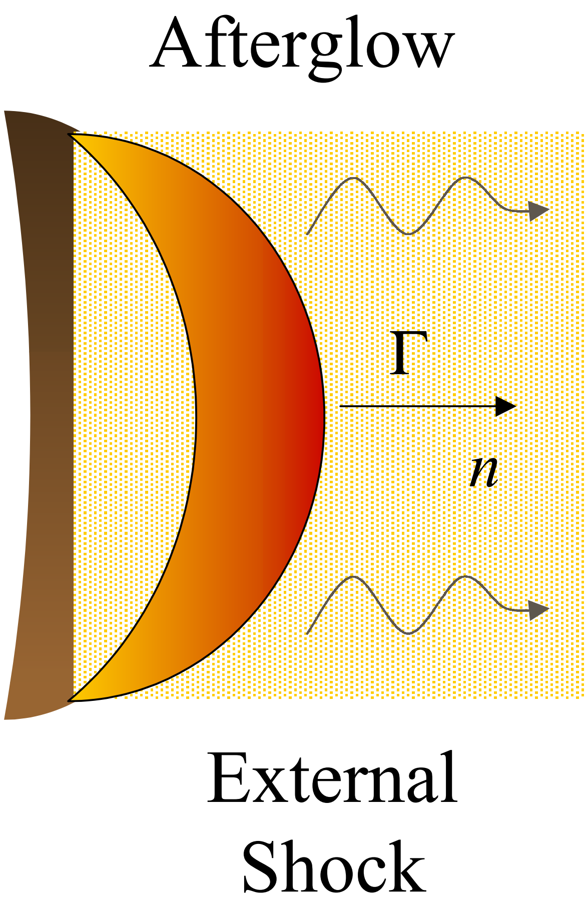


We don’t see this.

We see this.

Zooming in on the shock

- The shock moves in to medium with number density, *n*, with a Lorentz factor, Γ.
- The energy behind the shock is $4\Gamma^2 n m_p c^2$.
- The number density behind the shock is $4\Gamma n$.
- The bulk of the energy initially lies with protons.



Equipartition

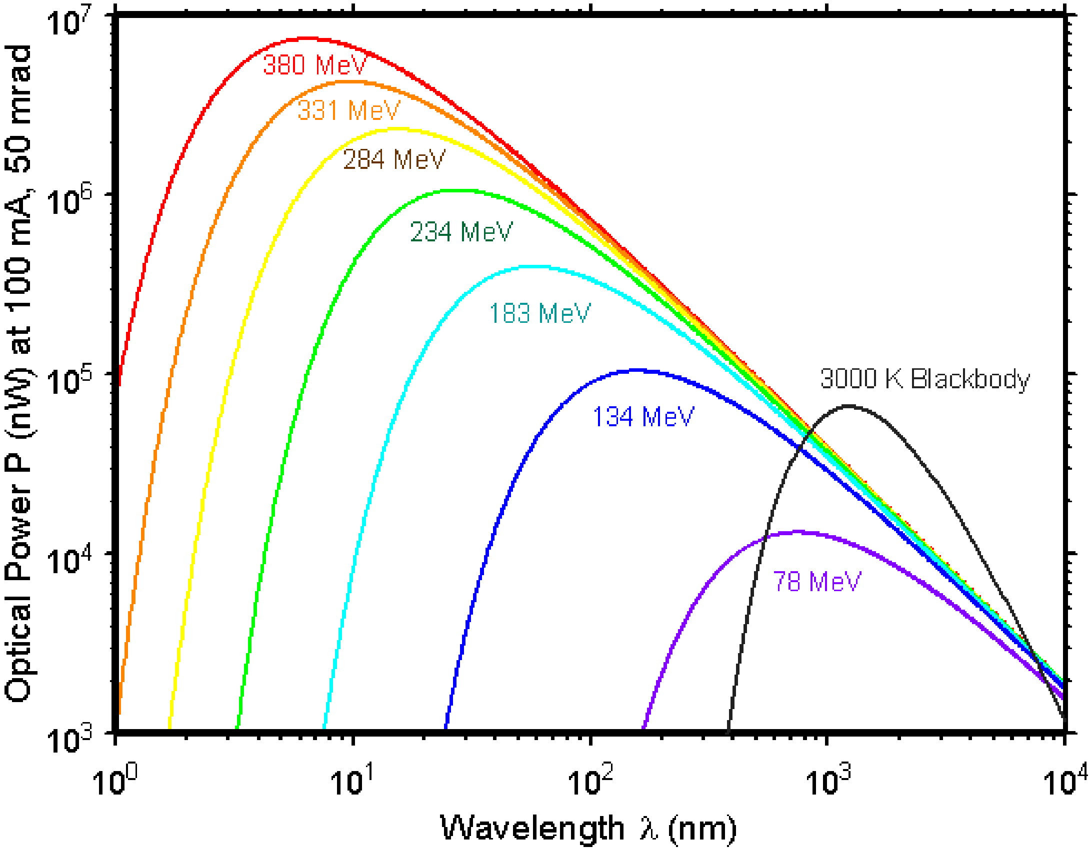
- "The energy of a system in equilibrium is shared among the various degrees of freedom of the system."
  - Protons
  - Electrons: $\epsilon_e(4\Gamma^2 n m_p c^2) = \int_{\gamma_m}^{\infty} A\gamma^{-p}\gamma m_e c^2 d\gamma = \frac{A\gamma_m^{2-p}}{p-2} m_e c^2$

$$4\Gamma n = \int_{\gamma_m}^{\infty} A\gamma^{-p} d\gamma = \frac{A\gamma_m^{1-p}}{p-1}, \gamma_m = \epsilon_e\left(\frac{p-2}{p-1}\right)\frac{m_p}{m_e}\Gamma \approx 610\epsilon_e\Gamma$$

  - Magnetic field: $\epsilon_B(4\Gamma^2 n m_p c^2) = \frac{B^2}{8\pi}, B = (32\pi\epsilon_B n m_p)^{1/2}\gamma c$

Radiation from one e$^-$ (1)

