Vortex-boson duality in 3+1 dimensions: cuprates meet string theory

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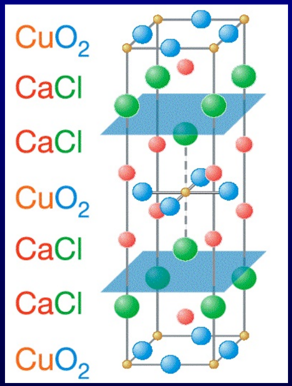
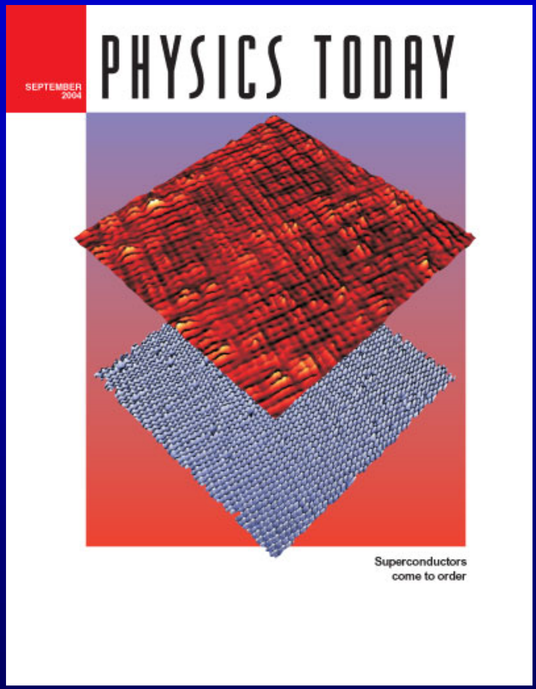


$\longleftrightarrow$

In collaboration with: T. Pereg-Barnea (UT Austin)

Two Experiments: Cuprate superconductors



Pair Wigner crystal in
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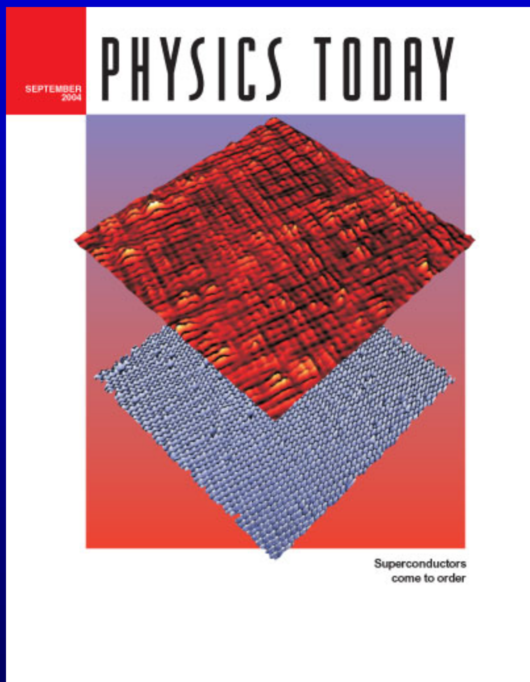

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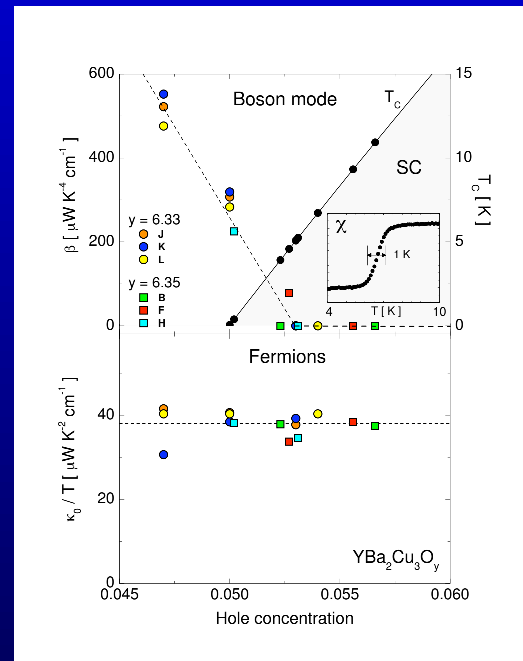
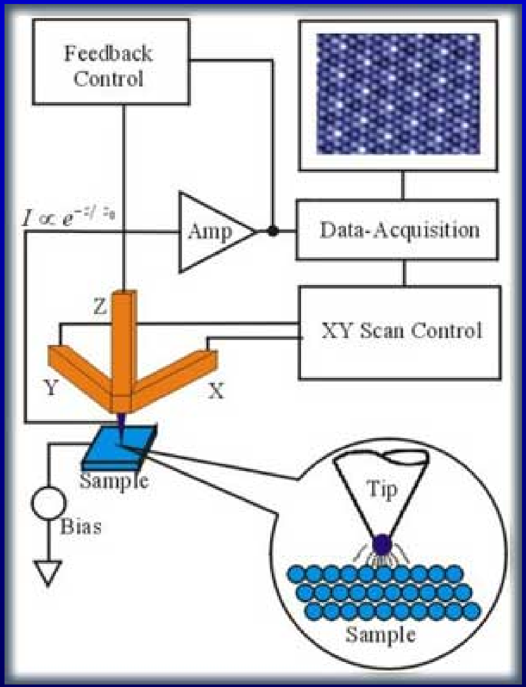


Mysterious bosonic mode in $YBa_2Cu_3O_{6+x}$

[Taillefer et. al, cond-mat/0606645]

Cooper pair Wigner crystal in underdoped cuprates



Scanning Tunneling Microscopy

-images topography and local density of states

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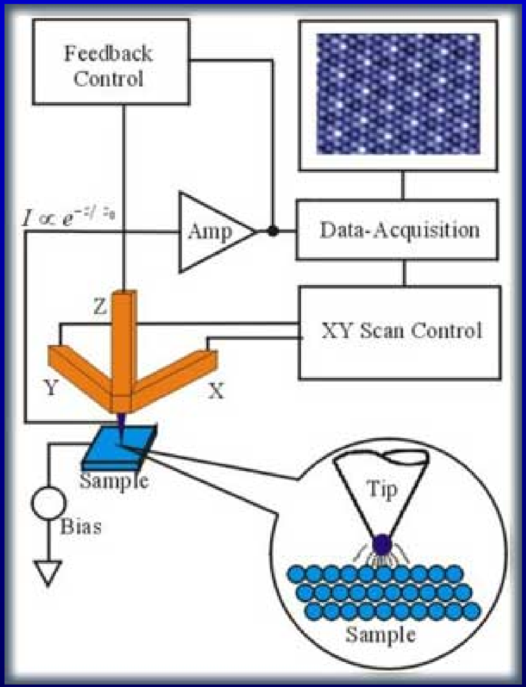

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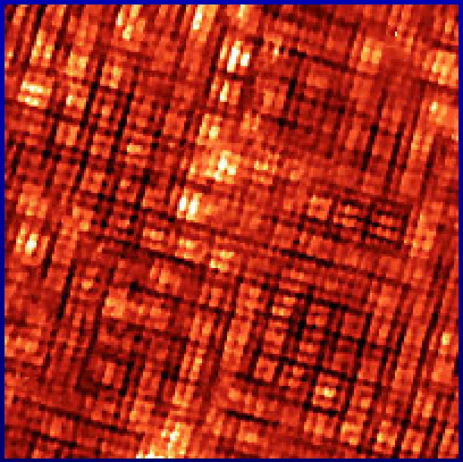
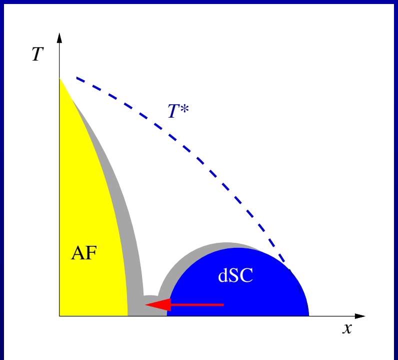
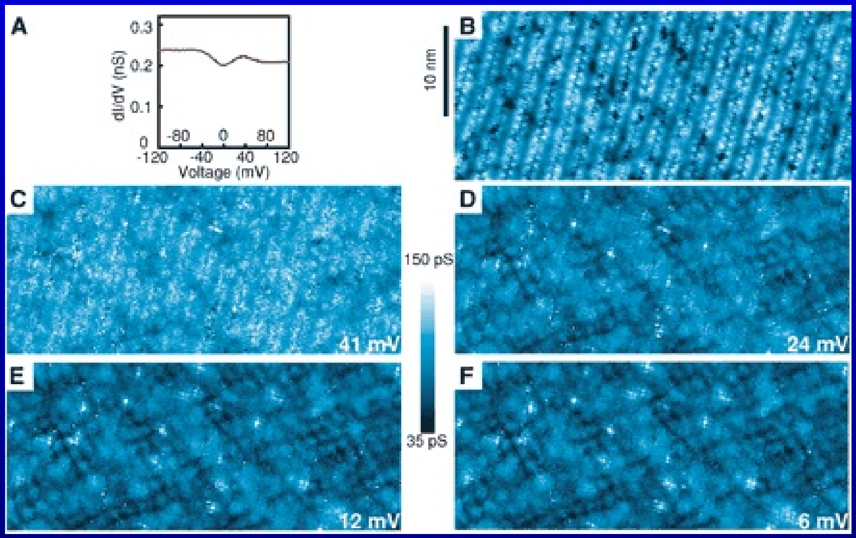


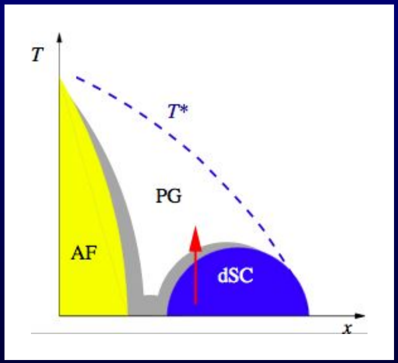


Checkerboard pattern in LDOS of NaCCOC

[Hanaguri *et. al*, Nature 2004]

