

Experiment 14

Hall Effect and Resistivity Measurements in Doped GaAs

Note: This laboratory manual is based on a manual for a very similar experiment from the Physics 111 Advanced Undergraduate Laboratory at UC Berkeley (see references).

14.1 Resistivity and the van der Pauw Method

To determine the electrical resistance of an object, one measures the relationship between two quantities: the current run through the object, and the voltage that arises due to that resistance. In this experiment, a four-probe technique is used, in which the current source and voltage measurement contacts are separated. One device is used to generate current, and run that current through the contact resistances as well as the resistor you're trying to measure. Then a second high-input-impedance device, which draws very little current, is used to sample only the voltage across the resistor, without including the resistance of leads or contacts.

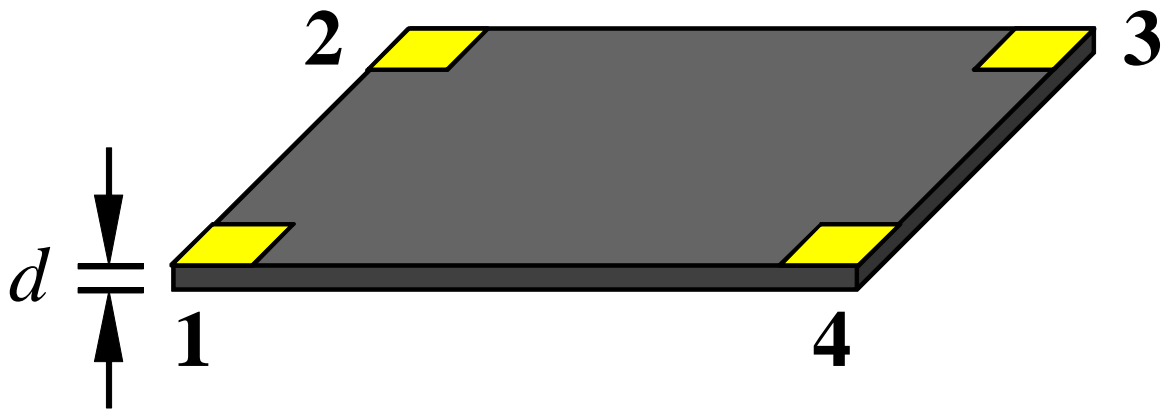


Figure 14.1: Contact arrangement and labelling for the GaAs sample.

The particular four-probe technique used in this experiment - the *van der Pauw method* - involves four contacts (connected to four leads) placed around the edge of a thin sample; these are labeled 1, 2, 3, and 4 in a clockwise manner, as shown in Fig. 14.1. To reduce the effects of unwanted thermoelectric voltages, all four leads should consist of the same material, and all four contacts should consist of the same material. The van der Pauw method is not only a useful resistivity measurement technique for materials research; it finds wide use in the semiconductor industry, where it is used to characterize semiconductor wafers.

(Note: for the analysis that follows, we assume that the sample be thin, isotropic, simply connected, and that the contacts are at the edge of the sample, in order to apply the van der Pauw theorem; fortunately, these are all satisfied here. See the Appendix for more details.)

We start by defining the following parameters:

- ρ = sample resistivity
- d = sample thickness
- I_{12} = positive dc current I injected into contact 1 and taken out of contact 2 (likewise for I_{14} , I_{21} , and I_{41})
- V_{43} = dc voltage measured between contacts 4 and 3 ($V_4 - V_3$) (likewise for V_{23} , V_{34} , and V_{32})

First consider the case where we drive the current I_{12} into contact 1 and out from contact 2, and measure the resulting voltage V_{43} between contacts 4 and 3. We can also reverse the polarity of the current (to I_{21}) and measure the reversed voltage V_{34} . In addition, we may repeat for the remaining two values V_{23} and V_{32} . For each of these configurations of current flow and voltage measurement, we will obtain a value for the *transresistance* denoted as, for example, $R_{ij,kl} = V_{kl}/I_{ij}$. The four such transresistances that will be used for this experiment are:

$$R_{12,43} \equiv \frac{V_{43}}{I_{12}}, \quad R_{14,23} \equiv \frac{V_{23}}{I_{14}}, \quad R_{21,34} \equiv \frac{V_{34}}{I_{21}}, \quad R_{41,32} \equiv \frac{V_{32}}{I_{41}} \quad (14.1)$$

From measurements of two such transresistances, the resistivity of the sample may then be determined by the van der Pauw equation,

$$\exp\left(-\frac{\pi d}{\rho} R_{12,43}\right) + \exp\left(-\frac{\pi d}{\rho} R_{14,23}\right) = 1. \quad (14.2)$$

