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Vortex-boson duality in four space-time dimensions

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Abstract – A continuum version of the vortex-boson duality in (3+1) dimensions is formulated and its implications studied in the context of a pair Wigner crystal in underdoped cuprate superconductors. The dual theory to a phase fluctuating superconductor (or superfluid) is shown to be a theory of bosonic strings interacting through a Kalb-Ramond rank-2 tensorial gauge field. String condensation produces Higgs mass for the gauge field and the expected Wigner crystal emerges as an interesting space-time analog of the Abrikosov lattice.

When particles in quantum many-body systems interact very strongly standard perturbation techniques break down and, in dimensions greater than 1, *dualities* often provide the only insights into the physics of such systems. Duality transformations, in general, map the strongly coupled sector of one theory onto the weakly coupled sector of another. The original Kramers-Wanier duality [1] for the Ising ferromagnet represents a prime example of such a mapping. Dualities permeate modern statistical, condensed matter and particle physics, and have emerged recently as a key tool in string theory.

In condensed-matter physics perhaps the most useful and influential duality is the one connecting vortices and bosons in two spatial dimensions [2–4]. This duality, hereafter referred to as Lee-Fisher duality, maps the system of interacting bosons in (2+1)D onto a fictitious superconductor in an external magnetic field whose flux in the temporal direction is proportional to the density of the original bosons. It shows that Mott insulator, proximate to the phase fluctuating boson condensate, can be viewed as the Abrikosov vortex lattice of the dual superconductor. This deep connection has been exploited in modeling systems ranging from quantum spins to fractional quantum Hall effect, and most recently cuprate superconductors.

In cuprates such considerations are motivated by the experimental findings of static checkerboard patterns in the charge density of very underdoped samples [5–7] which have been interpreted as evidence for a Cooper pair Wigner crystal (PWC) [8]. The latter can be most naturally understood by appealing to the Lee-Fisher duality [9–11]. However, recent analysis of the vibrational modes of such a PWC [12,13] indicates that it is 3-dimensional (in the sense that vibrations propagate in all 3 space dimensions) and it is thus unclear how the inherently two-dimensional Lee-Fisher duality applies to this situation. The problem can be stated as follows. A key role in the formulation of the Lee-Fisher duality is played by vortices which appear (in pairs of opposite vorticity) near the transition to the Mott insulating phase as quantum fluctuations of the system. The dual relationship between bosons and vortices however exists only in two spatial dimensions where the latter can be regarded as *point particles*. In three space dimensions vortices form oriented loops and can no longer be thought of as particles. The question thus arises how to understand the formation of a PWC in the three-dimensional phase fluctuating superconductor indicated by experiments [5–7].

In this letter we point a way out of this conundrum by constructing a (3+1)-dimensional implementation of vortex-boson duality using a representation of vortex loops as relativistic bosonic strings. We then show that string condensation indeed produces a ground state that can be characterized as an insulating crystal of Cooper pairs and discuss some of its unique properties.

We remark that the lattice formulation of such (3+1)D duality has been given long time ago [14,15] and was used recently to study exotic fractionalized phases [16,17] in (3+1)D. Here, by contrast, we formulate a continuum version which shows that a boson (Cooper pair) crystal can emerge from a phase fluctuating superfluid (superconductor) even in the absence of any underlying lattice structure. This is exactly the limit apparently relevant to cuprates [12]. On the formal side this continuum approach also reveals an intimate connection to the theory of bosonic strings and enables us to employ in our calculations some of the string theory technology.

We begin by considering a continuum theory of a superconductor in (3+1) space-time dimensions defined by the Euclidean partition function $Z=\int\mathcal{D}[\Psi,\Psi^*]\exp(-\int_0^\beta \mathrm{d}\tau\int \mathrm{d}^3x\mathcal{L})$ with $\Psi=|\Psi|e^{i\theta}$ a scalar order parameter and the Lagrangian density

$$\mathcal{L}=\frac{1}{2}\tilde{K}\left|(\partial_\mu-2ieA_\mu)\,\Psi\right|^2+U(|\Psi|^2). \tag{1}$$

The Greek index $\mu=0,1,2,3$ labels the temporal and spatial components of (3+1)-dimensional vectors, and we use natural units with $\hbar=c=1$. U is a potential function that sets the value of the order parameter Ψ in the superconducting state in the absence of fluctuations. The electromagnetic vector potential A is explicitly displayed in order to track the charge content of various fields. If we allowed A to fluctuate, then eq. (1) would coincide with the well-known Abelian Higgs model.