... more checkerboards





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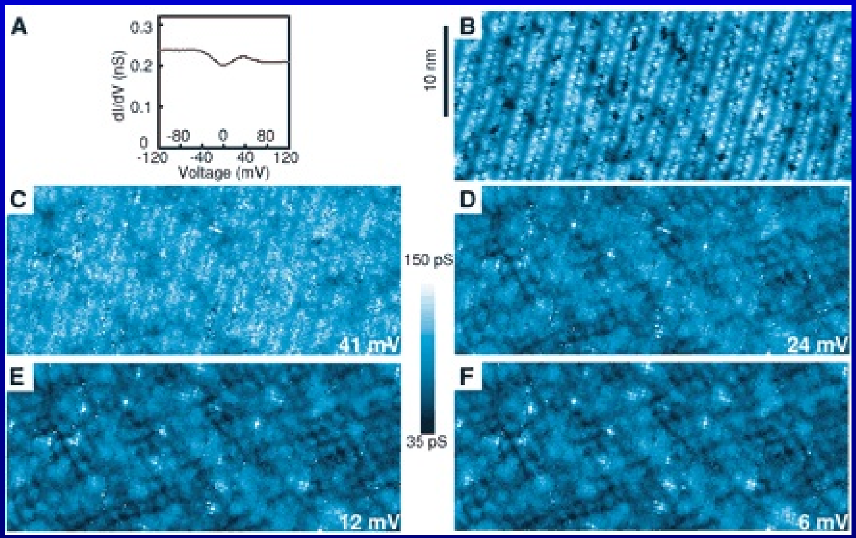

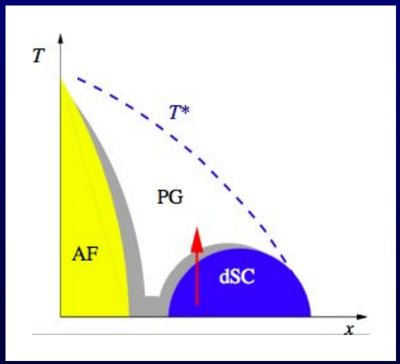

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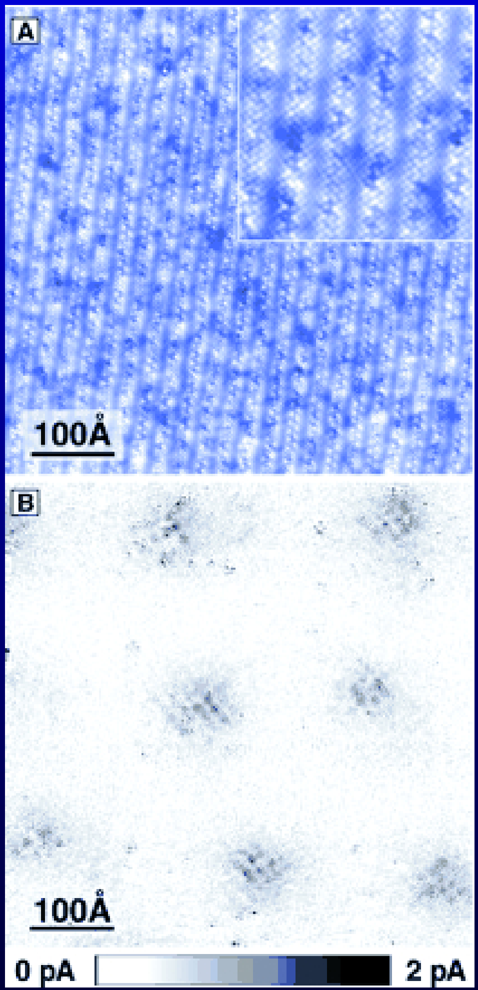
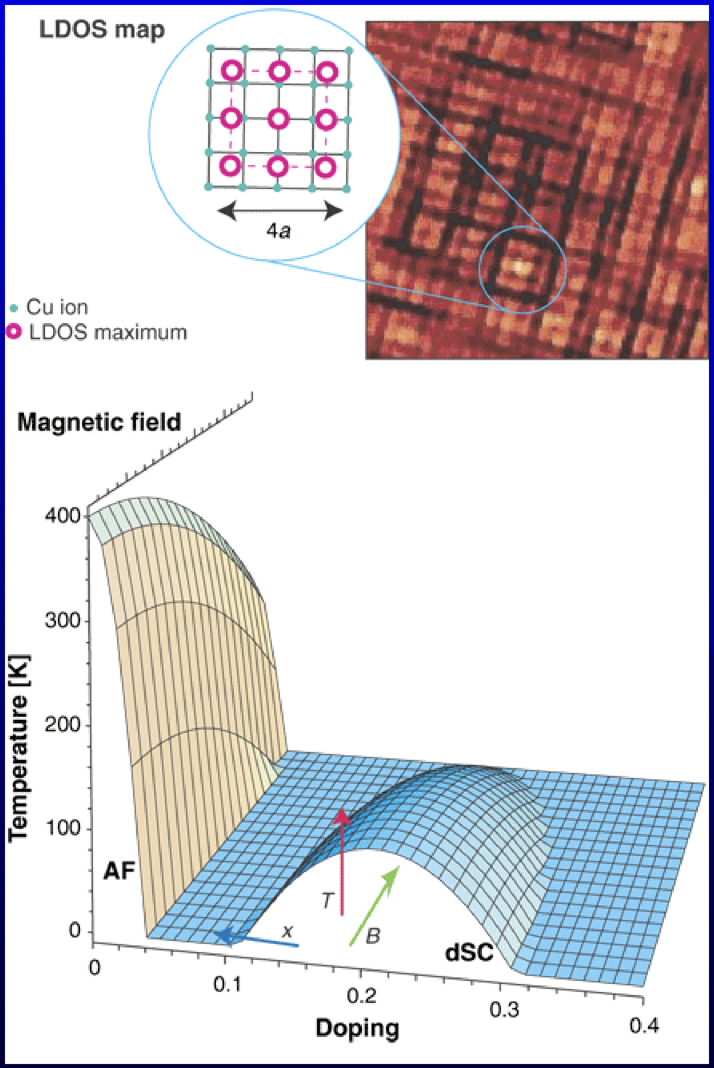


In Magnetic vortices

[Hoffman *et al.* Science 2002]



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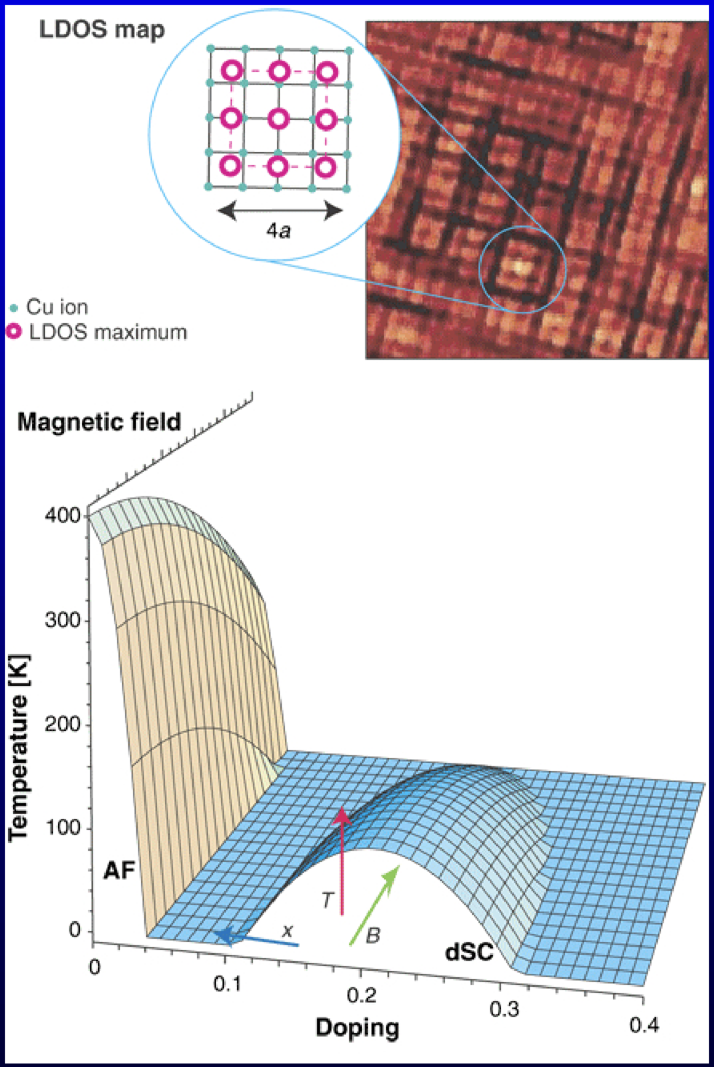
Checkerboard patterns in electron LDOS appear to be universal: encountered outside the superconducting dome in the underdoped region.



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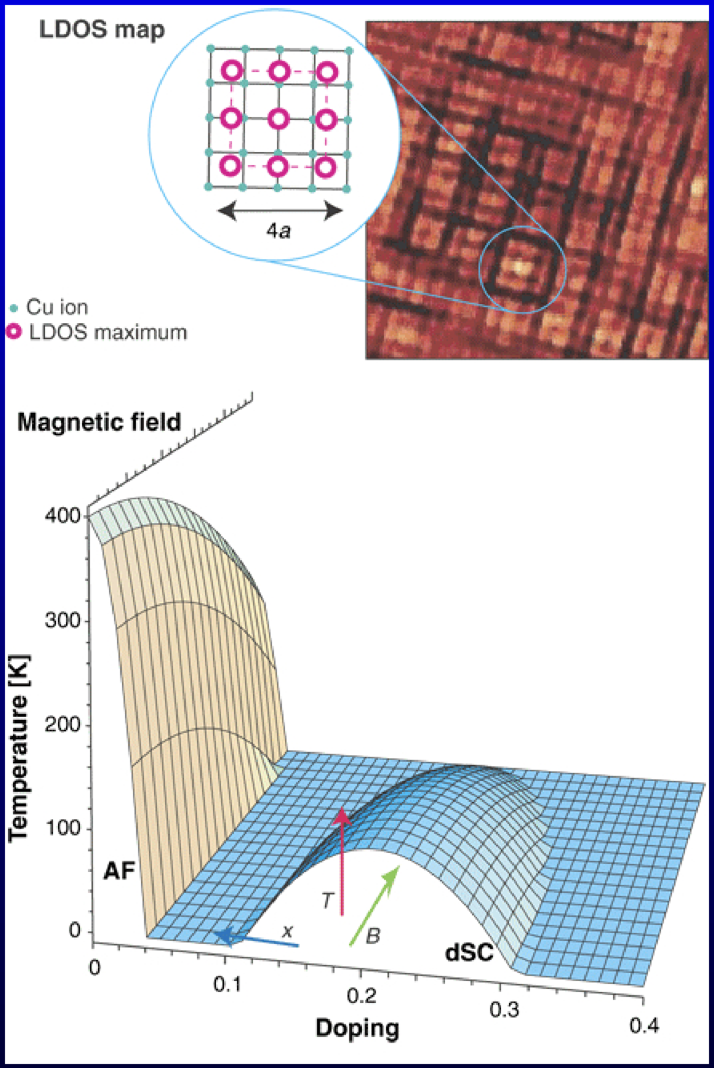

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A: Cooper pair Wigner crystal.