A detailed derivation of this expression is shown in the Appendix. Note that the same relation – along with Equation 14.3 below – holds for the pair $R_{21,34}$ and $R_{41,32}$. Further simplification of this expression (also shown in the Appendix) leads to an expression for the resistivity:

$$\rho = \frac{\pi d}{\ln 2} \left(\frac{R_{12,43} + R_{14,23}}{2} \right) \cdot f\left(\frac{R_{12,43}}{R_{14,23}}\right), \quad (14.3)$$

where the function $f(x)$ satisfies the equation

$$\exp\left(-\frac{\ln 2}{f(x)}\right) \cdot \cosh\left(\left(\frac{x-1}{x+1}\right) \frac{\ln 2}{f(x)}\right) = \frac{1}{2}. \quad (14.4)$$

This function $f(x)$ does not have a closed-form analytic expression, and must be solved numerically for each value of x . However, for small x , an approximate expression $f(x) \approx 1/\cosh(\ln(x)/2.403)$ may be used, with an error of less than 0.1% for $x < 2.2$ and less than 1% for $x < 4.3$.

14.2 The Hall coefficient

In 1879, E. H. Hall reported a method for determining the sign of the charge carriers in a conductor, making use of the fact that the deflection of current flowing in a magnetic field – the *Hall effect* – has a direction which depends on the polarity of the moving charge. Much like van der Pauw resistivity measurements, Hall effect measurements are widely used for materials characterization both in research and in industry. But perhaps of far greater importance are the widespread uses of Hall sensors – devices employing the Hall effect to measure magnetic fields – in countless applications, from engine cylinder timing in vehicles to digital compasses in cellphones. In this experiment, the combination of Hall effect measurements with resistivity measurements will allow the sign, density, and mobility of the charge carriers in your sample to be determined, as well as how they depend on temperature and magnetic field.

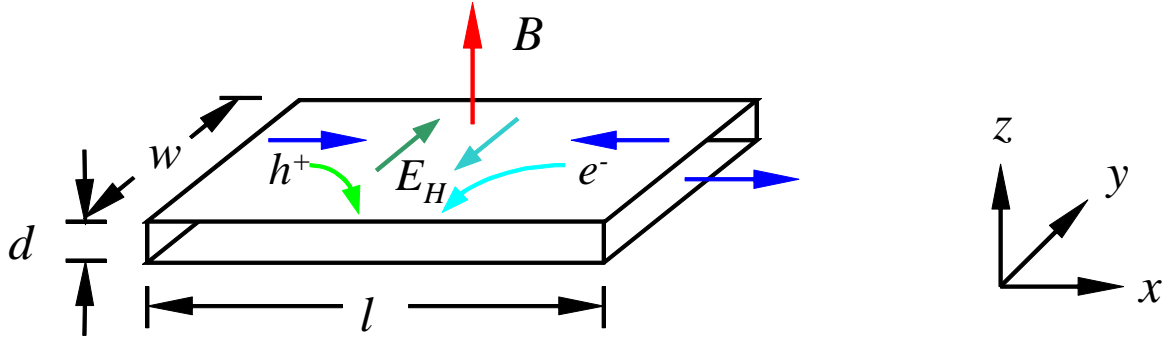


Figure 14.2: Schematic illustration of the Hall effect.

Consider an electrically conducting (or semiconducting) sample placed in a magnetic field \vec{B} pointed in the \hat{z} direction. Suppose we pass a current through that sample perpendicular to the magnetic field in the \hat{x} direction. Then the free carriers in the sample experience a force \vec{F} given by the Lorentz equation

$$\vec{F} = q(\vec{v} \times \vec{B}), \quad (14.5)$$

where q is the charge of a carrier and v its velocity. This Lorentz force deflects free carriers towards the $+\hat{y}$ direction, as shown in Figure 14.2. If the free carriers are electrons ($q = -e$), this results in an excess of negative charge on the $+\hat{y}$ side of the sample. This charge distribution results in an electric field \vec{E}_H pointing in the $-\hat{y}$ direction, which both balances the Lorentz force (so as to keep current flowing along the \hat{x} direction) and yields a voltage difference $V_H = w\vec{E}_H$ across a sample of width w . (Why is the current I_y zero in equilibrium?)

In equilibrium, when we consider the balance of the forces on the free carriers,

$$\vec{F} = q(\vec{E}_H + \vec{v} \times \vec{B}) = 0, \quad (14.6)$$

we obtain the expression for the electric field \vec{E}_H due to the Hall effect,

$$\vec{E}_H = -\vec{v} \times \vec{B}. \quad (14.7)$$

In a semiconductor sample, since V_H is proportional to E_H and the direction of E_H will depend on whether the charge carriers are predominantly negatively (**n-type**; electrons) or positively (**p-type**; holes) charged, by measuring the sign of the Hall voltage V_H we can determine whether we have an n -type or p -type semiconductor.