- The high-energy electrons are responsible for most of the radiation.
- A high-energy electron in a magnetic field emits synchrotron radiation. Its spectrum is given by



$$j(\omega) = \frac{3\sqrt{3}\sigma_T m_e c^2 B \sin\alpha}{16\pi^2 e} F\left(\frac{\omega}{\omega_c}\right), \omega_c = \frac{3}{2}\beta\gamma^2\left(\frac{eB}{m_e c}\right)\sin\alpha$$

Radiation from one e⁻ (2)

- How much power does the electron radiate?

$$P = \int_0^\infty j(\omega)d\omega = \frac{3\sqrt{3}\,\sigma_T m_e c^2 B \sin\alpha}{16\pi^2 e}\int_0^\infty F\left(\frac{\omega}{\omega_c}\right)d\omega$$

$$\text{Let } \ x = \omega/\omega_c$$

$$= \frac{9\sqrt{3}\,\sigma_T}{32\pi^2}(B\sin\alpha)^2\,\beta\gamma^2\int_0^\infty F(x)dx = \frac{4}{3}\sigma_T c\gamma^2\frac{B^2}{8\pi},$$

$$\nu(\gamma) = \gamma^2\frac{eB}{2\pi mc}$$

Boosting!

- All of these results are calculated in the frame of the shock. Let's go to the frame of the star that exploded.

$$P = \frac{4}{3}\sigma_T c \Gamma^2 \gamma^2 \frac{B^2}{8\pi}, \quad \nu = \Gamma \gamma^2 \frac{eB}{2\pi mc}, P_{\nu,\max} \approx \frac{P}{\nu} = \frac{mc^2\sigma_T}{3e}\Gamma B$$

- Although the initial distribution of electrons is a power-law, some have a chance to cool.

$$\Gamma \gamma_c mc^2 = Pt,$$

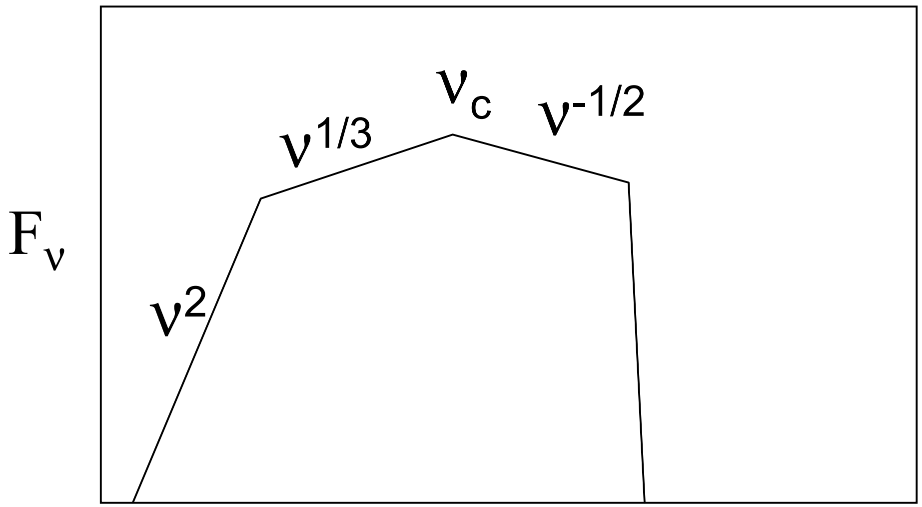
$$\gamma_c = \frac{6\pi mc}{\sigma_T \Gamma B^2 t} = \frac{3m}{16\epsilon_B \sigma_T m_p c}\frac{1}{t\Gamma^3 n}$$

Cooling!

- What does the spectrum of the electron look like as it cools?

$$E = \gamma m c^2, \quad \nu = \Gamma \left(\frac{E}{mc^2}\right)^2 \frac{eB}{2\pi mc}$$

$$E = mc^2 \left(\frac{2\pi mc}{eB\Gamma}\nu\right)^{1/2}, \frac{dE}{d\nu} = mc^2 \left(\frac{2\pi mc}{eB\Gamma}\right)^{1/2} \frac{1}{2\nu^{1/2}}$$



$$\nu_c = \Gamma \gamma_c^2 \frac{eB}{2\pi mc}$$

$$\nu_m = \Gamma \gamma_m^2 \frac{eB}{2\pi mc}$$

Radiation from many e⁻ (1)

- Let's say that $\gamma_m < \gamma_c$ (slow cooling). In this case there are some electrons that haven't had a chance to cool.
- There is a power-law distribution of electron energies: $N\,d\gamma = A\,\gamma^p\,d\gamma$. What is the total spectrum?

$$J(\omega) = QB\int_{\gamma_m}^{\infty} A\gamma^{-p}F\left(\frac{\omega}{\omega_c}\right)d\gamma, \text{Let } \; x = \omega/\omega_c, \gamma = \left[\frac{\omega}{xB}\,\frac{6mc}{9\beta e\sin\alpha}\right]^{1/2}$$

$$= PB^{(p+1)/2}\omega^{-(p-1)/2}\int_0^{x_m} x^{(p-3)/2}F(x)dx$$

Radiation from many e^- (2)

- The electrons with initial $\gamma > \gamma_c$ have had a chance to cool what is their spectrum.
- Again using the power-law distribution of electron energies: $N\, d\gamma = A\, \gamma^{-p}\, d\gamma$, we get

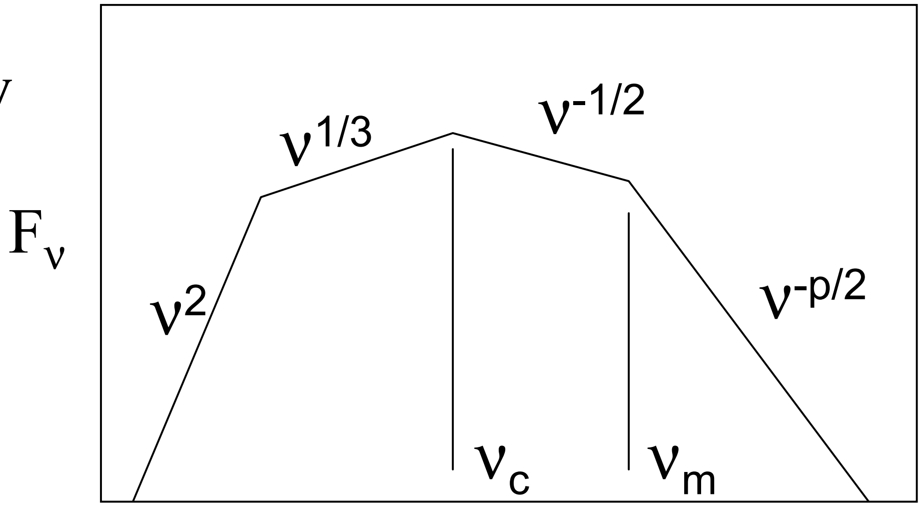
$$J(\omega) \sim \int \gamma^{-p} \nu^{-1/2} d\gamma \sim \int \nu^{-p/2} \nu^{-1/2} \nu^{-1/2} d\nu \sim \nu^{-p/2}$$