What is pair Wigner crystal?

Simple explanation: upon phase disordering Copper pairs in a superconductor can minimize their interaction energy by forming a crystal.

Number-phase uncertainty relation $\Delta N \cdot \Delta\varphi \geq 1$ also applies locally, e.g. in a lattice model of a superconductor,

$$\Delta n_i \cdot \Delta\varphi_j \geq \delta_{ij}.$$

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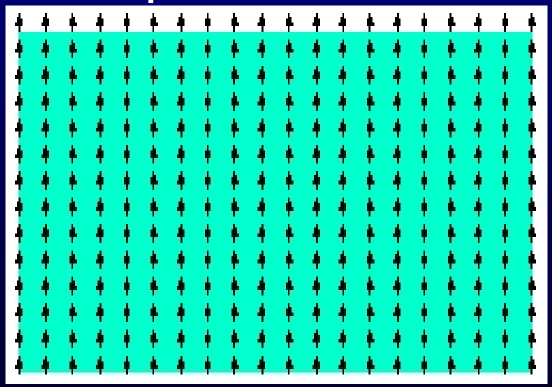

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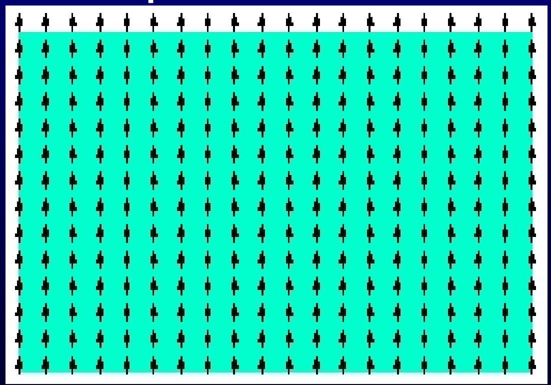

phase ordered $\leftrightarrow$ number uncertain

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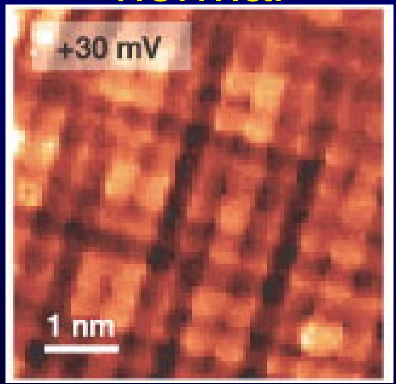

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Vortex-boson duality in (2+1)D

[Fisher & Lee, PRB **39**, 2756 (1989)]

Maps a Lagrangian for 2d phase-fluctuating superconductor

$$\mathcal{L} = \frac{1}{2} K_\mu \left| \left(\partial_\mu - 2ieA_\mu \right) \Psi \right|^2 + a|\Psi|^2 + \frac{1}{2} b|\Psi|^4,$$

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onto the fictitious dual superconductor

$$\mathcal{L}_{\rm dual} = \frac{1}{2}|(\partial_\mu - 2\pi i A_d^\mu)\chi|^2 + \mathcal{V}(|\chi|) - \frac{2\pi i}{\Phi_0} A\cdot(\partial \times A_d) + \frac{1}{2K}(\partial \times A_d)_\mu^2$$

in fictitious dual magnetic field

$$B_d = (\partial \times A_d)_0 = \rho,$$

with $\rho(\mathbf{r})$ the density of Cooper pairs.

When the average density ρ of Cooper pairs is non-zero then the pair Wigner crystal emerges as the

Abrikosov vortex lattice of the dual superconductor.

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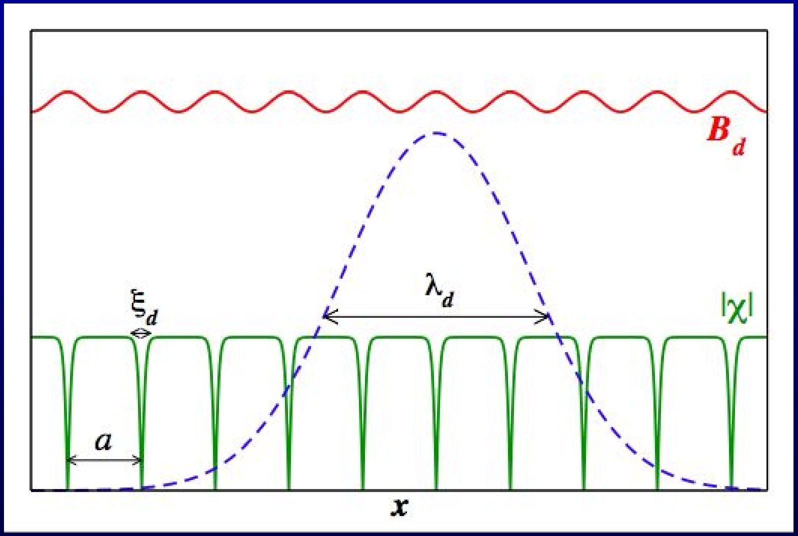

Dual vortex lattice

B_d: dual magnetic field
χ: dual order parameter

λ_d: dual penetration depth
ξ_d: dual coherence length

Dual order parameter χ

-describes *vortex condensate* in the original superconductor.

Two phases:
- $\langle\chi\rangle = 0$: vortices uncondensed $\longrightarrow$ superconductor
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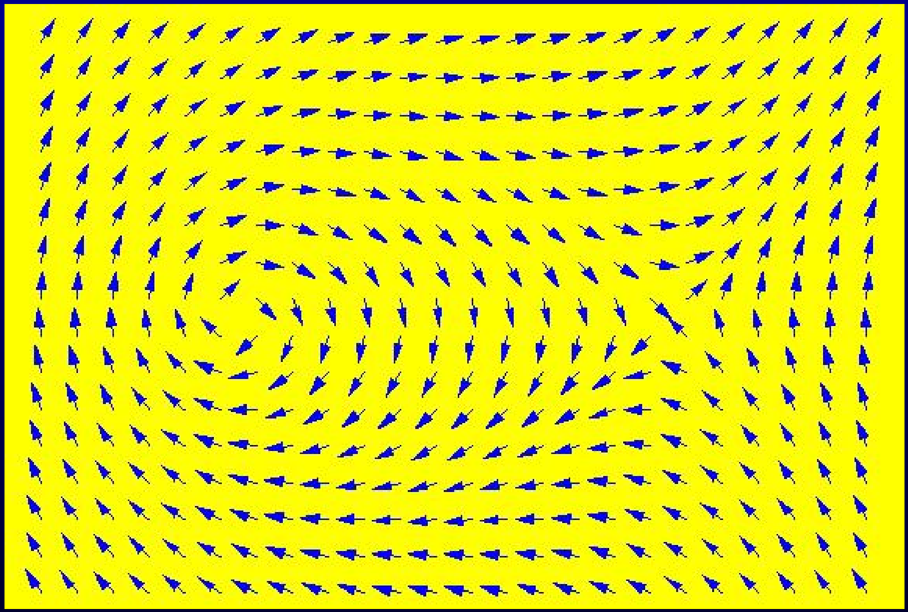


vortex-antivortex pair

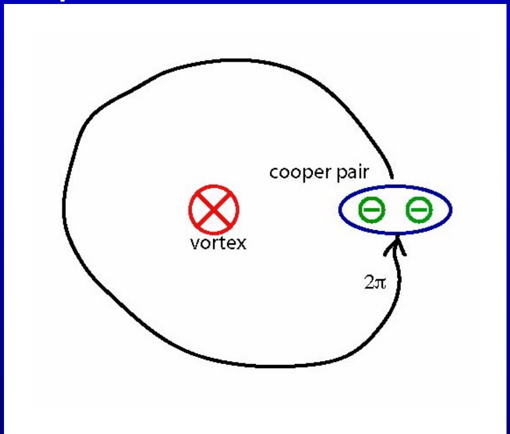
Vortices: Superconducting order parameter Ψ is a complex scalar field,

$$\Psi(r) = |\Psi(r)|e^{i\theta(r)}.$$

Vortices in the phase $\theta(r)$ are important topological excitations of the 2d superconductor.

Physical essence of vortex-boson duality

On encircling a vortex, a Cooper pair acquires phase 2π.