Suppose the charge carriers in our sample are electrons; *i.e.*, our semiconductor is n -type. Taking into account the dimensions of the sample, our total current is related to the density of electrons n and their **drift velocity** (their average velocity) v_x as

$$I_x = (-env_x) \cdot (wd), \quad (14.8)$$

which is just the product of the current density $J_x = -env_x$ and the cross-sectional area $A = wd$ of the sample. Combining Equations 14.7, 14.8, and the relation $V_H = wE_H$, we obtain the following expression by which the electron density may be determined:

$$V_H = -\frac{I_x B_z}{end}. \quad (14.9)$$

Using the definition of the Hall coefficient

$$R_H = \frac{E_H}{J_x B_z}, \quad (14.10)$$

we obtain the Hall coefficient for electrons

$$R_H = -\frac{1}{en}. \quad (14.11)$$

For a p -type semiconductor, the equation is similar to Equation 14.11; you should derive this equation as an exercise.

Apply the current I_{13} and measure voltage V_{24} (see Figure 14.1) with magnetic field parallel and antiparallel to the \hat{z} -axis as shown in Figure 14.2. (Note that, by this convention, \hat{z} is not vertical in this experiment.) Reverse the polarity of the current I_{31} and measure V_{42} with different magnetic field alignment. Similarly, you may obtain the other two values V_{31} and V_{13} with current flow I_{24} and I_{42} . The V_{24} , V_{42} , V_{31} , and V_{13} are usually not the Hall voltage, but the Hall voltage can be calculated by the values with magnetic field parallel and antiparallel as in Equation 14.12:

$$V_H = \frac{1}{2} \left((V_{24})_{\uparrow\uparrow} - (V_{24})_{\uparrow\downarrow} \right), \quad (14.12)$$

where $(V_{24})_{\uparrow\uparrow}$ and $(V_{24})_{\uparrow\downarrow}$ represent the value of V_{24} obtained from magnetic field parallel or antiparallel to \hat{z} as shown in Figure 14.2. (Why is the Hall voltage half the value of the difference between V_{24} with the field parallel and antiparallel alignments?) Similarly, the other three Hall voltages can be calculated from V_{42} , V_{31} , and V_{13} . The Hall voltage is averaged from V_{24} , V_{42} , V_{31} , and V_{13} , and is used to calculate the carrier density.

14.3 Mobility

Mobility is a quantity relating the drift velocity to the applied electric field across a material. For a semiconductor with both electrons and holes, the drift velocities of the electrons and holes in the \hat{x} direction are:

$$(v_e)_x = -\mu_e E_x \quad (14.13a)$$

and

$$(v_h)_x = +\mu_h E_x \quad (14.13b)$$

where $(v_e)_x$, $(v_h)_x$, μ_e , and μ_h are the drift velocities (along the \hat{x} direction) and mobilities for electrons and holes, respectively.

Since the current densities for electrons and holes are:

$$(J_e)_x = -en(v_e)_x \quad (14.14a)$$

and

$$(J_h)_x = +ep(v_h)_x \quad (14.14b)$$

where p is the the density of holes, the total current density in the \hat{x} direction is:

$$J_x = (J_e)_x + (J_h)_x = -en(v_e)_x + ep(v_h)_x. \quad (14.15)$$

Using Equations 14.13a, 14.13b, and 14.15, and using Ohm's Law in the form $\vec{J} = \sigma \vec{E}$, where σ is the conductivity, the following equation can be obtained:

$$\sigma \equiv \frac{1}{\rho} = e(n\mu_e + p\mu_h). \quad (14.16)$$

For samples in which the carriers are known to be either exclusively electrons or exclusively holes, only that particular corresponding term in Equation 14.16 needs to be considered.

14.4 The Experiment

In this lab, you will take measurements of Hall voltage and sample resistivity as a function of temperature (from room temperature to 120 °C), as well as calculate the Hall coefficient, carrier density, and carrier mobility, in addition to determining what type of material you are measuring: *n*-type or *p*-type. The sample in this lab is a doped single-crystal of the semiconductor GaAs(100),

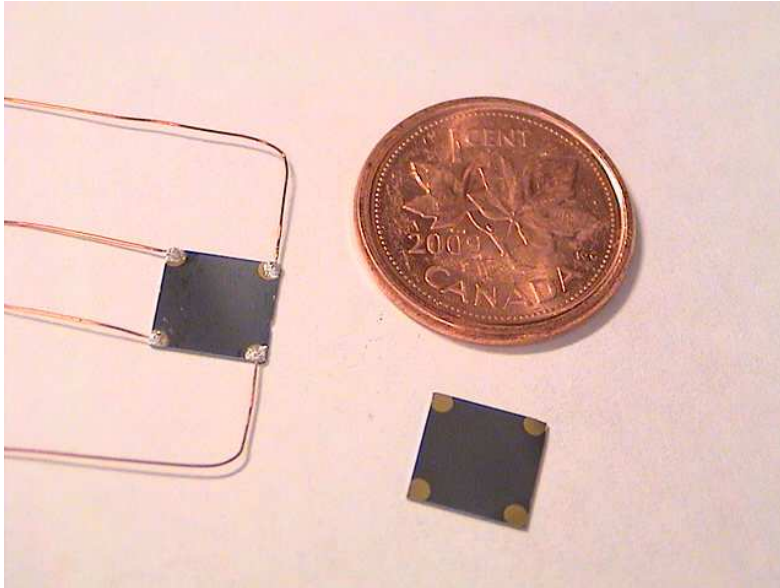


Figure 14.3: GaAs samples with and without attached leads.