Putting it together

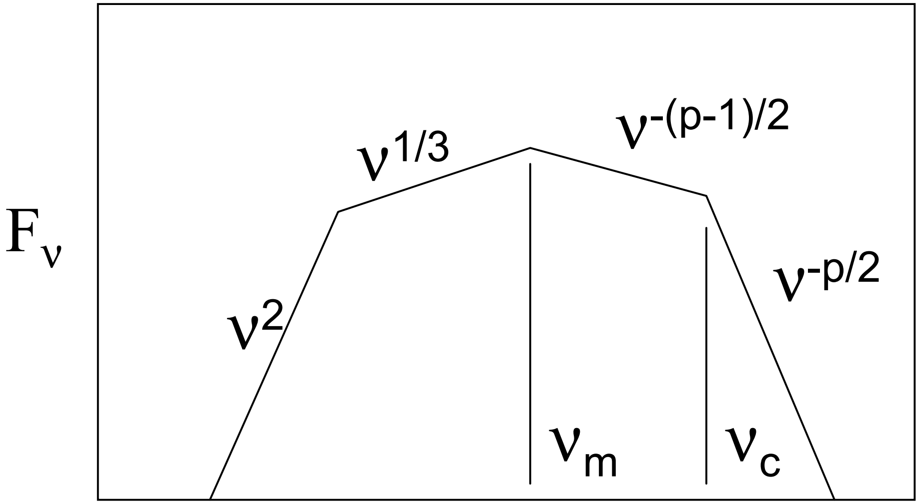
In the late regime (after a day typically) only a few of the accelerated electrons have had a chance to cool so

$\gamma_c > \gamma_m$.

Slow cooling



Fast cooling



In this early regime all of the accelerated electrons have had a chance to cool so

$\gamma_c < \gamma_m$.

How does the afterglow evolve?

- Previously we calculated how the shock itself grows with photon arrival time and local time. $\Gamma^2 = At^{-3}, \Delta t = t^4/(8A)$
- How do γ_m and γ_c change with time?

$$\gamma_m = 610\epsilon_e\Gamma = 610\epsilon_e A^{1/2}t^{-3/2} = 610\epsilon_e 8^{-3/8}A^{1/8}(\Delta t)^{-3/8}$$

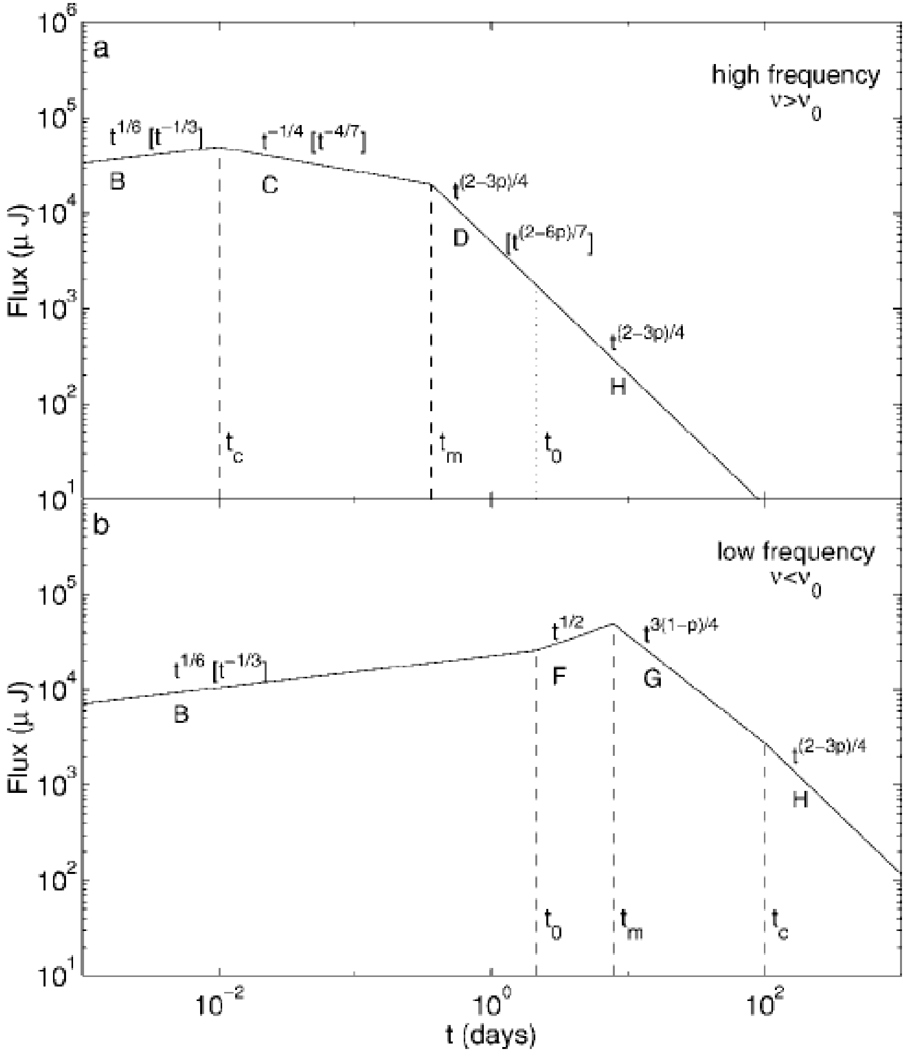
$$\gamma_c \propto \frac{1}{t^4\Gamma^3} \propto \frac{1}{t^4 t^{-9/2}} \propto t^{1/2} \propto (\Delta t)^{1/8}$$

$$\nu_m \propto \Gamma\gamma_m^2 B \propto \Gamma^2\gamma_m^2 \propto t^{-6} \propto (\Delta t)^{-3/2}$$