We now focus on the fluctuations in the phase θ by fixing the amplitude $|\Psi|=\Psi_0$ at the minimum of U,

$$\mathcal{L}=\frac{1}{2}K\left(\partial_\mu\theta-2eA_\mu\right)^2, \tag{2}$$

where $K=\tilde{K}\Psi_0^2$ represents the phase stiffness. The first few steps of the duality mapping proceed just as in (2+1)D. We first decouple the quadratic term with a real auxiliary field, W_μ, using the familiar Hubbard-Stratonovich transformation, obtaining

$$\mathcal{L}=\frac{1}{2K}W_\mu^2+iW_\mu(\partial_\mu\Theta-2eA_\mu)+iW_\mu(\partial_\mu\theta_s). \tag{3}$$

We have also decomposed the phase into a smooth part θ_s and singular part Θ containing vortex lines.

Gaussian integration over θ_s leads to a constraint

$$\partial_\mu W_\mu=0, \tag{4}$$

which reflects conservation of electric charge. In (2+1)D one enforces this constraint by expressing W_μ as a curl of an auxiliary gauge field. The curl operation, however, is meaningful only in 3 dimensions and herein lies the difficulty with higher-dimensional duality. In (3+1)D we may enforce the constraint (4) by writing

$$W_\mu=\epsilon_{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta}, \tag{5}$$

where $\varepsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor and $B_{\alpha\beta}$ is an antisymmetric rank-2 tensor gauge field. Substituting eq. (5) back into the Lagrangian and performing integration by parts in the term containing Θ, we obtain

$$\begin{aligned}\mathcal{L}\;=\;&\frac{H_{\alpha\beta\gamma}^2}{3K}-iB_{\alpha\beta}(\epsilon_{\alpha\beta\mu\nu}\partial_\mu\partial_\nu\Theta)\\&-i2e(\epsilon_{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta})A_\mu,\end{aligned} \tag{6}$$

where $H_{\alpha\beta\gamma}=\partial_\alpha B_{\beta\gamma}+\partial_\beta B_{\gamma\alpha}+\partial_\gamma B_{\alpha\beta}$ is the tensorial field strength which should be thought of as a generalization of the Maxwell field strength $F_{\mu\nu}$.

The above Lagrangian exhibits several notable features. First, it possesses invariance under the gauge transformation

$$B_{\alpha\beta}\to B_{\alpha\beta}+\partial_{[\alpha}\Lambda_{\beta]} \tag{7}$$

for an arbitrary smooth vector function Λ_μ. The square brackets represent antisymmetrization, *e.g.* $\partial_{[\alpha}\Lambda_{\beta]}=\partial_\alpha\Lambda_\beta-\partial_\beta\Lambda_\alpha$. This gauge invariance reflects conservation of vorticity in the original model. Second, from the last term in $\mathcal{L}$ we may immediately deduce that the electric four-current is related to B by $j_\mu=2e(\epsilon_{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta})$. The charge density, in particular, can be written as

$$\rho=j_0=2e(\epsilon_{ijk}\partial_i B_{jk}), \tag{8}$$

where Roman indices run over spatial components only.

The second term in $\mathcal{L}$ informs us that field B is minimally coupled to the "vortex loop current"

$$\sigma_{\alpha\beta}(x)=\epsilon_{\alpha\beta\mu\nu}\partial_\mu\partial_\nu\Theta(x), \tag{9}$$

which is an antisymmetric rank-2 tensor quantity. For a smooth function the right-hand side of eq. (9) would vanish since the derivatives would commute. In 3 spatial dimensions, single valuedness of $e^{i\Theta(\mathbf{x})}$ permits *line singularities* in $\Theta(\mathbf{x})$ such that it varies by an integer multiple of 2π along any line that encircles the singularity. These are the vortex loops.

To implement the duality transformation we now shift our point of view from the phase field $\Theta(x)$ to the vortex loop *worldsheets*. These describe the evolution of vortex loops in imaginary time and should be thought of in analogy with worldlines of point particles. The worldsheets are specified by a set of 2-parameter vector functions $X_\mu^{(n)}(\sigma_1,\sigma_2)$ where n labels the individual loops. We take σ_1 to be time-like, and correspondingly vary between 0 and the inverse temperature β, and σ_2 space-like, which by convention varies from 0 to 2π for closed loops. (Since vortices can only terminate on magnetic monopoles we consider only closed vortex loops here.) Clearly, given a set of worldsheets $X_\mu^{(n)}$ one can reconstruct the phase field $\Theta(x)$ up to any smooth contribution.