donated by Dan Beaton, a former Physics 409 teaching assistant. The geometry of the sample is a 7 mm \times 7 mm square with a thickness of $350 \pm 25 \mu\text{m}$. The four corners of the sample are coated with Ti/Pt/Au layers, onto which wires are pressed using indium as a contact material. (These contacts are very delicate - please do not touch them!) Figure 14.3 shows the samples before and after making the wire connection. The golden colour on the corners is the Ti/Pt/Au coating, which enhances the electrical contact to the sample.

A block diagram of the apparatus for this experiment is shown in Figure 14.4. The magnet is powered by the current from a KEPCO power supply. The temperature of the sample is controlled by the OMEGA CNi16D temperature controller, which monitors the sample temperature by thermocouple measurement and controls the GPS-3030DD heater power supply accordingly. The Agilent E3640A delivers the current to the sample and the Agilent 34901A measures the voltage from the sample through the Wire Connection and Agilent 34903A. Each device is controlled by the LabVIEW program on the computer, to which the Agilent devices are controlled by the computer through a GPIB connection, and the OMEGA CNi16D is connected through an RS-232 serial connection.

Figure 14.5 shows the sample connection with the current supply and multimeter through the Agilent 34903A and 34901A. The relay of the Agilent 34903A (Channels 1 through 8) in the figure is in the Normally Closed (NC) state. Using different combinations of open and closed states for Channels 1 through 8, obtain the required measurements. A detailed list of open and closed states for 34903A channels and the list of On and Off states for 34901A channels are shown in Table 14.1.

This lab is controlled by the LabVIEW program "Hall Effect.vi". Using this software, the

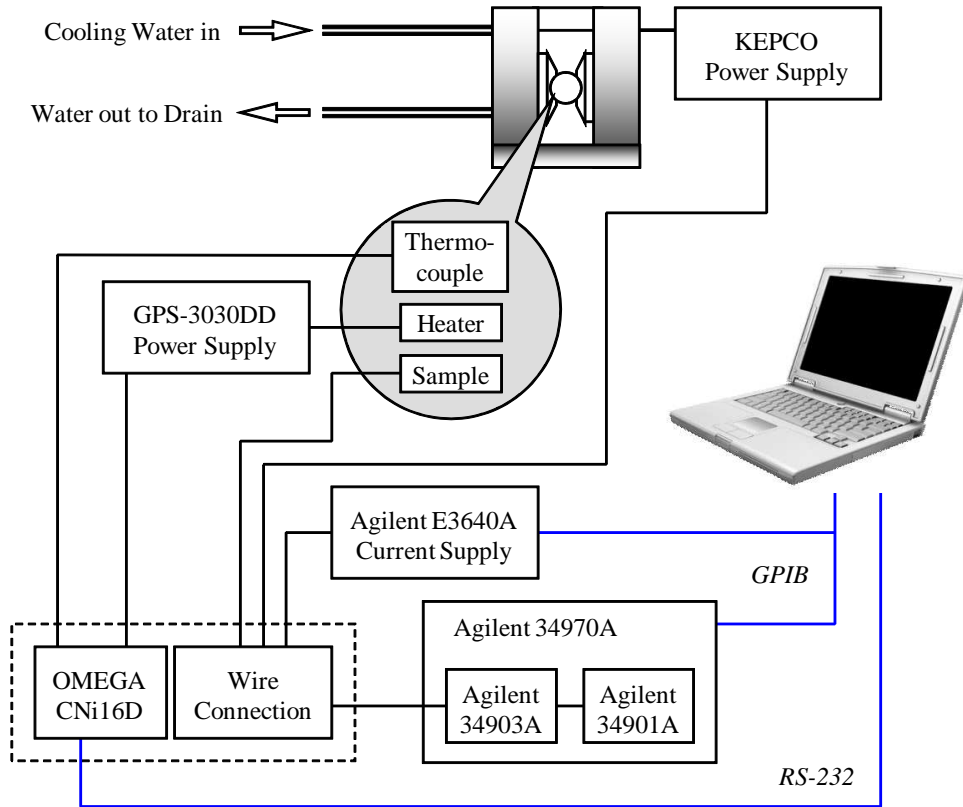


Figure 14.4: Block diagram of the resistivity and Hall coefficient apparatus.

channel selection and switching details for magnetic field, current, and voltage measurements are automated, as are temperature and magnet current control, based on the settings you provide. For more detailed information about how to use this software, please read the introduction on the front page of the program.