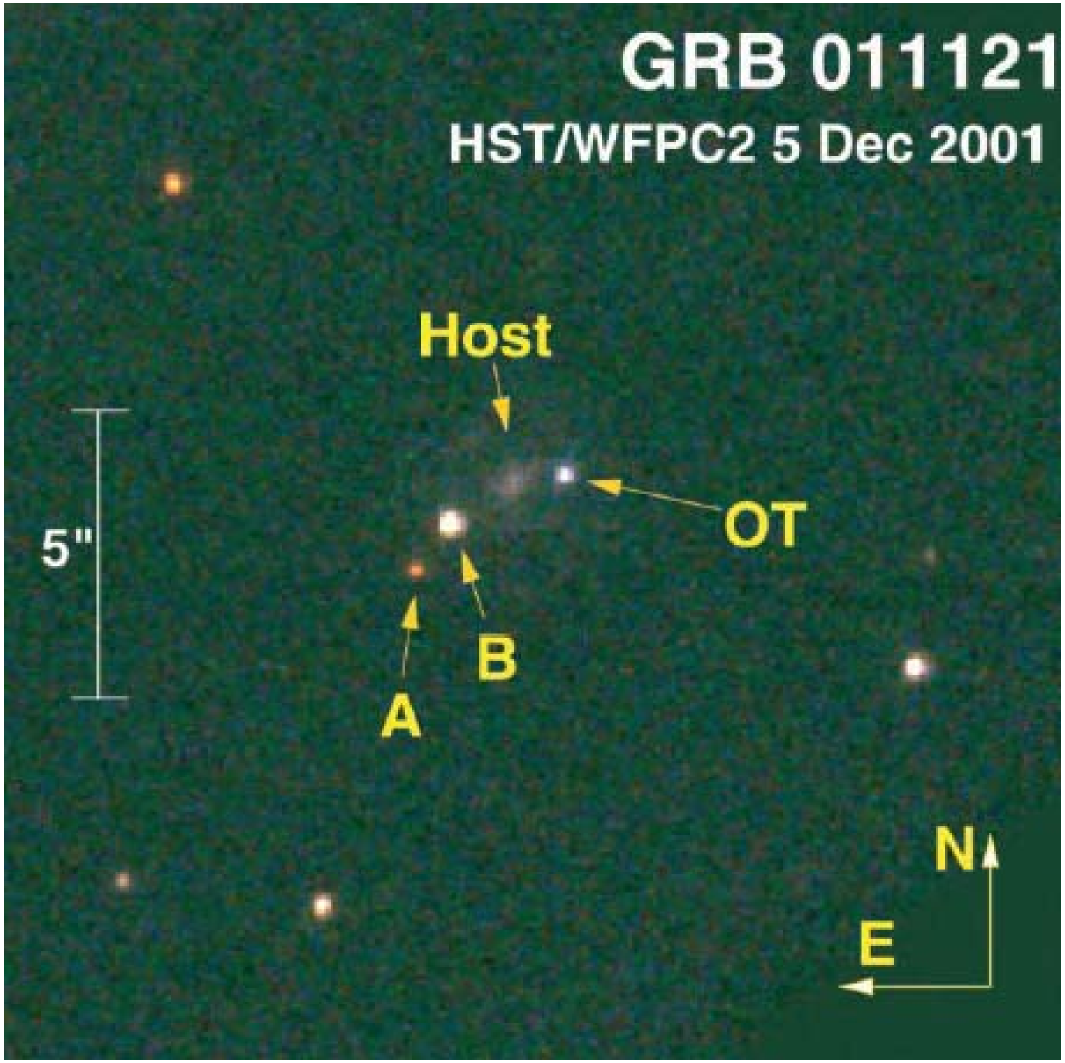
$$\nu_c \propto \Gamma\gamma_c^2 B \propto \Gamma^2\gamma_c^2 \propto t^{-2} \propto (\Delta t)^{-1/2}$$

