Phys529B: Topics of Quantum Theory

Lecture 9: interacting fermions and Non-Fermi liquid

instructor: Fei Zhou

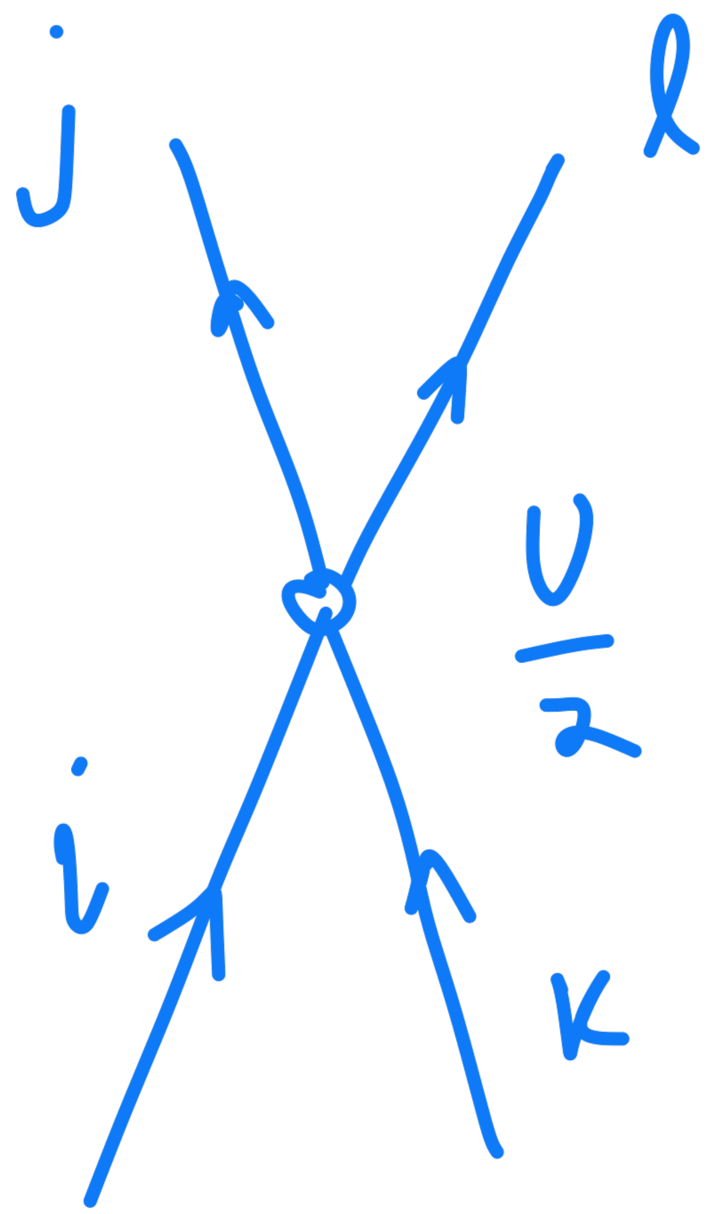
- NFL phenomenology

- 1) Z vanishes at Fermi surfaces and no quasi-particles.

- 2) Z is finite across Fermi surface but Z on two sides of the Fermi surface cancels. I.e. $Z(k, \omega > 0)$ is continuous across a Fermi surface near k=k_F. For given momentum and orbital, quasi-particle and hole co-exist.

HK Model (Hatsugai-Kohmoto)

$$H = -t \sum_{\langle ij \rangle} a_i^\dagger a_j + h.c. + \frac{U}{2} \sum_{ijkl} a_i^\dagger a_j \cdot a_k^\dagger a_l \cdot \delta_{i+k,\,j+l}$$