14.5 Procedure

- **Power on the electromagnet:**

To prevent the magnet from overheating, turn on the cooling water before you power on the current supply. The KEPCO power supply is set in the constant current mode and the current is between 0.6 and 8 A. For a simple coil setup (such as a Helmholtz coil pair), the magnetic field in the centre of the pair would be proportional to the current in the coil. However, in this lab we are using an electromagnet where the field is *not* exactly proportional to the current (why?). For more detailed information regarding this, please read the introduction to the software. Note also that for both the Hall coefficient and carrier type calculations, you will need to find a way to determine the direction of the magnetic field.

- **Temperature measurement and control:**

The temperature is measured by the OMEGA CNi16D temperature controller, using a type-K (chromel-alumel) thermocouple. If the measured temperature is lower than the setting temperature, there is an analog voltage output from the CNi16D to the Instek GPS-3030DD

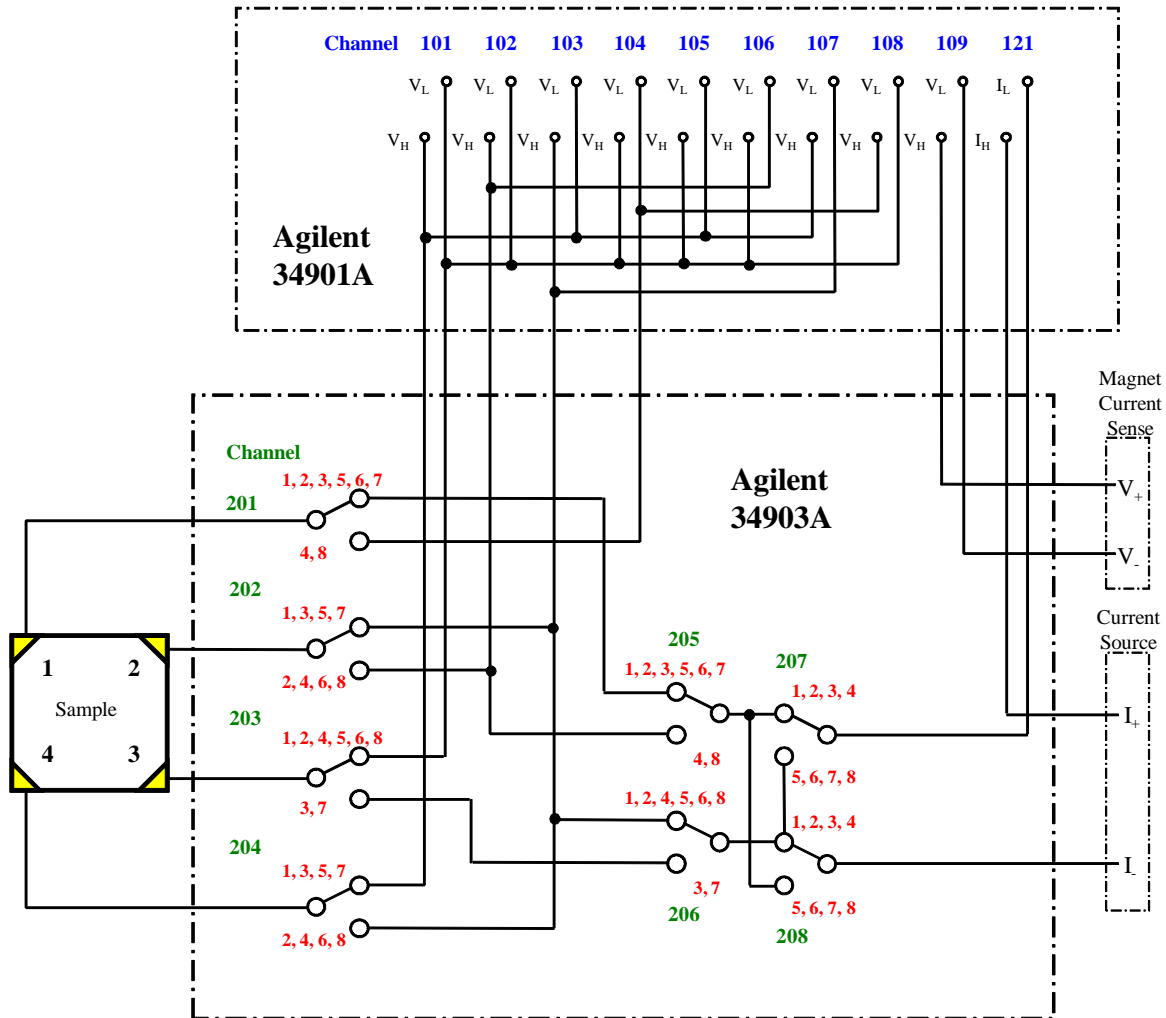


Figure 14.5: Current supply, multimeter, and sample connection through the Agilent 34903A and 34901A.

power supply. The larger the difference between the measured and set temperatures, the larger the analog signal to the heater. The heater is a $10\ \Omega$, 100 W max. power resistor, but as its temperature increases the maximum power rating of this resistor is quickly reduced linearly to 20 W at $120\ ^\circ\text{C}$. In this lab, the temperature is measured and controlled by the computer through the RS-232 port.