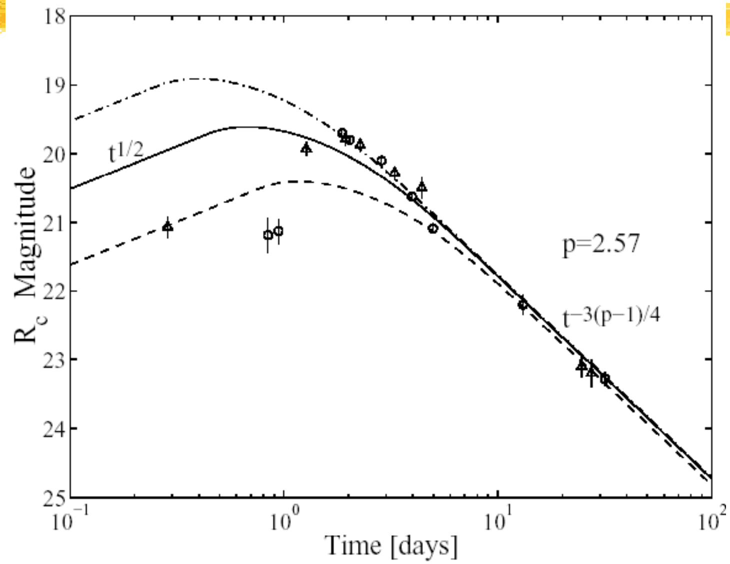
Light curve

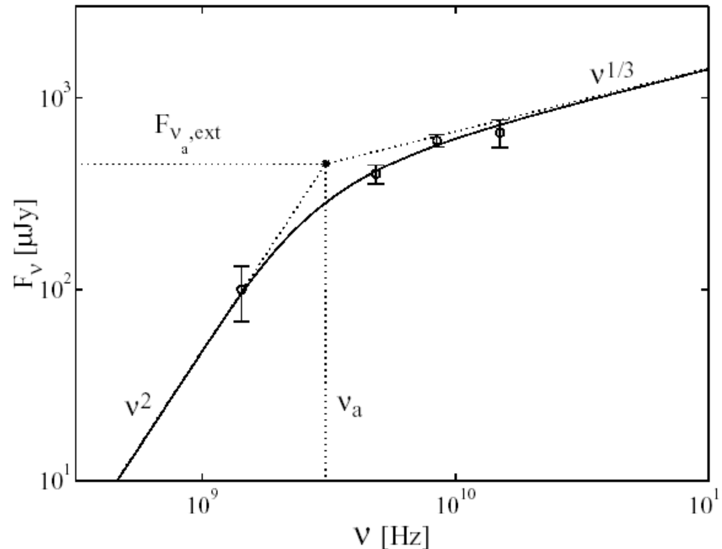
- This simple set of assumptions of a relativistic fireball radiating synchrotron radiation is sufficient to determine the lightcurve of the afterglow at all frequencies.



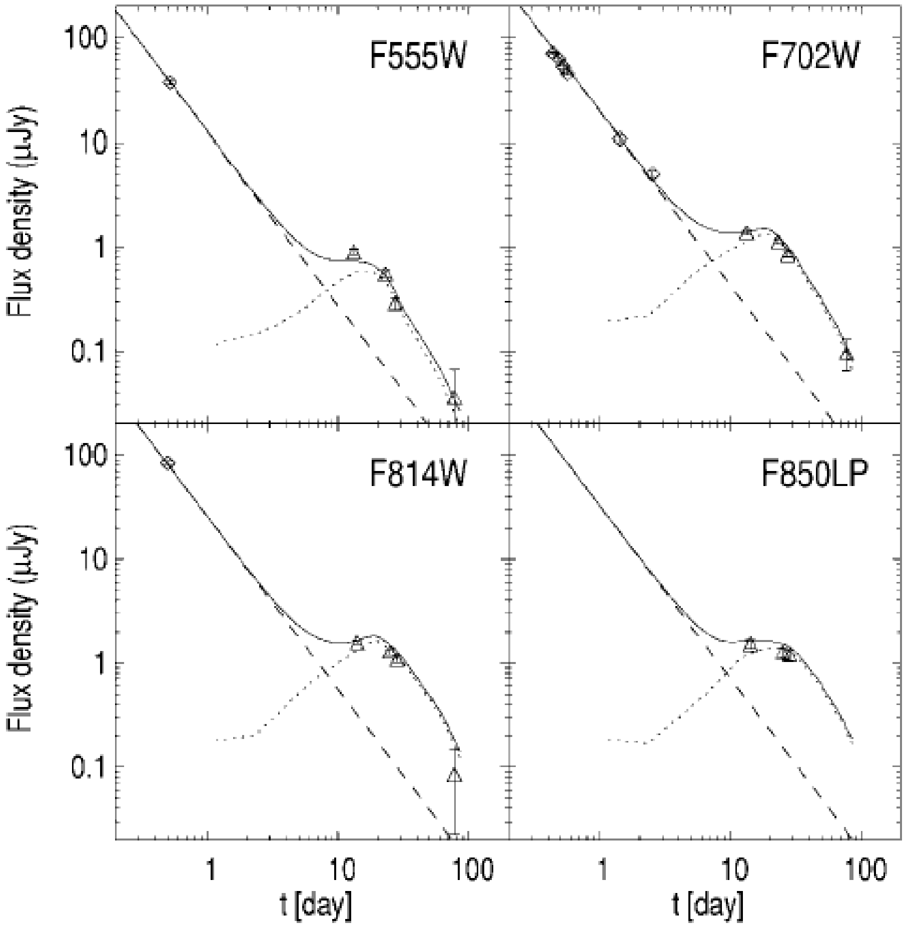
How well does it work?