local in k-space

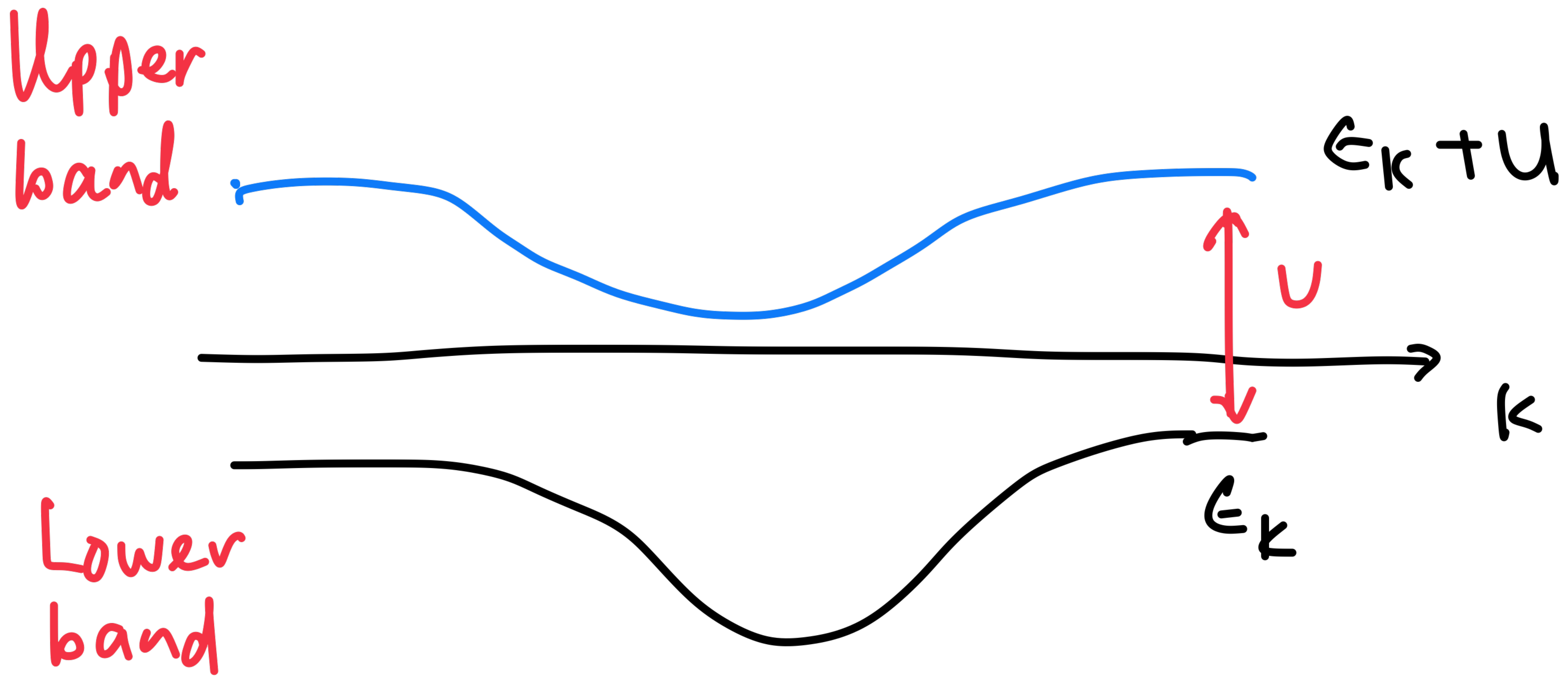
$$H = \sum_k a_k^\dagger a_k (\epsilon_k - \mu) + \sum_k \frac{U}{2} \underbrace{n_k n_k}$$