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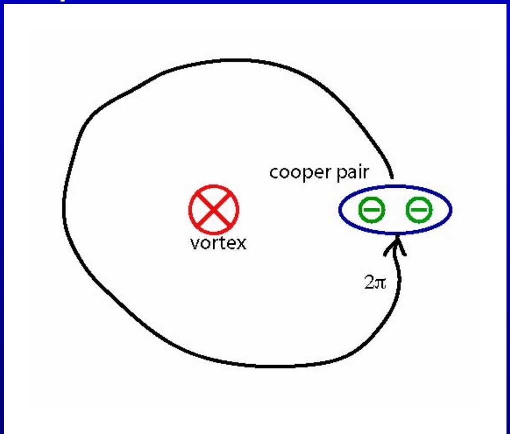

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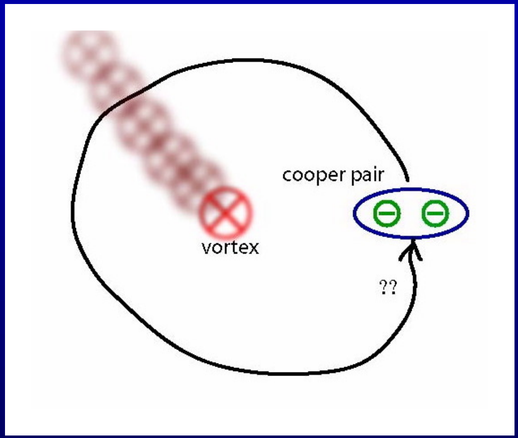

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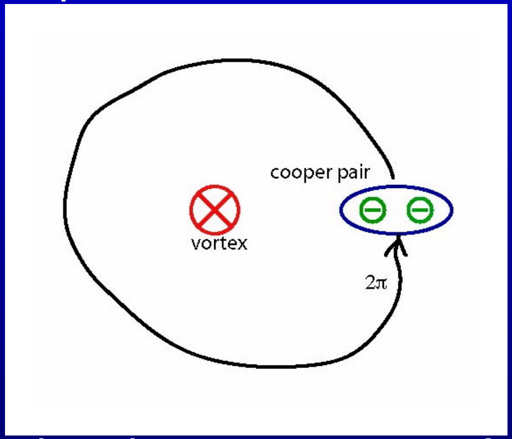

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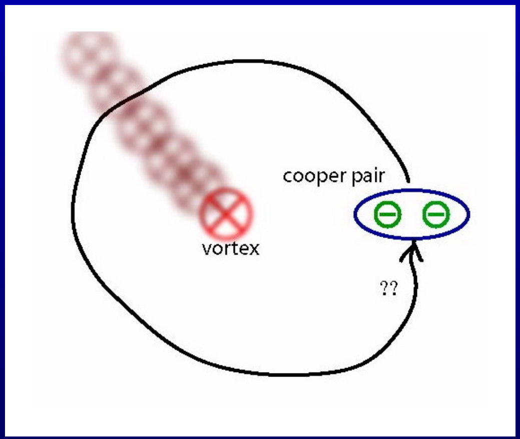
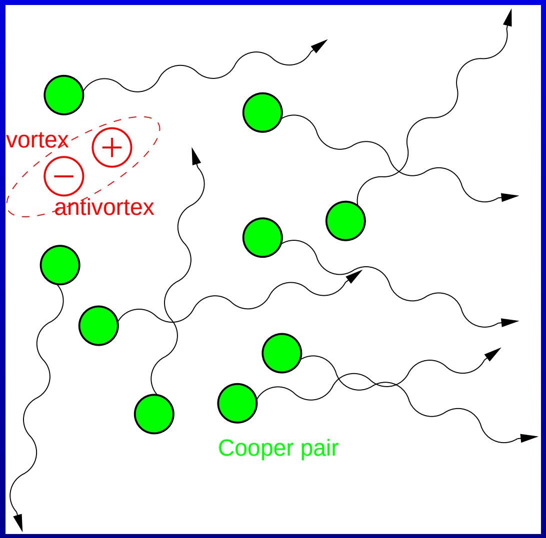


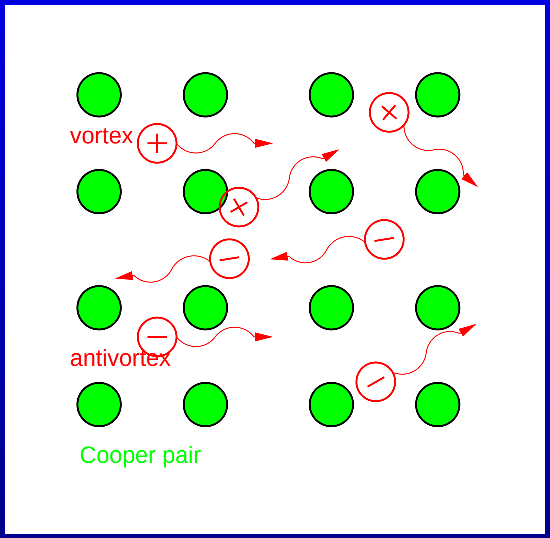
Coherent propagation of pairs is frustrated, superconducting order is suppressed.

Vortices and pairs cannot both propagate coherently.

Two possibilities:



superconductor



pair crystal

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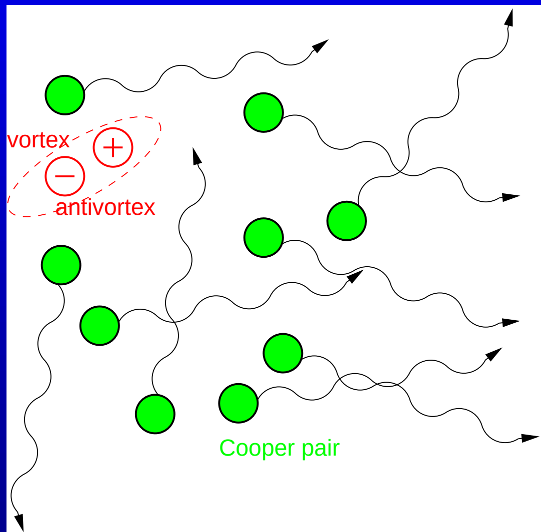

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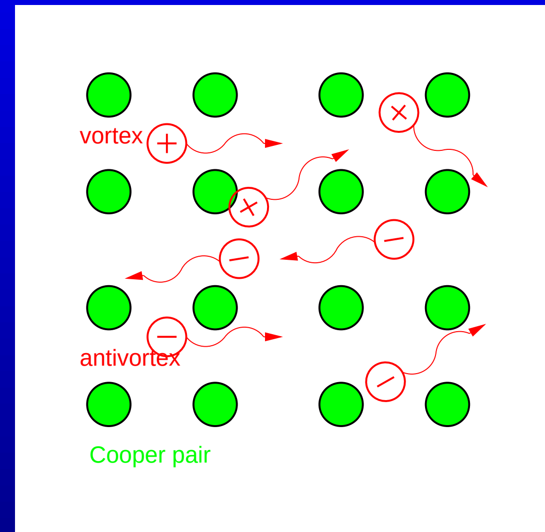

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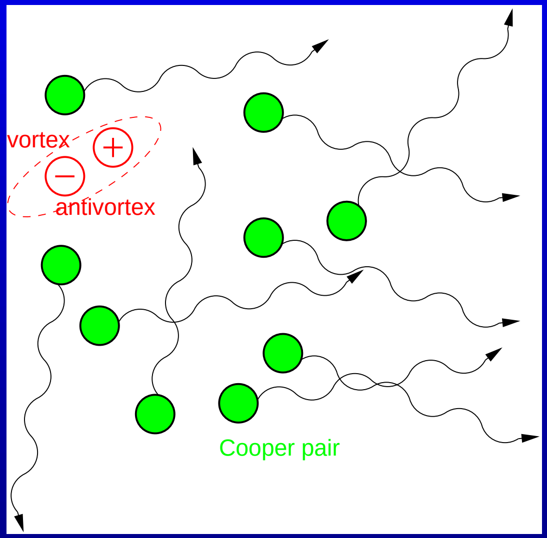

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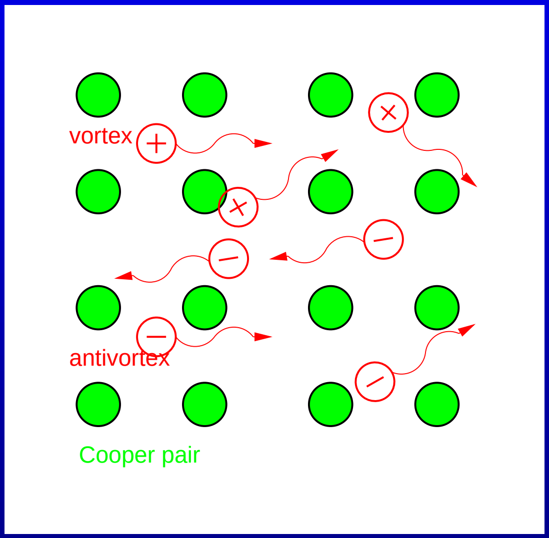

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A key formal point in vortex-boson duality is that vortices can be thought of as point particles with bosonic statistics.

Duality applied to cuprates

[Tesanovic, PRL 2004]

This is more complicated because of the underlying lattice and the d-wave symmetry of the order parameter. Nevertheless, one obtains checkerboard patterns in qualitative agreement with experiment.

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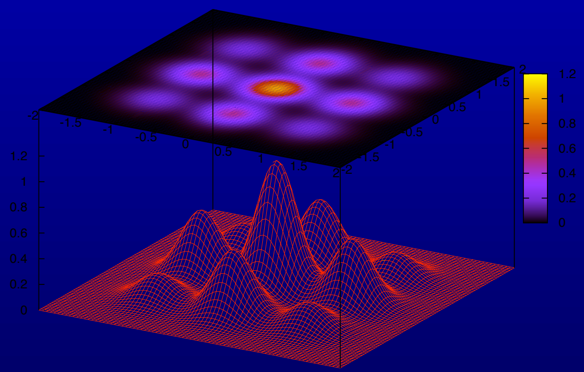