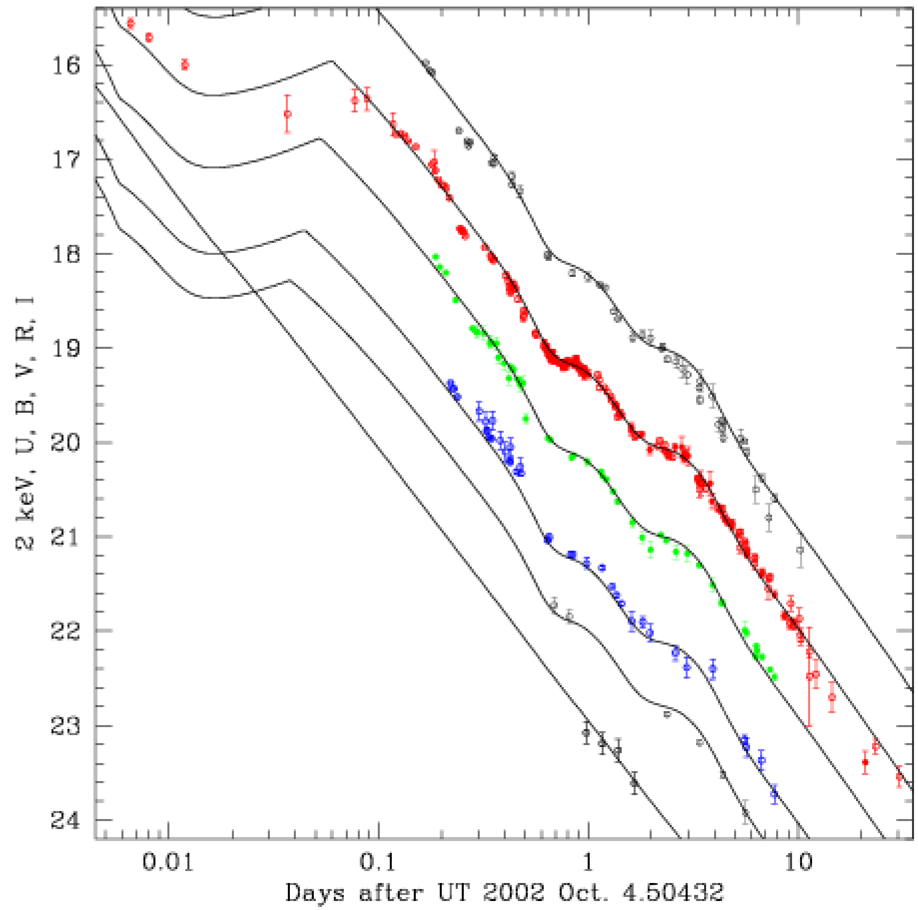






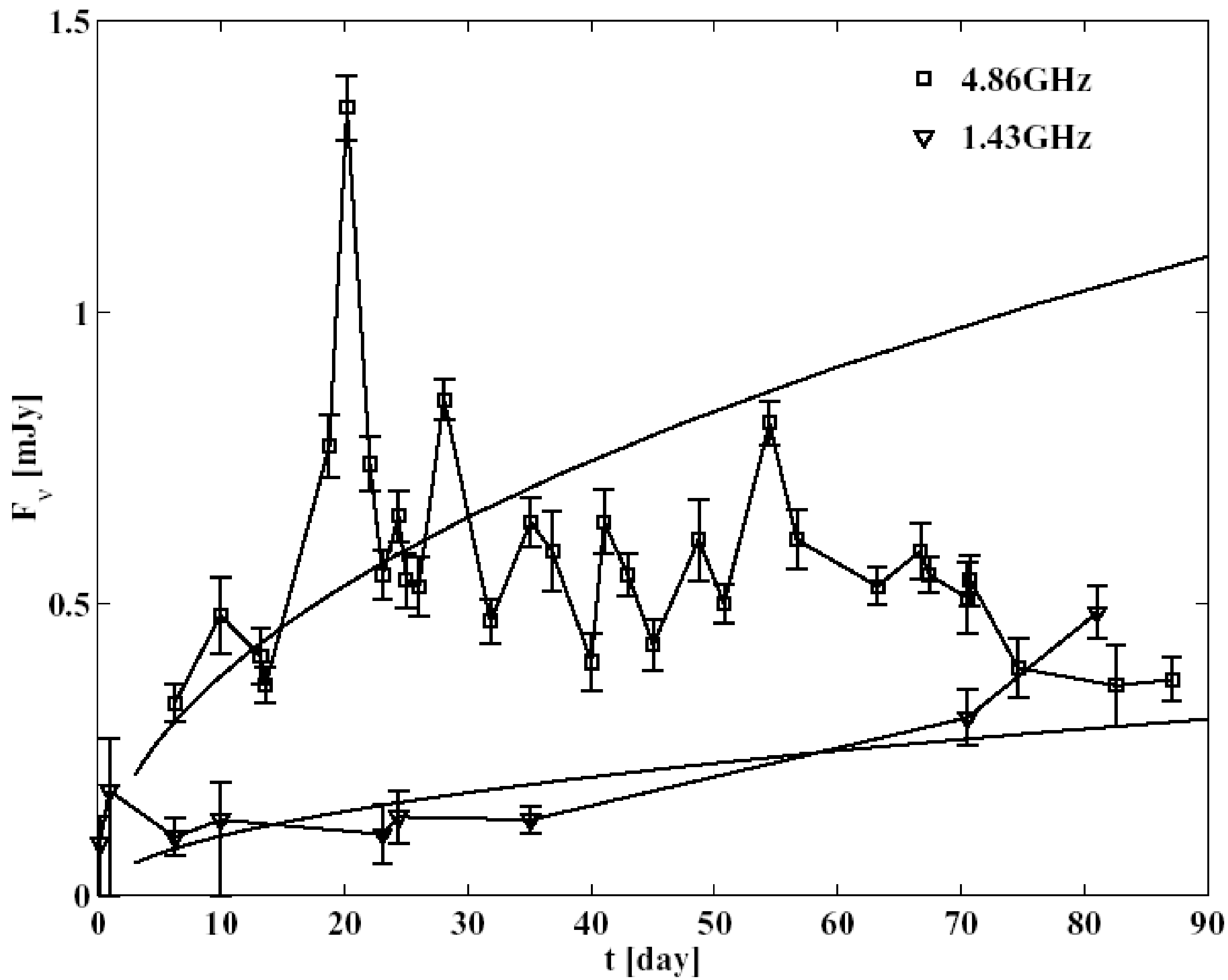
When it doesn't work… (1)