$$n_k (n_k - 1)$$

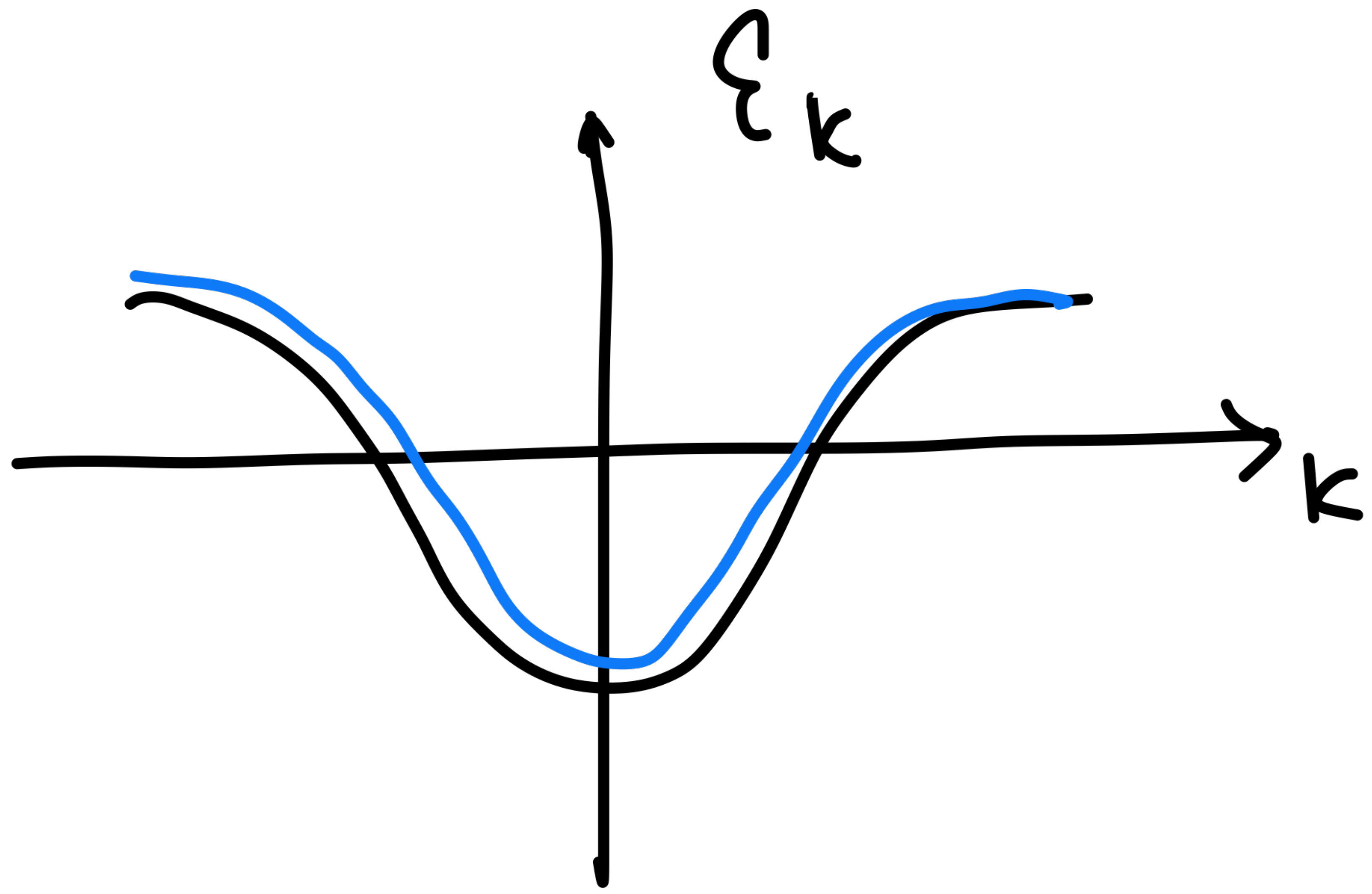
$$= \sum_k H_k , \qquad [H_k, H_{k'}] = 0$$

$$H = \sum_k a_k^\dagger a_k (\epsilon_k - \mu) + \sum_k \frac{U}{2} n_k (n_k - 1), \qquad [n_k, H] = 0$$

$$\langle n_k \rangle = 1, \text{ half-filling} \qquad \epsilon_k = -t(\cos k_x + \cos k_y + \cos k_z)$$

