All Mechanical Contact Force is Electrical

Physics 102 Class Note. Mark Halpern, 28 January 2003

In class today we calculated an electrical force between two Iron blocks, each one cubic centimeter, being pushed together with 1000 Newtons of force. I made a few algebra errors along the way, and this example is not in the text, so I thought I would re-do it here.

In an Iron block, the nuclei form a rigid lattice and the electrons float around this lattice like a fuzzy cloud. When two blocks of iron are brought into contact these electron clouds repel each other, and move apart slightly.

As we saw in class with our transparency-arrays of plusses and minusses, displacing the electron cloud compared to the protons leaves behind a surface charge density of protons. Therefore, we should expect a tiny positive surface charge density on the face of each of the iron blocks where they come into contact. In this note, we will calculate how large this surface density will be if we know the force applied to push the iron blocks together.

One square cm is 10^{-4} square metres, so our 1,000N force is $10^7\mathrm{N/m}^2$. Since the iron blocks are not being accelerated, this force must be balanced by the force produced by rearranging the electrons and protons of each block. If we calculate the surface charge required to produce $10^7\mathrm{N/m}^2$ we will know the surface charge induced on each iron block.

The blocks are the same as each other, so I presume that the same surface charge is induced in each. Let's call that surface charge density σ (Coulombs/square metre). The surface charge of the lower block makes an electric field $\vec{E} = \sigma\hat{z}/2\epsilon_\circ$ which causes a force density

$$\vec{F}/Area = \sigma\vec{E} = \sigma^2\hat{z}/2\epsilon_\circ.$$

(Verify that the force per area the upper block applies to the lower is identical to this.)Numerically,

$$10^7\mathrm{N/m}^2 = \frac{\sigma^2}{2\epsilon_\circ} = \frac{\sigma^2}{2 \times 8.85 \times 10^{-12}} \frac{(\mathrm{C/m}^2)^2}{(\mathrm{C}^2/\mathrm{Nm}^2)}$$

$$\sigma = 0.0133 \quad \text{Coulombs per square metre}$$

is induced on each surface by the force pushing the blocks together.

Now I will calculate how far the electrons are displaced in order to leave this charge density at the surface. I will use a slightly different approach than I used in class.

The surface charge density, σ, is the volume charge density of the electron cloud ρ in Coulombs per cubic metre times the distance the cloud moved, d, that is,

$$\sigma = \rho d$$

Verify that the units are the same on both sides of this equation.

What is the charge density of the electron cloud? Iron has a density of 9 grams per cubic cm, or 9 tonnes per cubic metre. The atomic weight is 57, so this amounts to 158,000 moles per cubic metre. At 26 electrons per atom, there are

$$\rho = 26 \times 158,000 \times 6.02 \times 10^{23} \times e = 2.5 \times 10^{30}\text{electrons/m}^3 \times 1.6 \times 10^{-19}\text{C/electron}$$

$$\rho = 4.0 \times 10^{11}\text{C/m}^3$$

This is a big sounding number, but it is about right. Remember we decided that the head of a pin contains one coulomb of protons? This is the same number. Anyhow, this lets us find how far the electron cloud has moved.

$$d = \sigma/\rho = 3.3 \times 10^{-14} m.$$

This is not very far at all. Perhaps this is why iron does not feel soft! (Just a joke–things are soft if the proton lattice is squishy. A pillow is soft because the whole pillow moves. The electrons in your head DO NOT penetrate the electron cloud in the pillow.)