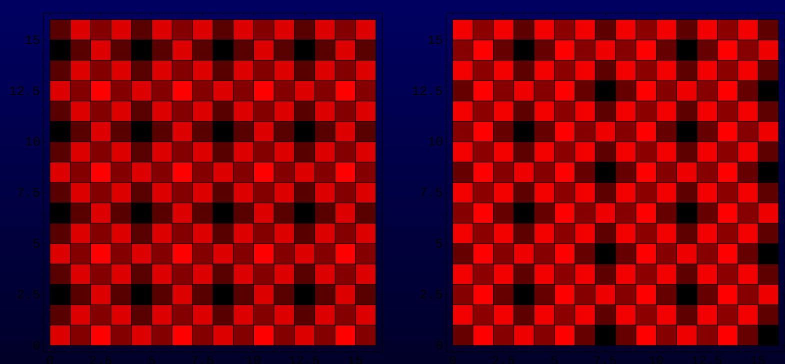

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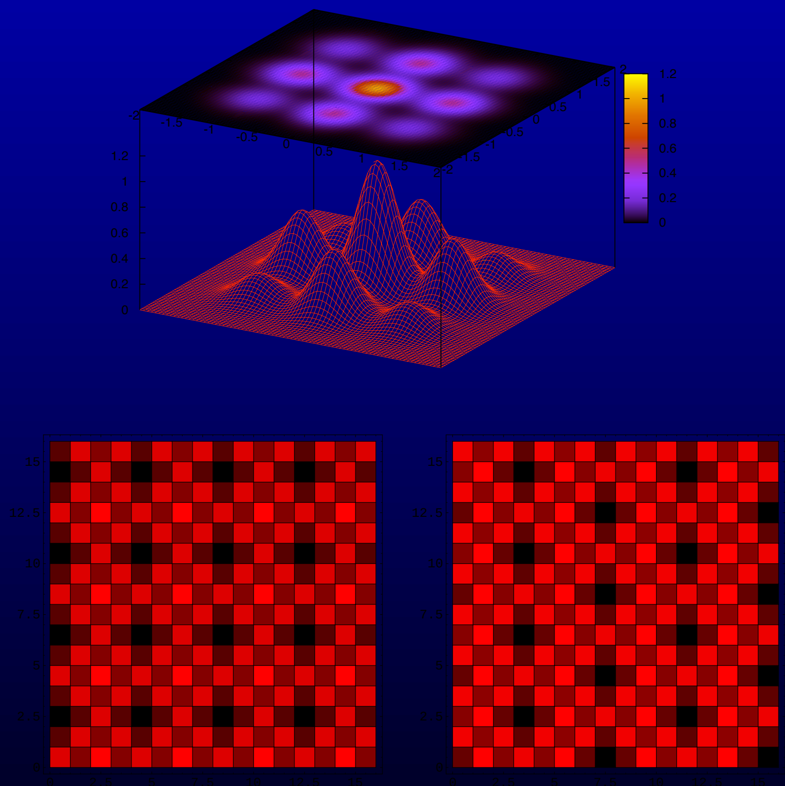

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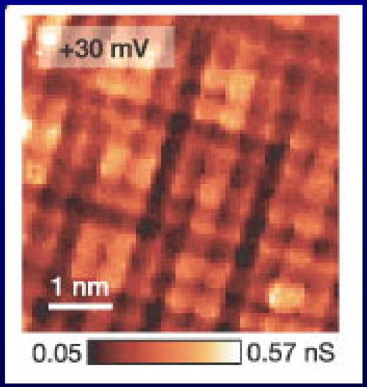
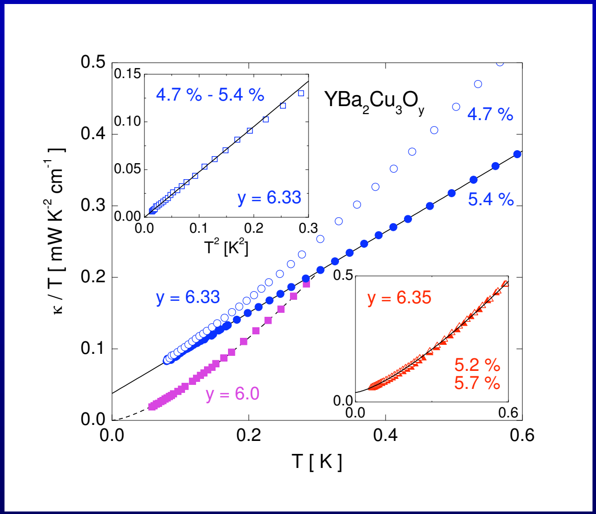


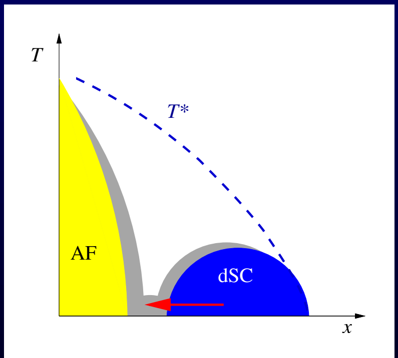
experiment
[Hanaguri *et al.* 2004]

The bosonic mode in underdoped YBCO

[Doiron-Leyraud et al., cond-mat 2006]

Thermal conductivity measurements reveal extra bosonic mode with T^3 temperature dependence.





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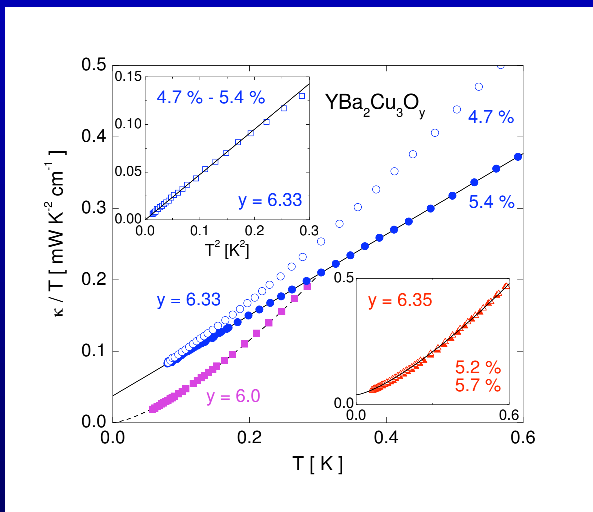

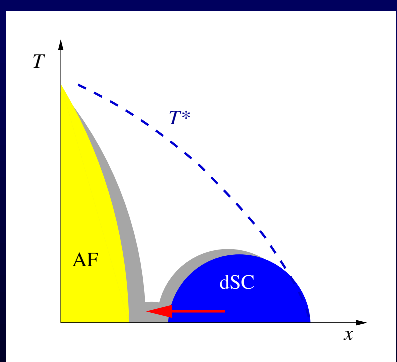

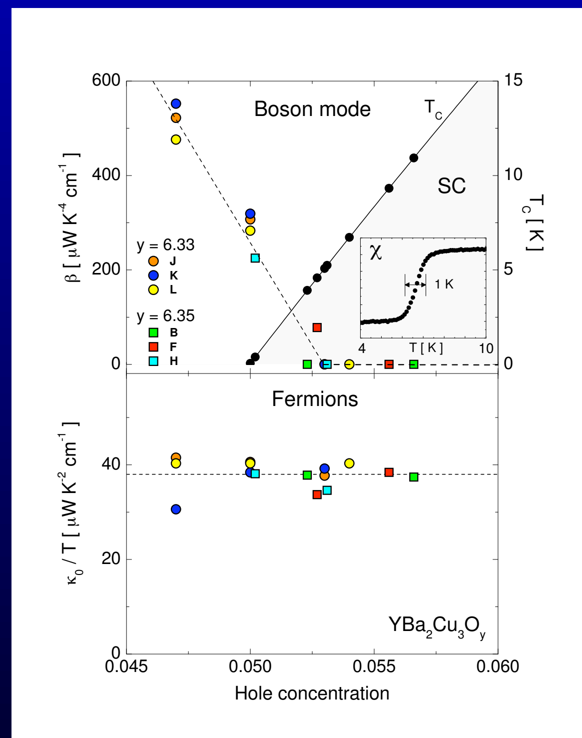

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The T^3 contribution is due to
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Analysis of such vibrational modes shows contribution to thermal conductivity $\kappa(T)$ of correct order of magnitude [Pereg-Barnea and MF, PRB 2006].

Important consequence: Since for bosonic modes

$$\kappa(T) \sim T^d$$

the vibrations propagate in 3 dimensions.

PWC is three dimensional!

Question:

How does fundamentally two-dimensional vortex-boson duality account for 3d pair crystal?

Is it possible to reformulate the duality for a 3d superconductor?

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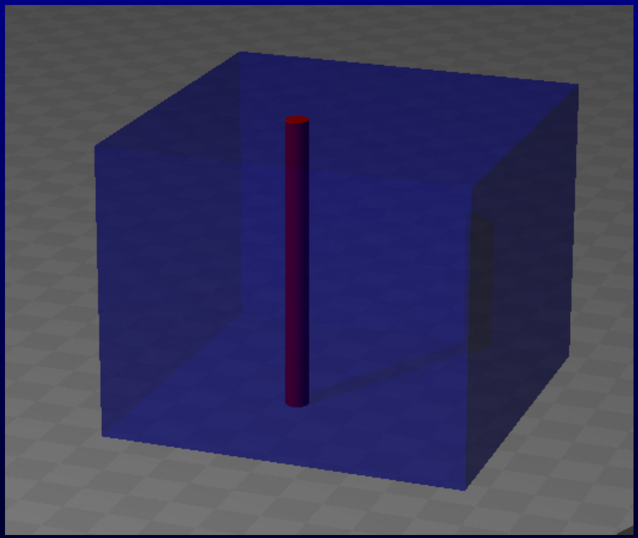

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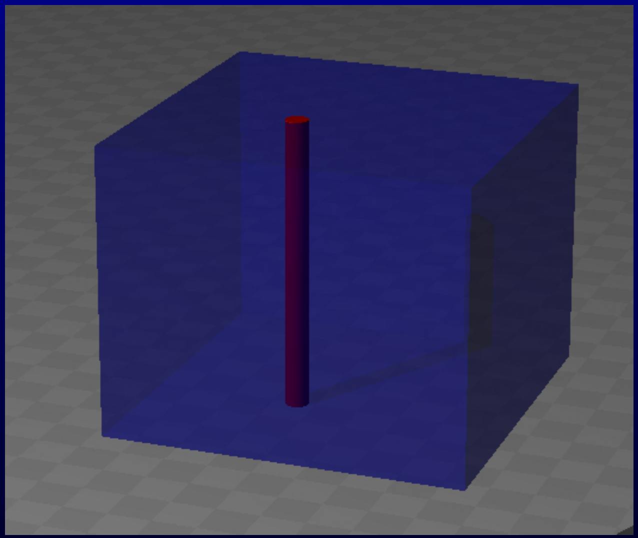