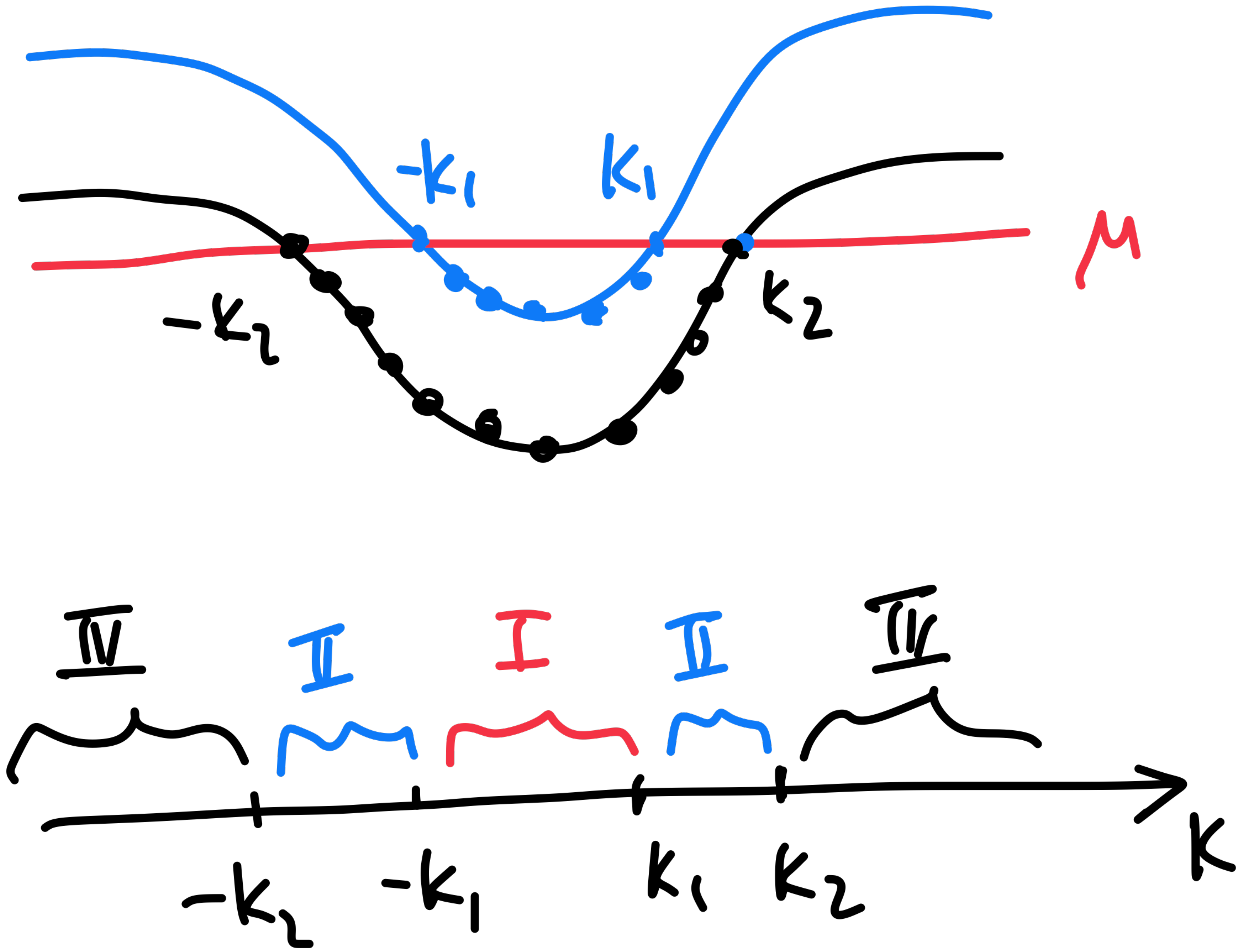


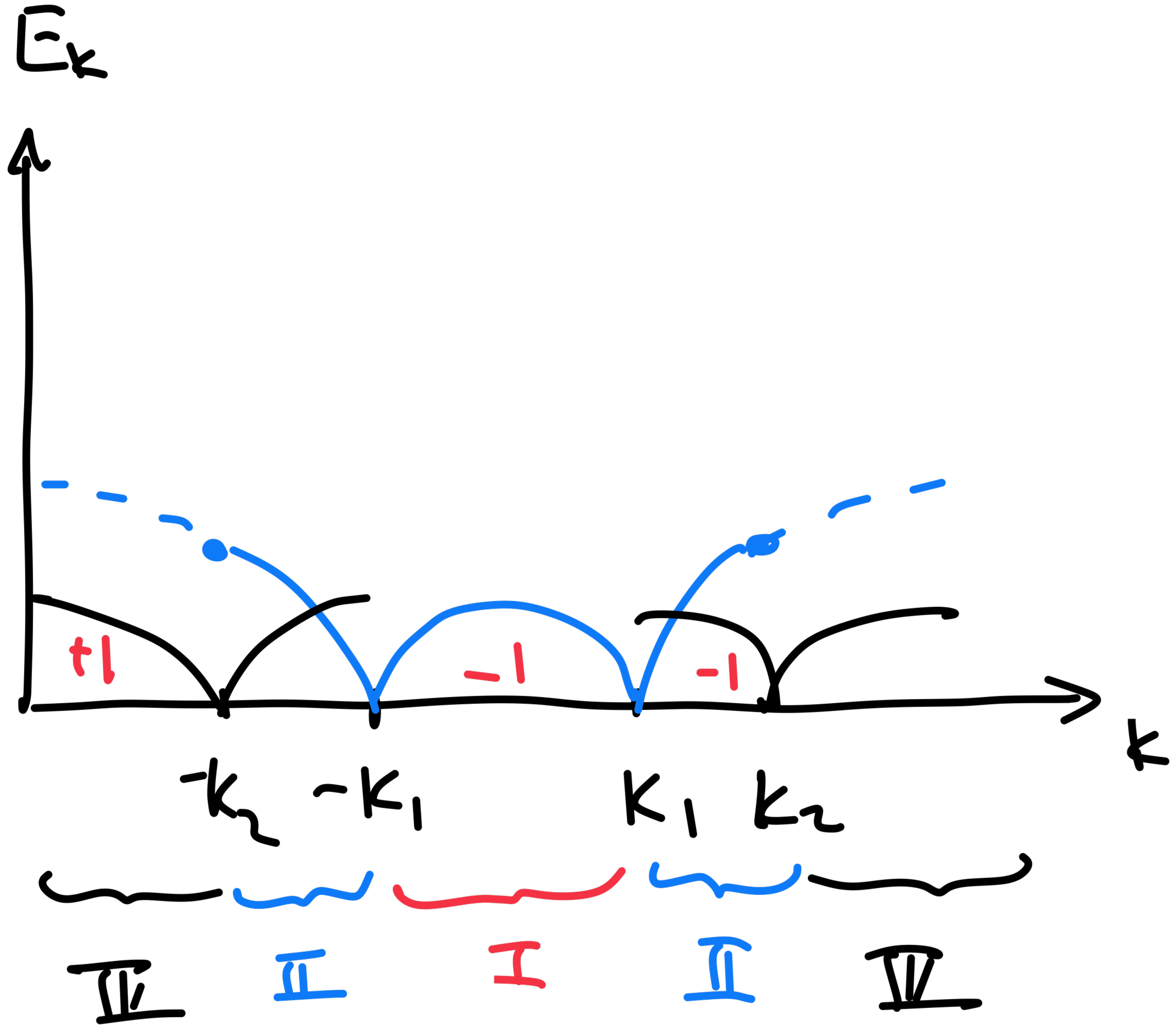
$U > 2t$



$U = 0$