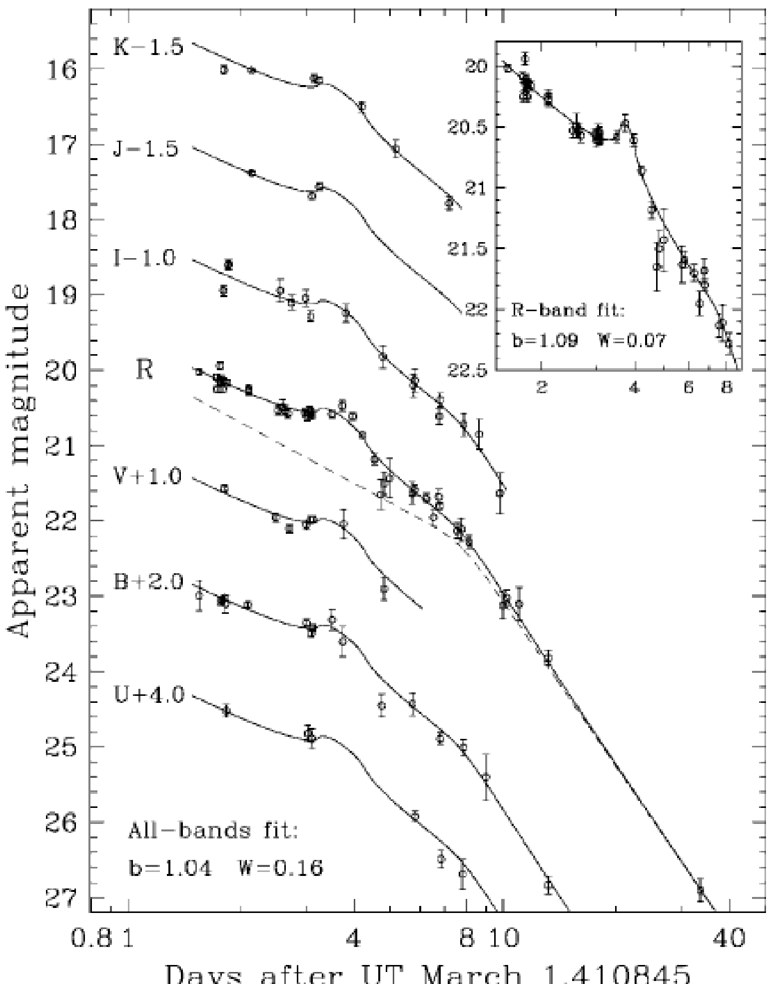


Chromatic bumps

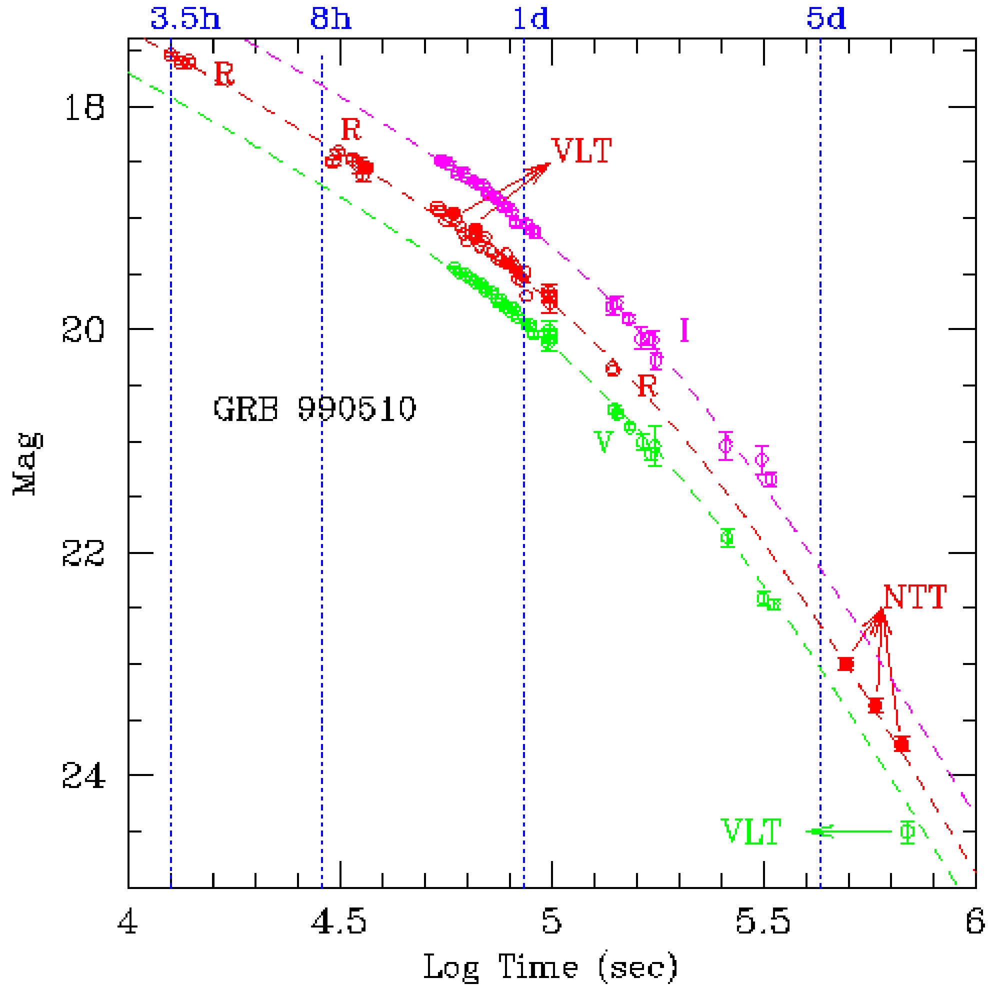
When it doesn't work... (2)



When it doesn't work… (3)