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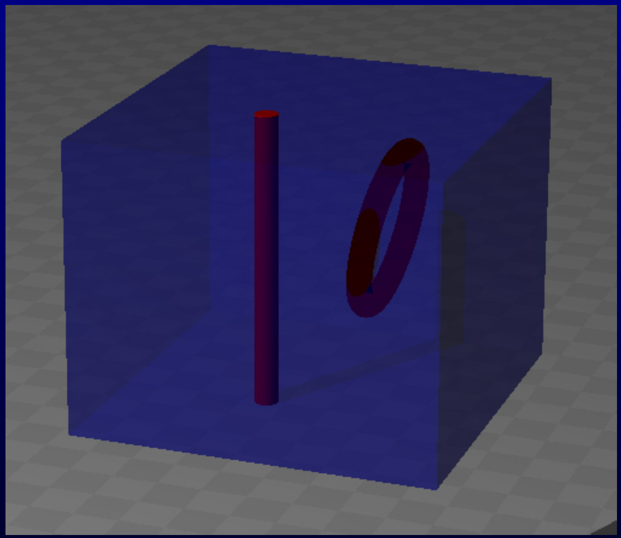
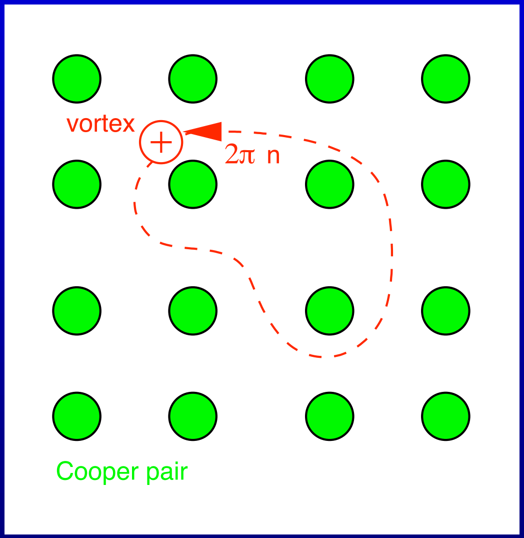


Vortex loop

Duality in 3 dimensions

Generalize the 2d concept:



2d: point-like vortices

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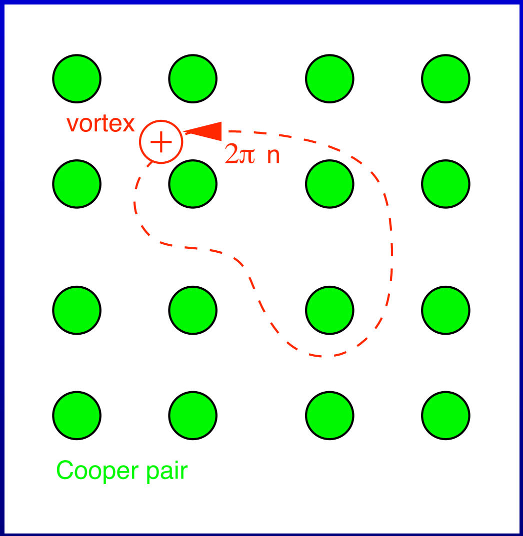

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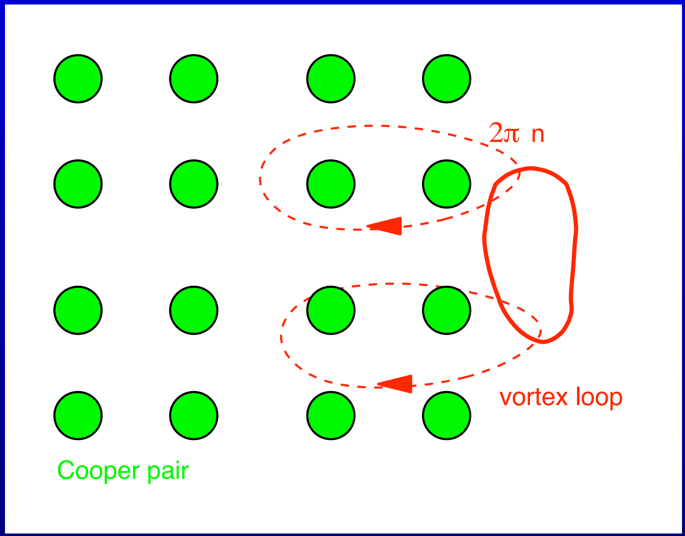

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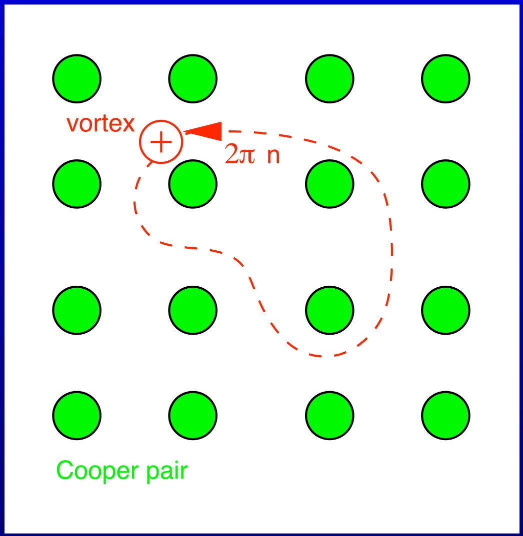

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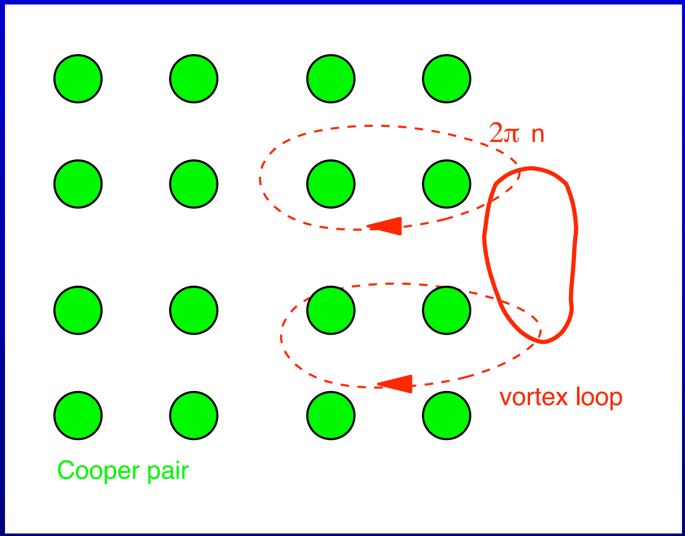

3d: vortex loops

The idea is clear, need to find mathematical formulation of (3+1)D vortex-boson duality. In fact, it will turn out to be a

vortex-loop – string duality

Formalism of the (3+1)D duality

[Franz, cond-mat/0607310]

Begin with a Lagrangian for 3d phase-fluctuating superconductor

$$\mathcal{L} = \frac{1}{2}\tilde{K}_\mu \left|(\partial_\mu - 2ieA_\mu)\,\Psi\right|^2 + a|\Psi|^2 + \frac{1}{2}b|\Psi|^4,$$

where $\Psi = |\Psi|e^{i\theta}$ is the order parameter.

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where $\Psi = |\Psi|e^{i\theta}$ is the order parameter. Consider London approximation, $\Psi(x) \simeq \Psi_0 e^{i\theta(x)}$,

$$\mathcal{L} = \frac{1}{2}K\left(\partial_\mu\theta - 2eA_\mu\right)^2,$$

where $K = \tilde{K}\Psi_0^2$ represents the phase stiffness.

Formalism of the (3+1)D duality

[Franz, cond-mat/0607310]

Begin with a Lagrangian for 3d phase-fluctuating superconductor

$$\mathcal{L} = \frac{1}{2}\tilde{K}_\mu \left|(\partial_\mu - 2ieA_\mu)\,\Psi\right|^2 + a|\Psi|^2 + \frac{1}{2}b|\Psi|^4,$$

where $\Psi = |\Psi|e^{i\theta}$ is the order parameter. Consider London approximation, $\Psi(x) \simeq \Psi_0 e^{i\theta(x)}$,

$$\mathcal{L} = \frac{1}{2}K\left(\partial_\mu\theta - 2eA_\mu\right)^2,$$

where $K = \tilde{K}\Psi_0^2$ represents the phase stiffness.

Write $\theta = \Theta + \theta_s$, where θ_s is the smooth part of the phase, and Θ contains vortex loops.

Formalism of the (3+1)D duality

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Write $\theta = \Theta + \theta_s$, where θ_s is the smooth part of the phase, and Θ contains vortex loops. Now decouple the quadratic term with a real auxiliary field, W_μ, using the familiar Hubbard-Stratonovich transformation, obtaining

$$\mathcal{L} = \frac{1}{2K}W_\mu^2 + iW_\mu(\partial_\mu\Theta - 2eA_\mu) + iW_\mu(\partial_\mu\theta_s).$$

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However, curl operation is only meaningful in 3 dimensions. In (3+1)D we may enforce the constraint by writing

$$W_\mu = \epsilon_{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta},$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor and $B_{\alpha\beta}$ is antisymmetric rank-2 tensor gauge field.

The Lagrangian becomes

$$\mathcal{L} = \frac{H_{\alpha\beta\gamma}^2}{3K} - iB_{\alpha\beta}(\epsilon_{\alpha\beta\mu\nu}\partial_\mu\partial_\nu\Theta) - 2ie(\epsilon_{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta})A_\mu,$$

where $H_{\alpha\beta\gamma} = \partial_\alpha B_{\beta\gamma} + \partial_\beta B_{\gamma\alpha} + \partial_\gamma B_{\alpha\beta}$ is the tensorial field strength.

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The above Lagrangian possesses invariance under the gauge transformation

$$B_{\alpha\beta} \to B_{\alpha\beta} + \partial_{[\alpha}\Lambda_{\beta]}$$

for an arbitrary smooth vector function Λ_μ; and $\partial_{[\alpha}\Lambda_{\beta]} = \partial_\alpha\Lambda_\beta - \partial_\beta\Lambda_\alpha$.

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The electric four-current is related to $B_{\mu\nu}$ by $j_\mu = 2e(\epsilon_{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta})$. The charge density, in particular, can be written as

$$\rho = j_0 = 2e(\epsilon_{ijk}\partial_i B_{jk}),$$

where Roman indices run over spatial components only.