A surface element of a worldsheet is characterized by a rank-2 antisymmetric tensor

$$\Sigma_{\mu\nu}^{(n)}=\frac{\partial X_{[\mu}^{(n)}}{\partial\sigma_1}\frac{\partial X_{\nu]}^{(n)}}{\partial\sigma_2}. \tag{10}$$

It is straightforward to show that the loop current (9) is related to the worldsheet by

$$\sigma_{\mu\nu}(x)=2\pi\sum_n\int \mathrm{d}^2\sigma\Sigma_{\mu\nu}^{(n)}\delta\left(X^{(n)}-x\right). \tag{11}$$

This relation allows us to rewrite the partition function as a functional integral over the vortex loop worldsheets $X_\mu^{(n)}$. We thus have $Z=\int\mathcal{D}[X]\exp(-\mathcal{S})$ with

$$\mathcal{S} = \sum_n \int \mathrm{d}^2\sigma \left[\mathcal{T}\sqrt{\Sigma_{\mu\nu}^{(n)}\Sigma_{\mu\nu}^{(n)}} - 2\pi i \Sigma_{\mu\nu}^{(n)} B_{\mu\nu}(X^{(n)})\right] + \frac{1}{3K}\int \mathrm{d}^4x H_{\alpha\beta\gamma}^2 + \mathcal{S}_{\mathrm{int}} + \mathcal{S}_{\mathrm{Jac}}. \quad (12)$$

We recognize the first line as the celebrated Nambu-Goto action [18] for bosonic strings propagating in the presence of a background Kalb-Ramond gauge field $B_{\mu\nu}$ [19]. The first term can be interpreted as the reparametrization invariant surface area of the string worldsheet with the string tension $\mathcal{T}$. Although such term does not appear explicitly in eq. (6) it would arise in a more careful treatment of our starting theory (1) had we retained the cost of the suppression of the order parameter amplitude $|\Psi|$ near the vortex core. The second and the third terms follow directly from eq. (6) and describe long-range interactions between strings mediated by the superflow, now represented by the Kalb-Ramond gauge field.

$\mathcal{S}_{\mathrm{int}}$ contains short-range interactions between strings that would also arise from a more careful treatment of the core physics. Finally, $\mathcal{S}_{\mathrm{Jac}}$ represents the Jacobian of the transformation from phase variable Θ to string worldsheets $X_\mu^{(n)}$. This last term plays an important role in the quantization of our string theory. It is well known that a fundamental string can be consistently quantized only in the critical dimension, which for bosonic string is $D=26$ [20]. A question then arises as to how we quantize our vortex strings in (3+1)D; after all we started from a well-defined field theory (1) and we expect the string theory (12) derived from it to also be well behaved. The answer lies in the fact that our strings are *not* fundamental; rather they are Nielsen-Olesen–type strings [21] with intrinsic thickness defined by the core size. It was shown by Polchinski and Strominger [22] that terms in $\mathcal{S}_{\mathrm{Jac}}$, which would be absent in the case of a fundamental string, precisely cancel the conformal anomaly responsible for the high critical dimension. Vortex strings are indeed well behaved in the physical dimension.

We are now ready to complete the duality mapping. Our main goal will be to understand the string-condensed phase, analogous to the vortex-condensed phase in the (2+1)D Lee-Fisher duality. To this end we must pass to second quantized string theory, a "string-field theory" This can be done rigorously in the so-called light-cone gauge [23] or using the Becchi-Rouet-Stora-Tyutin (BRST) procedure [24]. Here we opt for a less rigorous but physically much more transparent procedure devised in ref. [25] which provides a straightforward route towards the description of the string-condensed phase.

The central concept in the string-field description is the wave *functional* $\Phi[X]$, a complex-valued functional defined on the space of one-parameter string trajectories $\{X_\mu(\sigma_2), \sigma_2=(0,2\pi)\}$, which we regard as cross-sections of the string worldsheet $X_\mu(\sigma_1,\sigma_2)$ at fixed value of σ_1. The physical significance of $\Phi[X]$ is most readily visualized in the so-called static parametrization: in a chosen Lorentz frame of reference take $X_0=\sigma_1=\tau$ and $\mathbf{X}=\mathbf{X}(\tau,\sigma)$, with τ the imaginary time. $\Phi[\tau,\mathbf{X}]$ then represents the quantum-mechanical amplitude for finding the string in configuration $\{\mathbf{X}(\sigma),\sigma=(0,2\pi)\}$ at time τ.