Achromatic bump

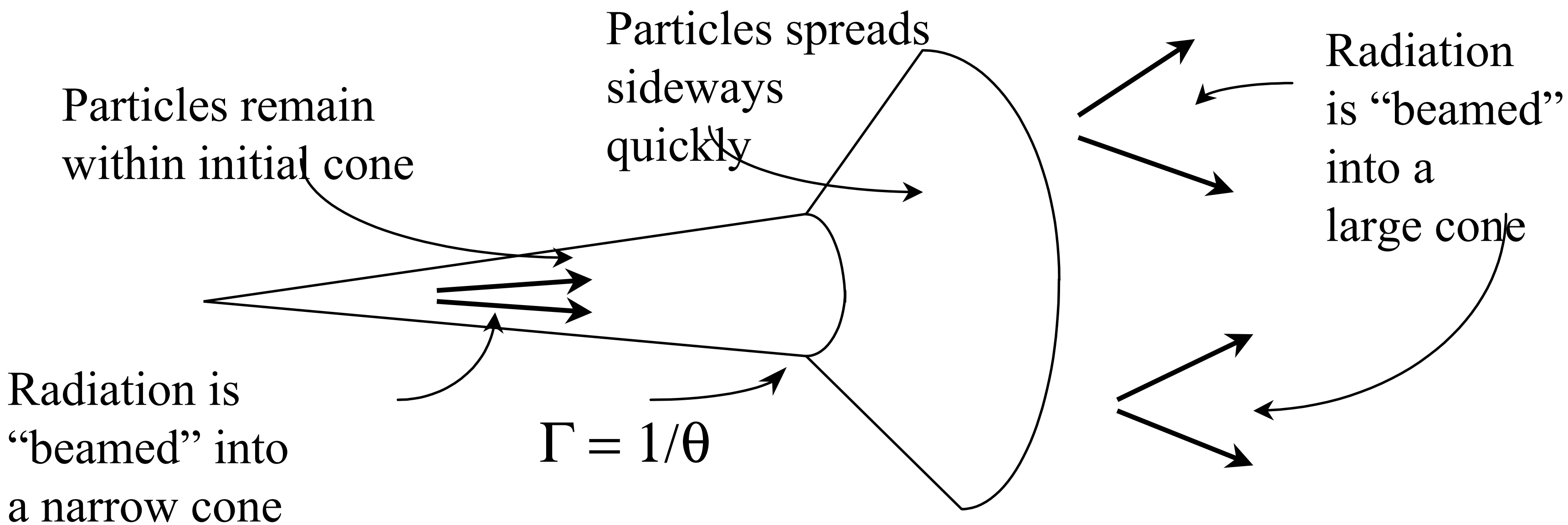


Achromatic break

Gamma-Ray Burst Energies

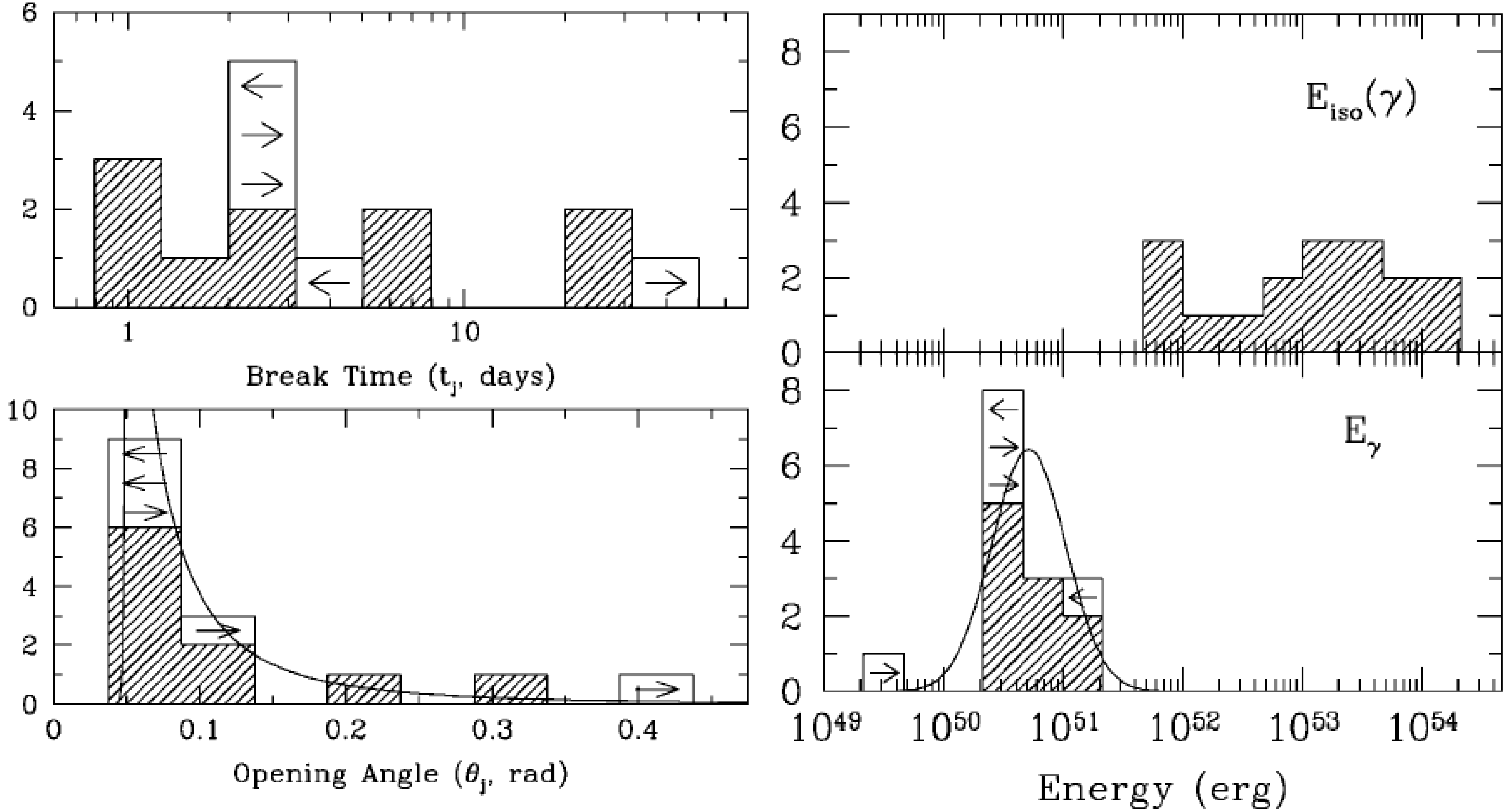
GRB	F_γ	z	d_L	$E_{iso}(\gamma)$	t_j	θ_j	E_γ
970228	11.0	0.695	1.4	22.4	...	...	...
970508	3.17	0.835	1.8	5.46	25	0.293	0.234
970828	96.0	0.958	2.1	220	2.2	0.072	0.575
971214	9.44	3.418	9.9	211	>2.5	>0.056	>0.333
980613	1.71	1.096	2.5	5.67	>3.1	>0.127	>0.045
980703	22.6	0.966	2.1	60.1	7.5	0.135	0.544
990123	268	1.600	3.9	1440	2.04	0.050	1.80
990506	194	1.30	3.0	854	...	...	...
990510	22.6	1.619	4.0	176	1.20	0.053	0.248
990705	93	0.84	1.8	270	~1	0.054	0.389
990712	6.5	0.433	0.8	5.27	>47.7	>0.411	>0.445
991208	100	0.706	1.4	147	<2.1	<0.079	<0.455
991216	194	1.02	2.3	535	1.2	0.051	0.695
000131	41.8	4.500	13.7	1160	<3.5	<0.047	<1.30
000301C	4.1	2.034	5.3	46.4	5.5	0.105	0.256
000418	20.0	1.119	2.5	82.0	25	0.198	1.60
000926	6.2	2.037	5.3	297	1.45	0.051	0.379

Beaming (1)



The flux suddenly drops off achromatically. The afterglow models give Γ as a function of time, so the break tells you what θ is.

Beaming (2)



6
4
2
0
1
10
Break Time (t$_j$, days)
10
8
6
4
2
0
0
0.1
0.2
0.3
0.4
Opening Angle (θ_j, rad)
8
6
4
2
0
$E_{iso}(\gamma)$
8
6
4
2
0
E_γ
10^{49}
10^{50}
10^{51}
10^{52}
10^{53}
10^{54}
Energy (erg)