$B_{\mu\nu}$ is minimally coupled to the "vortex loop current"

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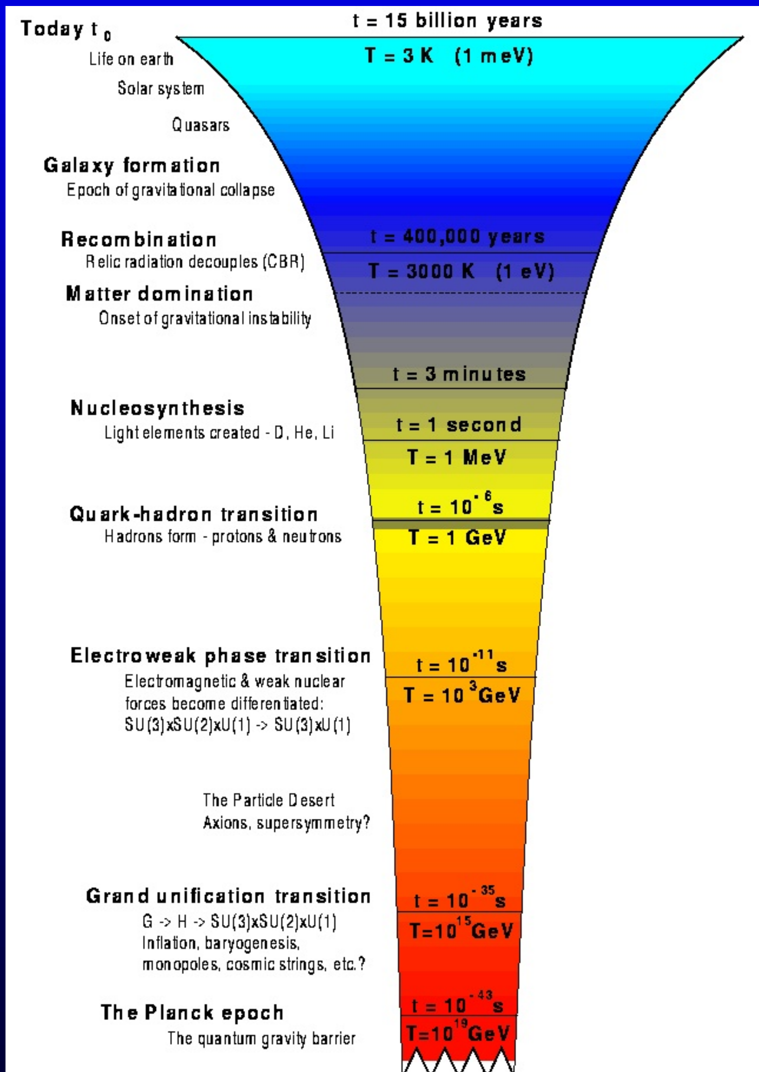
This is only non-zero when $\Theta(x)$ is multiply valued.

In 3d single valuedness of $e^{i\Theta(\mathbf{x})}$ permits *line singularities* in $\Theta(\mathbf{x})$ such that it varies by an integer multiple of 2π along any line that encircles the singularity. These are the vortex loops.

Connection to string theory



Theory of everything ...

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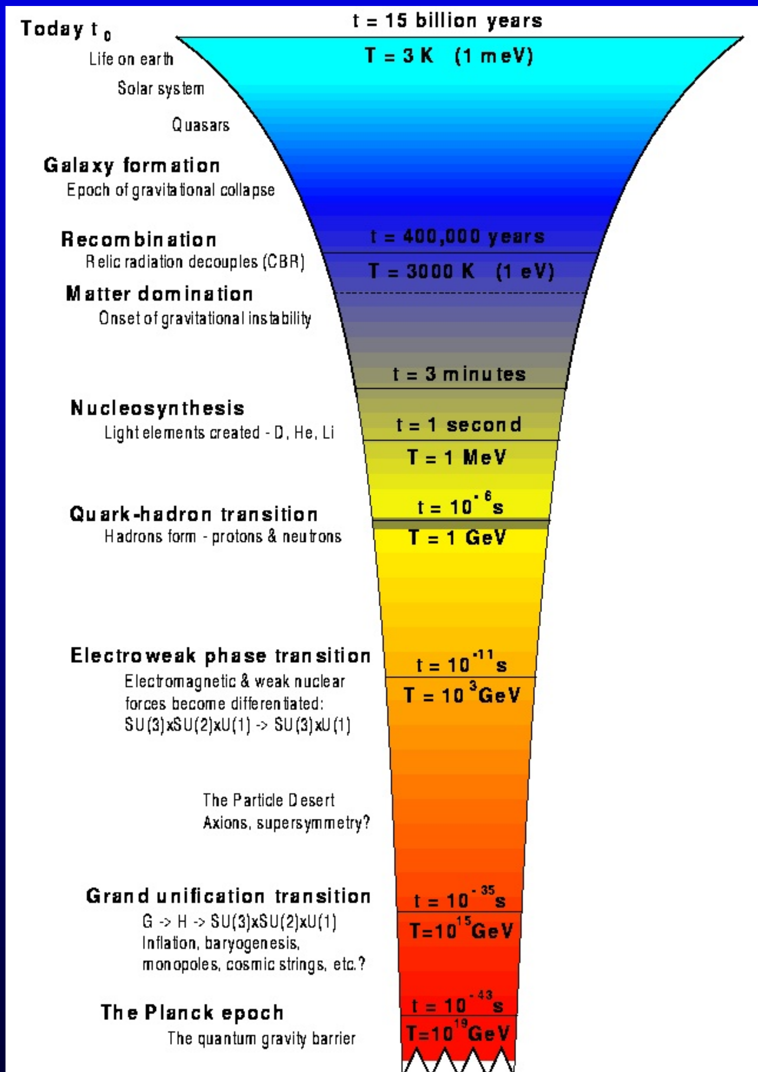

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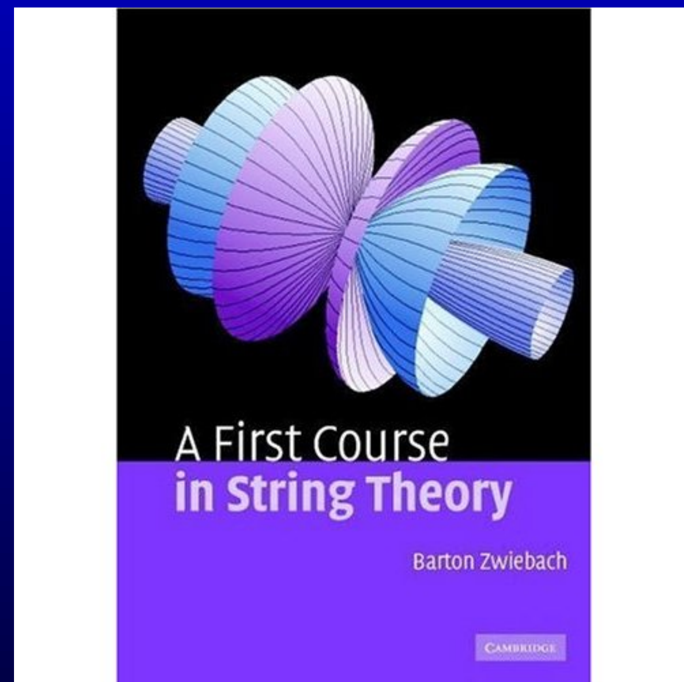
... but Zwiebach comes to rescue:



The worldsheet construction

In string theory strings are described by *worldsheets*.

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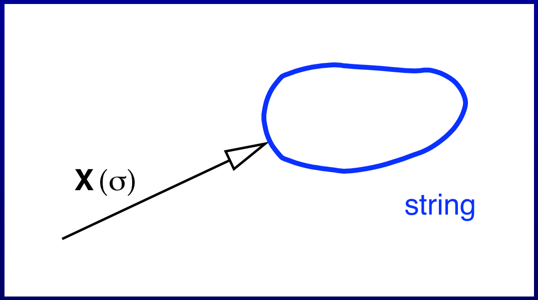
In 3d space a piece of static string can be described by a 3-vector $\mathbf{X}(\sigma)$ where $\sigma = (0, 2\pi)$ parametrizes the string.



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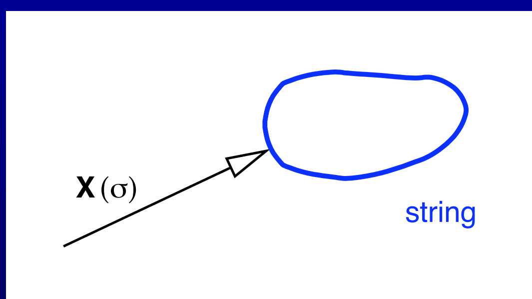
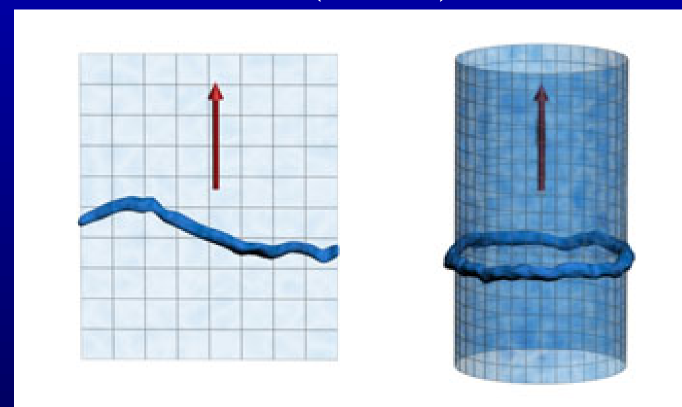


A moving string can be parametrized by 3-vector $\mathbf{X}(\tau, \sigma)$ where τ is the (imaginary) time and again $\sigma = (0, 2\pi)$.