- **Resistivity and Hall coefficient measurement:**

Fortunately, in this experiment both the resistivity and Hall voltage can be measured by the LabVIEW program "Hall Effect.vi"; this software handles the complicated interconnection details described above, and automates the temperature control, field control, and data collection. Even though some of these details are taken care of for you, make sure you understand what is going on inside the "black box". For more detailed information about how to use this software, please read the introduction on the front page of the program.

	1	2	3	4	5	6	7	8	B*	I†
34901A										
Ch. 101	On									
Ch. 102		On								
Ch. 103			On							
Ch. 104				On						
Ch. 105					On					
Ch. 106						On				
Ch. 107							On			
Ch. 108								On		
Ch. 109									On	
Ch. 121										On
34903A‡										
Ch. 201	NC	NC	NC	NO	NC	NC	NC	NO	When measuring magnetic field and current, all switches maintain their former states.	
Ch. 202	NC	NO	NC	NO	NC	NO	NC	NO		
Ch. 203	NC	NC	NO	NC	NC	NC	NO	NC		
Ch. 204	NC	NO	NC	NO	NC	NO	NC	NO		
Ch. 205	NC	NC	NC	NO	NC	NC	NC	NO		
Ch. 206	NC	NC	NO	NC	NC	NC	NO	NC		
Ch. 207	NC	NC	NC	NC	NO	NO	NO	NO		
Ch. 208	NC	NC	NC	NC	NO	NO	NO	NO		
Measurement										
Current	I_{12}	I_{14}	I_{13}	I_{24}	I_{21}	I_{41}	I_{31}	I_{42}	–	I
Voltage	V_{43}	V_{23}	V_{24}	V_{31}	V_{34}	V_{32}	V_{42}	V_{13}	V_{mag}	–
Trans-R	$R_{12,43}$	$R_{14,23}$	–	–	$R_{21,34}$	$R_{41,32}$	–	–	–	–

Table 14.1: Channel configuration map for the Agilent 34901A and 34903A, as used for particular measurements.

*Magnet current measurement

†Sample current measurement

‡NC = Normally Closed (*i.e.* connected by default); NO = Normally Open.

14.6 Experimental Goals

In this experiment, the goal is to extract as much information about the sample using the quantities you can measure: resistivity and Hall voltage as functions of temperature and magnetic field. In particular, you should aim to do the following:

- Measure the resistivity ρ and Hall coefficient R_H of the sample at room temperature for several magnetic field values.
- Measure the resistivity and Hall coefficient of the sample as a function of temperature.
- Determine what type of material you have measured: *p*-type or *n*-type.
- Extract the electron or hole concentrations for the sample as a function of temperature.
- Extract the Hall coefficient R_H and the mobility μ as a function of temperature.
- Measure the *magnetoresistance*, the change in resistance as a function of magnetic field.

14.7 Appendix: The van der Pauw Theorem

14.7.1 Conditions of applicability

The van der Pauw theorem allows one to calculate the resistivity of a sample from two 4-point transresistance measurements from the van der Pauw equation, without knowing *any specifics* about the geometry or contact positions, as long as the following conditions are satisfied:

1. The sample is “flat”; i.e., planar, of uniform thickness d , and sufficiently thin ($d \ll l, w$);
2. The sample shape is simply connected (no holes or interior boundaries);
3. The sample is homogeneous and isotropic (resistivity is the same everywhere and in every direction; $\rho_{xx} = \rho_{yy} = \rho_{zz} = \rho$);
4. The electrical contacts are on (or near) the periphery of the sample.

If these conditions hold, then by the use of *conformal mapping* – a topic you may have encountered in a complex analysis course – the problem of calculating the electrostatic potentials for one shape can be mapped by appropriate transformations onto any other shape satisfying these conditions. While the solution shown below is for the case of contacts on the edge of a thin infinite half-plane (which is clearly not the situation we have in this experiment!), it can be shown via conformal mapping that it is true for contacts on the periphery of any simply connected finite shape as well. Thus the solution – the van der Pauw *equation* – applies to any sample satisfying the above conditions; this is the van der Pauw *theorem*. (Where the above conditions are violated, the van der Pauw equation can often be used with modifications, but the simple, geometry-independent solution may no longer hold).

Note that the Hall coefficient calculation does not rely on this theorem, but rather on the fact that all current traversing the sample from, say, contact 1 to contact 3 must pass between the other two contacts, across which the Hall voltage is measured. Can you see why?