The second quantized action for the string functional takes the form [25]

$$\mathcal{S} = \int\mathcal{D}[X]\int \mathrm{d}\sigma\sqrt{h}\left[\left|(\delta/\delta\Sigma_{\mu\nu} - 2\pi i B_{\mu\nu})\Phi[X]\right|^2 + \mathcal{M}_{\mathrm{eff}}^2\left|\Phi[X]\right|^2\right] + \frac{1}{3K}\int \mathrm{d}^4x H_{\alpha\beta\gamma}^2 + \mathcal{S}'_{\mathrm{int}}. \quad (13)$$

The plaquette derivative $\delta/\delta\Sigma_{\mu\nu}$ quantifies the variation of the functional $\Phi[X]$ upon modifying the path X by an infinitesimal loop ΔX which sweeps the surface element $\delta\Sigma_{\mu\nu}$. The loop space metric $h=(\partial X_\mu(\sigma)/\partial\sigma)^2$ is needed to preserve the reparametrization invariance of the action. $\mathcal{S}'_{\mathrm{int}}$ represents short-range string interactions and contains terms cubic and higher order in $|\Phi|$. All contributions quadratic in $|\Phi|$ have been folded into the effective string mass $\mathcal{M}_{\mathrm{eff}}$. The action (13) remains invariant under the gauge transformation (7) if we require Φ to transform as

$$\Phi[X]\to\Phi[X]e^{-4\pi i\int \mathrm{d}X_\mu\Lambda_\mu}. \quad (14)$$

String condensation occurs when $\mathcal{M}_{\mathrm{eff}}^2$ becomes negative. The string functional then develops nonzero vacuum expectation value, $\langle 0|\Phi[X]|0\rangle\neq 0$. The simplest case is that of a uniform string condensate,

$$\langle 0|\Phi[X]|0\rangle = \Phi_0 = \mathrm{const.} \quad (15)$$

Physically, this simply means that *any* string configuration is equally probable. This ansatz, however, cannot describe a phase disordered superconductor. To see this note that substituting eq. (15) into action (13) produces a mass term for the Kalb-Ramond gauge field. Such a mass term then leads to the Meissner effect: the gauge field is expelled from the interior of the sample, $B_{\mu\nu}=0$. In view of eq. (8), this corresponds to complete expulsion of charge from the system, which is not the situation we are interested in.

What we seek is the analog of the Abrikosov vortex state in which the field can penetrate in quantized increments. We thus consider a more general ansatz, which allows both the amplitude and the phase of $\Phi[X]$ to vary:

$$\langle 0|\Phi[X]|0\rangle = \Phi_0 e^{\int \mathrm{d}\sigma[\zeta\sqrt{X'^2}\ln f(X) + 2\pi i X'_\mu\cdot\Omega_\mu(X)]}. \quad (16)$$

Here $X=X(\sigma)$, $X'=\partial_\sigma X(\sigma)$, $f(x)$ and $\Omega_\mu(x)$ are real scalar and vector functions parametrizing the functional, and ζ is a parameter with the dimension of inverse length. $f(x)$ is nonnegative and should be thought of as the space-time-dependent amplitude of the string condensate. Specifically, $f=1$ corresponds to uniform condensate amplitude while $f>1$ ($f<1$) describes its local enhancement (depletion). $\Omega_\mu(x)$ determines the phase of the string condensate. Substituting eq. (16) to (13) we obtain $\mathcal{S}=\int \mathrm{d}^4x\mathcal{L}$ with

$$\mathcal{L} = \frac{\Phi_0^2}{2}\left[\pi^2 f^2(\partial_{[\mu}\Omega_{\nu]}-2B_{\mu\nu})^2 + \zeta^2(\partial_\mu f)^2+\mathcal{V}(f^2)\right]+\frac{1}{3K}H^2_{\alpha\beta\gamma}. \tag{17}$$

We observe that any smooth part of Ω can be eliminated by the gauge transformation (14). Thus, only the singular part of Ω has physical significance. Indeed we note that it is permissible for Ω to be *multiply valued* as long as the wave functional (16) remains single valued. A configuration of specific interest to us contains *monopoles* in the spatial part of $\Omega=(\Omega_0,\mathbf{\Omega})$,

$$\nabla\cdot(\nabla\times\mathbf{\Omega})=\sum_a Q_a\delta^{(3)}(\mathbf{x}-\mathbf{x}_a), \tag{18}$$

where $\mathbf{x}_a$ and Q_a label the position and the charge of the a-th monopole. Single valuedness of (16) demands that Q_a be *integer*. We shall see that such singularities represent sources for $B_{\mu\nu}$, just as vortices in a superconductor act as sources for the magnetic field.