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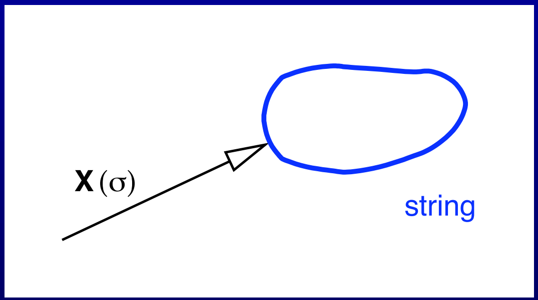

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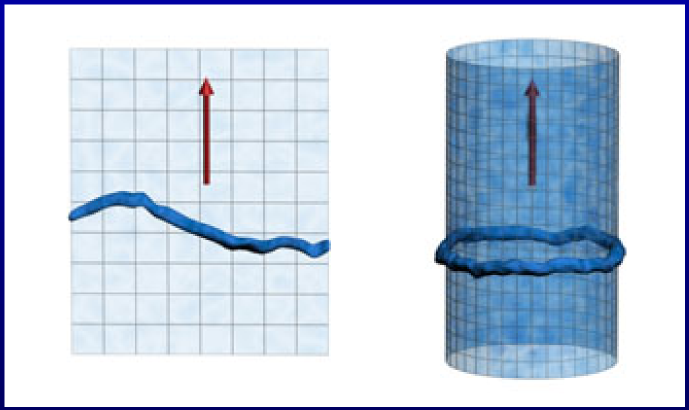

A Lorentz invariant description of a relativistic string is obtained by using a Lorentz 4-vector

$$X_\mu(\sigma_1, \sigma_2) = [X_0(\sigma_1, \sigma_2), \mathbf{X}_\mu(\sigma_1, \sigma_2)]$$

where σ_1 is time-like and σ_2 spacelike parameter.

A surface element of a worldsheet is characterized by a rank-2 antisymmetric tensor

$$\Sigma^{(n)}_{\mu\nu} = \frac{\partial X^{(n)}_{[\mu}}{\partial\sigma_1}\frac{\partial X^{(n)}_{\nu]}}{\partial\sigma_2}.$$

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It is straightforward to show that the loop current is related to the worldsheet by

$$\sigma_{\mu\nu}(x) = 2\pi \sum_n \int d^2\sigma \Sigma_{\mu\nu}^{(n)} \delta\left(X^{(n)} - x\right).$$

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This relation allows us to rewrite the partition function as a functional integral over the vortex loop worldsheets $X_\mu^{(n)}$. We thus have $Z = \int \mathcal{D}[X] \exp(-\mathcal{S})$ with

$$\begin{aligned} \mathcal{S} &= \sum_n \int d^2\sigma \left[\mathcal{T}\sqrt{\Sigma_{\mu\nu}^{(n)}\Sigma_{\mu\nu}^{(n)}} - 2\pi i \Sigma_{\mu\nu}^{(n)} B_{\mu\nu}(X^{(n)})\right] \\ &+ \frac{1}{3K} \int d^4x H_{\alpha\beta\gamma}^2 + \mathcal{S}_{\text{int}} + \mathcal{S}_{\text{Jac}}. \end{aligned}$$

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Nambu-Goto action for bosonic string minimally coupled to Kalb-Ramond rank-2 tensor gauge field $B_{\mu\nu}$.

String condensation

The second-quantized string action takes the form

$$\mathcal{S} \;=\; \int \mathcal{D}[X] \int d\sigma \sqrt{h} \Big[\big| (\delta/\delta\Sigma_{\mu\nu} - 2\pi i B_{\mu\nu}) \Phi[X] \big|^2 + \mathcal{M}^2_{\rm eff} \big| \Phi[X] \big|^2 \Big] + \frac{1}{3K} \int d^4x H^2_{\alpha\beta\gamma} + \mathcal{S}'_{\rm int}.$$

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Here $\Phi[X]$ is the string annihilation operator.

String condensation occurs when $\mathcal{M}_{\text{eff}}^2$ becomes negative and $\Phi[X]$ acquires finite vacuum expectation value:

$$\langle \Phi[X] \rangle \neq 0.$$

The simplest ansatz of “uniform string condensate”

$$\langle \Phi[X] \rangle = \Phi_0 = \text{const.}$$

leads to “Meissner state” for Kalb-Ramond gauge field: $B_{\mu\nu} = 0$ in the interior meaning that the charge is expelled from the sample.

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We seek the analog of "Abrikosov state" for $B_{\mu\nu}$. Consider the ansatz

$$\langle \Phi[X] \rangle = \Phi_0 \exp \left\{ \int d\sigma [\zeta \sqrt{X'^2} \ln f(X) + 2\pi i X'_\mu \cdot \Omega_\mu(X)] \right\}.$$

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Substituting this to the action we get

$$\mathcal{L} = \frac{\Phi_0^2}{2} \left[\pi^2 f^2 (\partial_{[\mu} \Omega_{\nu]} - 2B_{\mu\nu})^2 + \zeta^2 (\partial_\mu f)^2 + \mathcal{V}(f^2) \right] + \frac{1}{3K} H^2_{\alpha\beta\gamma}.$$

Last page with formulas ...

Configurations with *monopoles* in the spatial part of $\Omega = (\Omega_0, \mathbf{\Omega})$,

$$\nabla \cdot (\nabla \times \mathbf{\Omega}) = \sum_a Q_a \delta^{(3)}(\mathbf{x} - \mathbf{x}_a),$$

where $\mathbf{x}_a$ and Q_a label the position and the charge of the a-th monopole.

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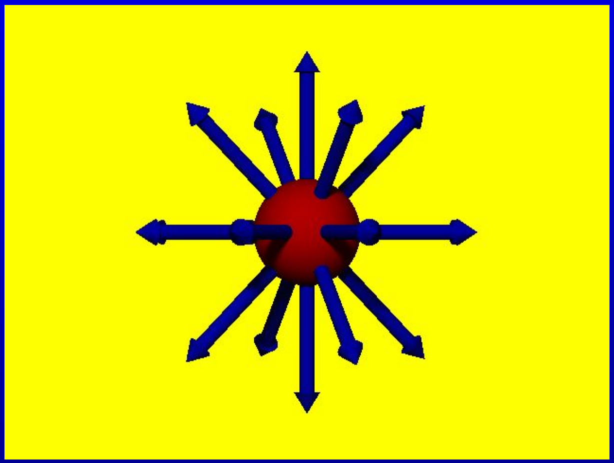

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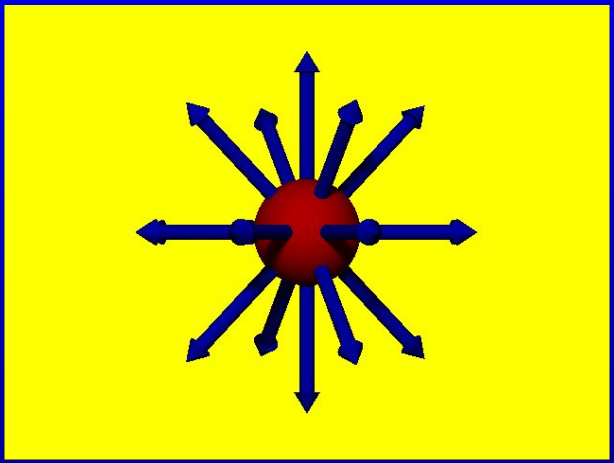

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Minimizing the action with respect to B_{ij} leads to a London-like equation for the Cooper pair charge density

$$\boxed{\rho - \lambda_d^2 \nabla^2 \rho = 2e\nabla \cdot (\nabla \times \boldsymbol{\Omega})}$$

with $\lambda_d^{-2} = 2\pi^2 \Phi_0^2 K$ a dual “penetration depth”.

In 2d the analogous dual London equation is

$$\rho - \lambda_d^2 \nabla^2 \rho = 2e \sum_a \delta^{(2)}(\mathbf{x} - \mathbf{x}_a)$$

and leads to Abrikosov lattice of dual vortices (Cooper pairs) $\rightarrow$ Cooper pair Wigner crystal.

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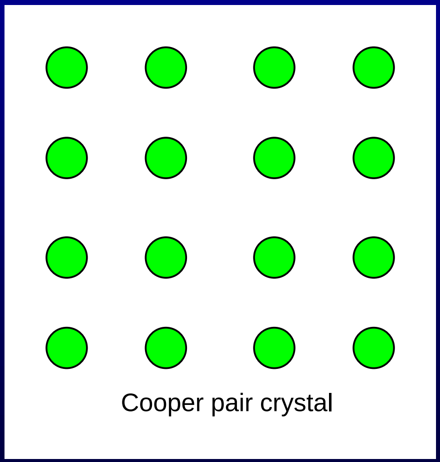
In essence the lattice forms because of repulsive interactions between dual vortices mediated by the dual superflow.

[The lattice would be triangular in continuum; the square lattice can arise due to square anisotropies inherent to cuprates (band structure, d-wave order parameter...)]



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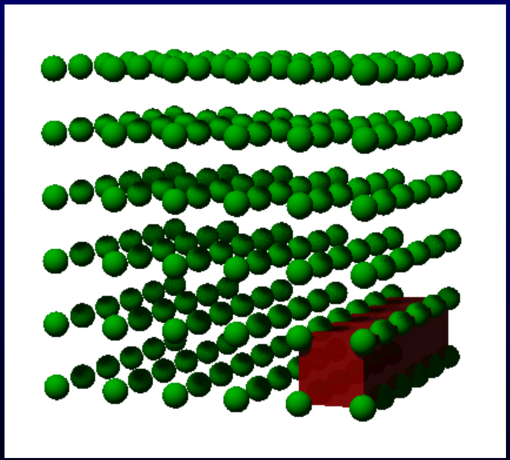
Dual vortices interact by a repulsive Yukawa-type interaction, $\sim e^{-r/\lambda_d}/r$, and will form a a 3d crystal

$\longrightarrow$ Pair Wigner Crystal in 3 space dimensions



Testable prediction

The details of PWC crystalline structure depend on various second order effects (e.g., anisotropies in the material, pining to the ionic lattice, ...).

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However, once the $\{\mathbf{x}_a\}$ are *fixed* then duality gives a universal prediction for the charge distribution,

$$\rho(\mathbf{x}) = 2e \sum_a Q_a \frac{e^{-|\mathbf{x}-\mathbf{x}_a|/\lambda_d}}{4\pi\lambda_d^2|\mathbf{x}-\mathbf{x}_a|}.$$

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This could be, at least in principle, extracted from STM or X-ray scattering data.

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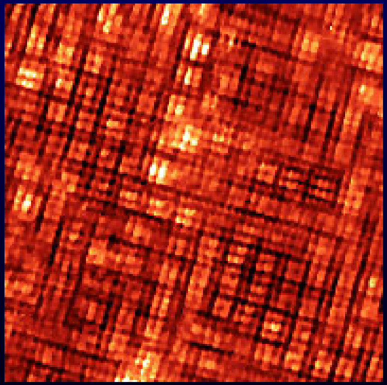
$\Longrightarrow$



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