14.7.2 Derivation of the van der Pauw equation

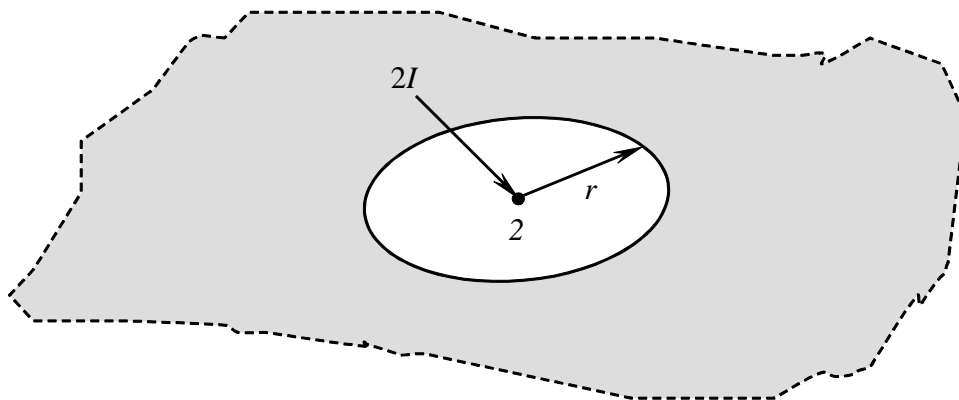


Figure 14.6: Infinite sample with a point labelled **2**.

Let a sample be infinite in all directions. Then apply a current $2I$ to a point **2** as shown in Figure 14.6. This current flows away from **2** with radial symmetry out to infinity. Let d be the

thickness of the sample and ρ the resistivity. At a distance r from point **2** the current density is:

$$\vec{J} = \frac{2I}{2\pi r d} \hat{r}. \quad (14.17)$$

The electric field \vec{E} is radially oriented, and according to Ohm's law,

$$\vec{E} = \rho \vec{J} = \frac{\rho I}{\pi r d} \hat{r}. \quad (14.18)$$

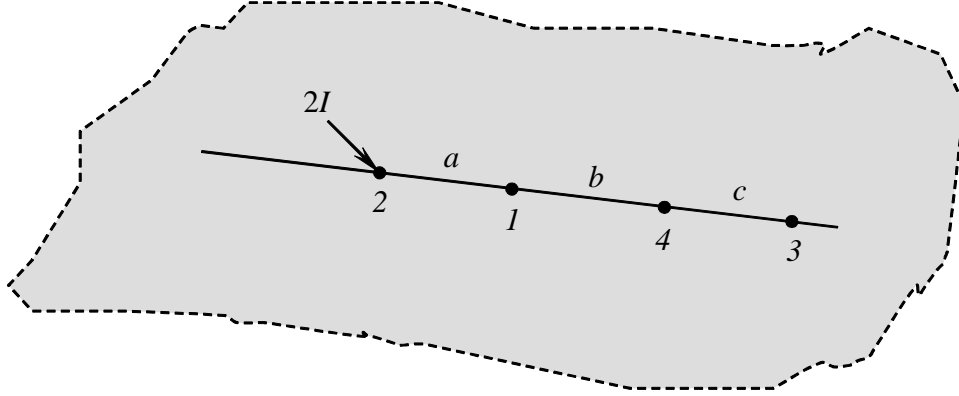


Figure 14.7: Current through sample points on a line.

Suppose there are also points **3** and **4** lying on a straight line with **2** as shown in Figure 14.7. The potential difference V_{43} between points **4** and **3** with a current flow into point **2** is:

$$(V_3 - V_4)_{\text{in}} = - \int_4^3 \vec{E} \cdot d\vec{l} = - \frac{\rho I}{\pi d} \int_4^3 \frac{dr}{r} = - \frac{\rho I}{\pi d} \ln \left(\frac{a+b+c}{a+b} \right). \quad (14.19)$$

Note that no current flows perpendicular to the line; therefore, the result is also valid if the half of the sample on one side of this line is removed, yielding an infinite half-plane - with half the current (I instead of $2I$). This is the geometry applicable to measurements involving contacts at the edge of a sample, and we will consider this case going forward.

Now consider a current I drained from another point **1** lying on the same line. The potential difference between point **4** and **3** with a current flow out point **1** is:

$$(V_3 - V_4)_{\text{out}} = - \int_4^3 \vec{E} \cdot d\vec{l} = + \frac{\rho I}{\pi d} \int_4^3 \frac{dr}{r} = \frac{\rho I}{\pi d} \ln \left(\frac{b+c}{c} \right). \quad (14.20)$$

Now combining Equations 14.19 and 14.20, we find that the potential difference between points **4** and **3**, with current flowing into the sample at point **2** and out from the sample at point **1**, is:

$$(V_3 - V_4)_{\text{total}} = (V_3 - V_4)_{\text{in}} + (V_3 - V_4)_{\text{out}} = \frac{\rho I}{\pi d} \ln \left(\frac{(b+c)(a+b)}{b(a+b+c)} \right). \quad (14.21)$$