We now analyze the action (17) in the presence of static monopole configurations in $\mathbf{\Omega}$. To this end we adopt a dual mean-field approximation (DMFA) which neglects quantum fluctuations of all the fields. We emphasize that in terms of the original phase degrees of freedom, DMFA describes a highly nontrivial quantum fluctuating state. In addition, we perform a dual "London" approximation, $f(\mathbf{x})=1$, which should be adequate as long as the monopoles are relatively dilute. (This approximation fails in the small region near the monopole center where $f\to 0$.) The ground state energy of the system can then be written as

$$\mathcal{E}=\frac{1}{2}\int \mathrm{d}^3x\left[\pi^2\Phi_0^2(\partial_{[i}\Omega_{j]}-2B_{ij})^2+\frac{1}{K}(\epsilon_{ijk}\partial_i B_{jk})^2\right]. \tag{19}$$

Minimizing with respect to B_{ij} leads to the Euler-Lagrange equation

$$\pi^2\Phi_0^2(2B_{ij}-\partial_{[i}\Omega_{j]})-\frac{1}{2eK}\epsilon_{ijk}\partial_k\rho=0, \tag{20}$$

where we used eq. (8). Next, acting on all terms by $\varepsilon_{ijl}\partial_l$ and defining a dual "penetration depth" $\lambda_d^{-2}=2\pi^2\Phi_0^2K$, we obtain an equation for charge density $\rho(\mathbf{x})$

$$\rho-\lambda_d^2\nabla^2\rho=2e\nabla\cdot(\nabla\times\mathbf{\Omega}). \tag{21}$$

This equation resembles the London equation for the z-component of magnetic field in the presence of an Abrikosov lattice of vortices and can be analyzed by similar methods. The key difference is that, in light of eq. (18), the right-hand side describes a collection of *point sources* in three space dimensions, whereas Abrikosov vortices are line singularities described by $\delta^{(2)}$. Below we briefly summarize some main results of this analysis and the relevant details will be given elsewhere [26].

Equation (21) can be solved for an arbitrary arrangement of monopole positions and charges to obtain

$$\rho(\mathbf{x})=2e\sum_a Q_a\frac{e^{-|\mathbf{x}-\mathbf{x}_a|/\lambda_d}}{4\pi\lambda_d^2|\mathbf{x}-\mathbf{x}_a|}. \tag{22}$$

It is easy to show that the total electric charge associated with a monopole is $2eQ_a$; the charge is quantized in the units of $2e$, as expected. At finite charge density, monopoles with like charges repel by Yukawa potential $\sim e^{-r/\lambda_d}/r$ and the ground state is a Bravais lattice of elementary ($Q_a=1$) monopoles. This leads to periodic modulation in $\rho(\mathbf{x})$ with charge $2e$ per unit cell: a pair Wigner crystal in three space dimensions.

An appealing overall picture thus emerges. Vortex loops in a (3+1)-dimensional superconductor (or superfluid) can be efficiently described as bosonic strings interacting through a rank-2 tensorial Kalb-Ramond gauge field $B_{\mu\nu}$. In the non-superconducting phase, strings proliferate and condense, producing Higgs mass for the gauge field. In the Higgs phase the only way for $B_{\mu\nu}$ to penetrate into the bulk of the system is to set up quantized monopole-like singularities in the phase Ω_μ of the string condensate wave functional. These singularities then act as point sources for $B_{\mu\nu}$. The associated electric charge density ρ, which is closely related to the Kalb-Ramond field strength $H_{\mu\nu\lambda}$, is then governed by a London-like equation (21). For a periodic array of point sources, such as will form at finite charge density, a 3-dimensional pair Wigner crystal emerges with charge distribution given by eq. (22).

The duality discussed above establishes vortex-loop condensation as a concrete mechanism for the formation of a pair Wigner crystal in a 3-dimensional quantum phase fluctuating superconductor. It explains how a 3d PWC can form in underdoped cuprates and allows for detailed computations of its structure and vibrational modes [26] which are of direct experimental interest.

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