for $t_1 > t_3 = t_4 > t_2$ we would get

$$X = 2e^{-i\Omega_0(t_1 - t_2)}$$

Please check that same answer is obtained by direct evaluation.

LECTURE 6

Evaluation of phonon GF by Feynman-Dyson perturbation theory

$$H_1(t) = \frac{1}{2}M \sum_{i \neq j} D_{ij}\, u_i(t)\, u_j(t)$$

We need to evaluate exp. values of the form

$$\langle T[u_i(t)\, u_j(0)\, H_1(t_1) \dots H_1(t_n)] \rangle_0 \quad \text{in } n\text{-th order}$$

We can apply Wick's theorem directly to u_i's

- Zeroth order

$$\langle T[u_i(t)\, u_j(t')] \rangle_0 = i\, G^0_{ij}(t - t') = i\, G^0(t - t')\, \delta_{ij}$$

– unperturbed GF, already evaluated previously.

First order $(n=1)$

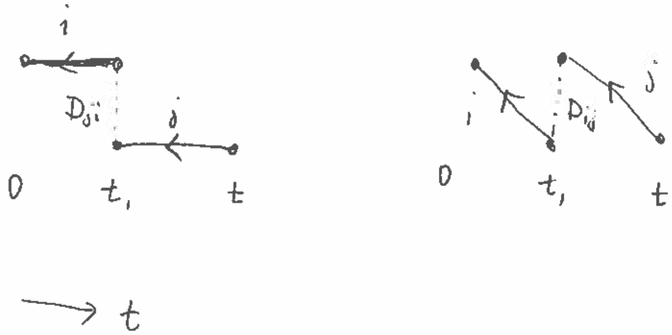
$$\langle T[u_i(t)\, u_j(0)\, H_1(t_1)]\rangle_0 = \tfrac{1}{2} M \sum_{i_1 \neq j_1} D_{i_1 j_1} \langle T[u_i(t)\, u_j(0)\, u_{i_1}(t_1)\, u_{j_1}(t_1)]\rangle \ominus$$

Note $\overline{u_{i_1}(t_1)\, u_{j_1}(t_1)} = 0$ because $G^0_{ij} \sim \delta_{ij}$ and $i_1 \neq j_1$

$$\ominus\ \tfrac{1}{2} M \sum_{i_1 \neq j_1} D_{i_1 j_1} \{ iG^0(t-t_1)\delta_{i j_1}\, iG^0(0-t_1)\delta_{j i_1} + iG^0(t-t_1)\delta_{i i_1}\, iG^0(0-t_1)\delta_{j j_1} \}$$

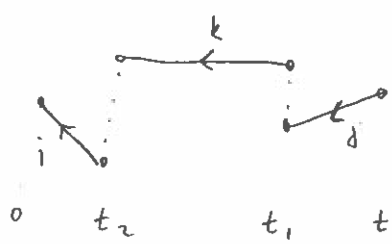
$$= \tfrac{1}{2} M \{ iG^0(t-t_1)\, D_{ji}\, G^0(t_1) + iG^0(t-t_1)\, D_{ij}\, iG^0(t_1) \}$$

[Note that $G^0(-t) = G^0(t)$, also $D_{ji} = D_{ij}$]



Second order $(n=2)$

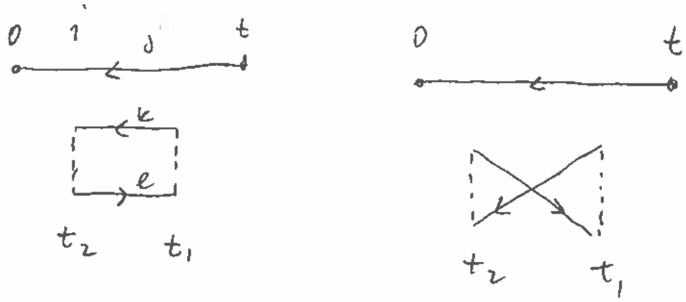
$$(\tfrac{1}{2} M)^2 \sum_k iG^0(t-t_1)\, D_{jk}\, iG^0(t_1-t_2)\, D_{ik}\, G^0(t_2)$$



$2^n n! \to 4 \times 2 = 8$ diagrams

Disconnected diagrams and the linked-cluster theorem

- for $n \geq 2$ we have "disconnected" or "unlinked" diagrams:

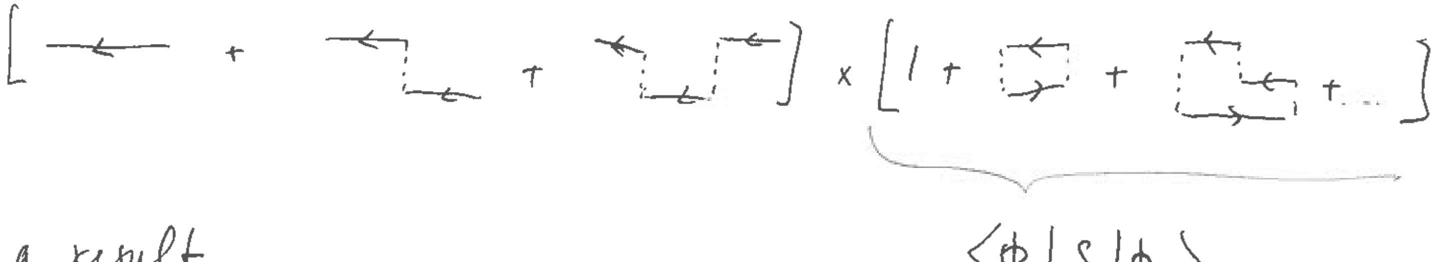


No class this Th

Pls read Sec. 3.3 and 3.4 (finite-T FD exp.)

[But these get EXACTLY CANCELED by the $\langle \Phi_0 | S | \Phi_0 \rangle$ denominator.]

Pictorially, the numerator $\langle T[u_i(t) u_j(0) S] \rangle_0$ factorizes as follows:



As a result

$$\boxed{G_{ij}(t) = -i \langle \Phi_0 | T[u_i(t) u_j(0) S] | \Phi_0 \rangle_{\text{linked}}}$$

$$G_{ij}(t) = G^0(t)\,\delta_{ij} + \sum_{n=1}^{\infty} (\lambda M)^n \sum_{i_1 \dots i_{n-1}} \int_{-\infty}^{\infty} dt_1 \dots dt_n \; G^0(t-t_1)$$

$$\times \; D_{i i_1} G^0(t_1 - t_2) D_{i_1 i_2} G^0(t_2 - t_3) \dots D_{i_{n-1} j} G^0(t_n)$$

SCATTERING OF FERMIONS BY A LOCALIZED PERTURBATION

Relevant to electrons in metals with impurities.

1. Formulation of the problem

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \sum_i U(\vec{x}_i)$$

first-quantized Hamiltonian of N electrons in potential $U(\vec{r})$

We now reformulate in second-quantized formalism [Appendix 1].

Define electron creation & annihilation operators c_p^+, c_p

c_p^+ - creates electron in a plane-wave state

$$u_p(\vec{x}) = \frac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{x}}$$

V - "normalization" volume

Satisfy anticommutation rules:

$$\{c_p^+, c_q\} = \delta_{pq}\,, \qquad \{c_p^+, c_q^+\} = \{c_p, c_q\} = 0$$

$n_p = c_p^+ c_p$ is the number operator.

Since $n_p^2 = n_p$ (check!) it has eigenvalues 0 or 1

i.e. the state with momentum $\vec{p}$ can be either empty or occ

The many-particle states can be created from the vacuum state by applying c_p^+ operators:

$$|n_{p_1} n_{p_2} \cdots n_{p_N} \cdots\rangle = (c_{p_1}^+)^{n_{p_1}} (c_{p_2}^+)^{n_{p_2}} \cdots (c_{p_N}^+)^{n_{p_N}} \cdots |0\rangle \qquad n_{p_j} = 0, 1$$

- because c_p^+ anticommute one has to postulate some definite ordering in momenta $\vec{p}$.

The second quantized Hamiltonian has the form

$$H = \sum_{pp'} \langle p' | h | p \rangle c_{p'}^+ c_p$$

$$\langle p' | h | p \rangle = \int_V d^3x \; u_{p'}^*(\vec{x}) \left[\frac{\hat{\vec{p}}^2}{2m} + U(\vec{x}) \right] u_p(\vec{x})$$

$$= \varepsilon_p \delta_{p'p} + U(\vec{p}' - \vec{p}) \qquad \varepsilon_p = \frac{\vec{p}^2}{2m}$$

$$U(\vec{q}) = \frac{1}{V} \int_V U(\vec{x}) e^{-i\vec{q}\cdot\vec{x}} d^3x \qquad U(\vec{x}) = \sum_q U(\vec{q}) e^{i\vec{q}\cdot\vec{x}}$$

$$\boxed{H = \sum_p \varepsilon_p c_p^+ c_p + \sum_q U(\vec{q}) \sum_p c_{p+q}^+ c_p}$$

ground-state energy for example

$$E_G = \langle \Psi_G | H | \Psi_G \rangle = \sum_{p,p'} \left[\delta_{pp'} \varepsilon_p + U(\vec{p} - \vec{p}') \right] \langle \Psi_G | c_p^+ c_{p'} | \Psi_G \rangle$$