The transresistance $R_{21,34}$ is then:

$$R_{21,34} = \frac{V_{34}}{I_{21}} = \frac{(V_3 - V_4)_{\text{total}}}{I_{21}} = \frac{\rho}{\pi d} \ln \left(\frac{(b+c)(a+b)}{b(a+b+c)} \right). \quad (14.22)$$

For measurement consistency, the result should be the same if we reverse the current from I_{21} to I_{12} and measure the voltage V_{43} instead of V_{34} ; that is, $R_{21,34} = R_{12,43}$. We now manipulate Equation 14.22 (after making this substitution) to get:

$$\exp\left(-\frac{\pi d}{\rho}R_{12,43}\right) = \frac{b(a+b+c)}{(b+c)(a+b)}. \quad (14.23)$$

If we instead consider a current I flowing into point **1** and out from point **4**, and measure the potential difference between points **3** and **2**, we obtain a similar expression in terms of $R_{14,23}$:

$$\exp\left(-\frac{\pi d}{\rho}R_{14,23}\right) = \frac{ac}{(b+c)(a+b)}. \quad (14.24)$$

Summing Equations 14.23 and 14.24 yields a simple expression, known as the *van der Pauw equation*:

$$\boxed{\exp\left(-\frac{\pi d}{\rho}R_{12,43}\right) + \exp\left(-\frac{\pi d}{\rho}R_{14,23}\right) = 1} \quad (14.25)$$

To simplify the algebra that follows, we define:

$$x_1 \equiv \pi d R_{12,43} \quad \text{and} \quad x_2 \equiv \pi d R_{14,23}. \quad (14.26)$$

We note that in place of x_1 and x_2 we can make the following substitutions:

$$x_1 = \frac{1}{2}((x_1 + x_2) + (x_1 - x_2)) \quad \text{and} \quad x_2 = \frac{1}{2}((x_1 + x_2) - (x_1 - x_2)). \quad (14.27)$$

Now combining Equations 14.25 and 14.26 and making the substitutions in Equation 14.27, we get:

$$\exp\left(-\frac{1}{2\rho}((x_1 + x_2) + (x_1 - x_2))\right) + \exp\left(-\frac{1}{2\rho}((x_1 + x_2) - (x_1 - x_2))\right) = 1. \quad (14.28)$$

This can be factored to obtain:

$$\exp\left(-\frac{x_1 + x_2}{2\rho}\right) \cdot \left(\exp\left(-\frac{x_1 - x_2}{2\rho}\right) + \exp\left(\frac{x_1 - x_2}{2\rho}\right)\right) = 1. \quad (14.29)$$

Using the hyperbolic trig identity $\cosh x = \frac{1}{2}(e^x + e^{-x})$, this becomes:

$$\exp\left(-\frac{x_1 + x_2}{2\rho}\right) \cdot \cosh\left(\frac{x_1 - x_2}{2\rho}\right) = \frac{1}{2}. \quad (14.30)$$

We now define the quantity f , where

$$f \equiv \ln 2 \cdot \frac{2\rho}{x_1 + x_2}; \quad (14.31)$$

substituting this into Equation 14.30 gives:

$$\exp\left(-\frac{\ln 2}{f}\right) \cdot \cosh\left(\frac{x_1 - x_2}{x_1 + x_2} \cdot \frac{\ln 2}{f}\right) = \frac{1}{2}. \quad (14.32)$$

Finally, dividing both the numerator and denominator in the cosh by x_2 , and using that $x_1/x_2 = R_{12,43}/R_{14,23}$ (see Equation 14.26), we get:

$$\boxed{\exp\left(-\frac{\ln 2}{f}\right) \cdot \cosh\left(\frac{R_{12,43}/R_{14,23} - 1}{R_{12,43}/R_{14,23} + 1} \cdot \frac{\ln 2}{f}\right) = \frac{1}{2}} \quad (14.33)$$

Equation 14.33 defines f implicitly as a function of the ratio $R_{12,43}/R_{14,23}$; there is no exact closed-form solution for f , which must be solved numerically.

After solving for $f\left(\frac{R_{12,43}}{R_{14,23}}\right)$, which depends only on $R_{12,43}/R_{14,23}$ and not ρ , we combine Equations 14.26 and 14.31 to solve for ρ :

$$\rho = \frac{\pi d (R_{12,43} + R_{14,23})}{2 \ln 2} \cdot f\left(\frac{R_{12,43}}{R_{14,23}}\right) \quad (14.34)$$

Using Equation 14.34 along with the solution for f from Equation 14.33, we can calculate the resistivity ρ of any thin, uniform sample using only measurements of $R_{12,43}$ and $R_{14,23}$ and the thickness d . Note that $R_{12,43}$ and $R_{14,23}$ are not special here; any equivalent complementary pair of transresistances may be used.

As previously mentioned, this solution for the thin infinite half-plane can be mapped by appropriate conformal transformations onto arbitrary shapes satisfying certain conditions. A proof of this is beyond the scope of this lab, but if you're interested, you can find details in most textbooks on complex analysis or partial differential equations.

14.8 References

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