$U < 2t$





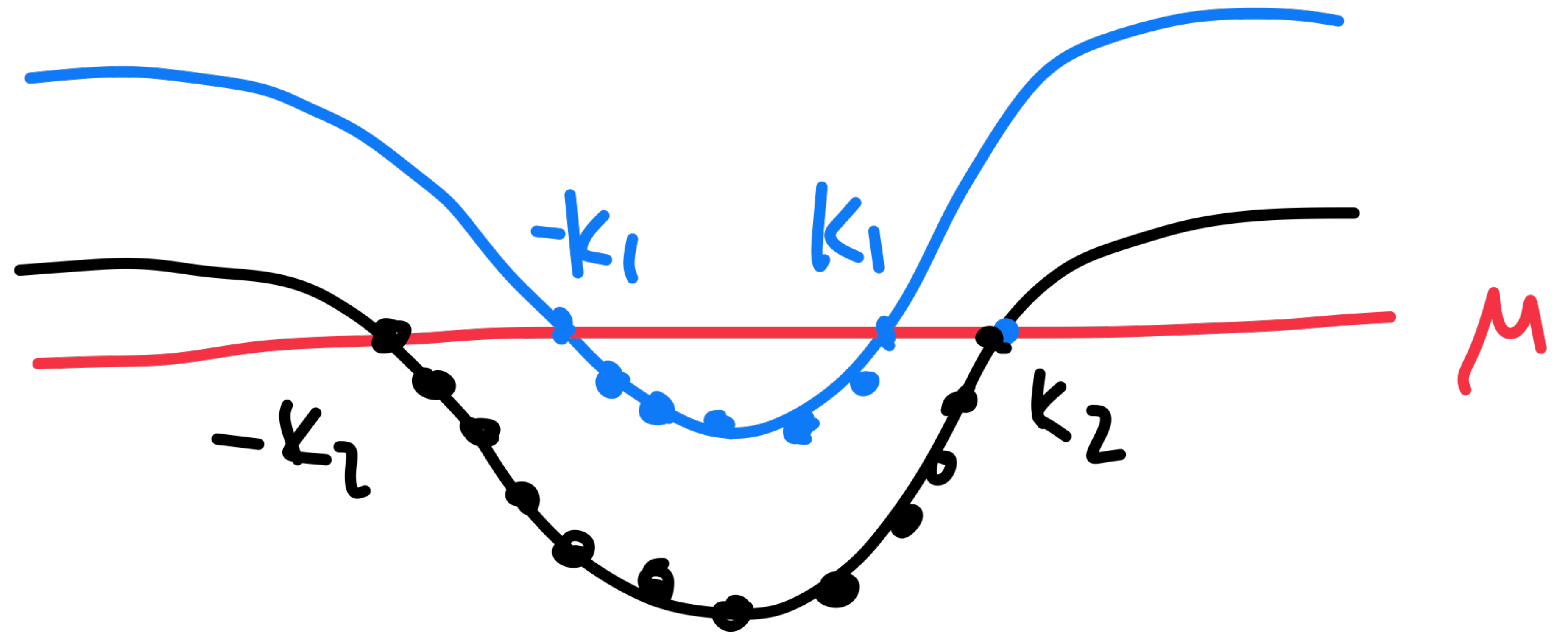
$Q = \pm 1, \quad \pm 1, \quad -1, \quad \pm 1 \quad +1$

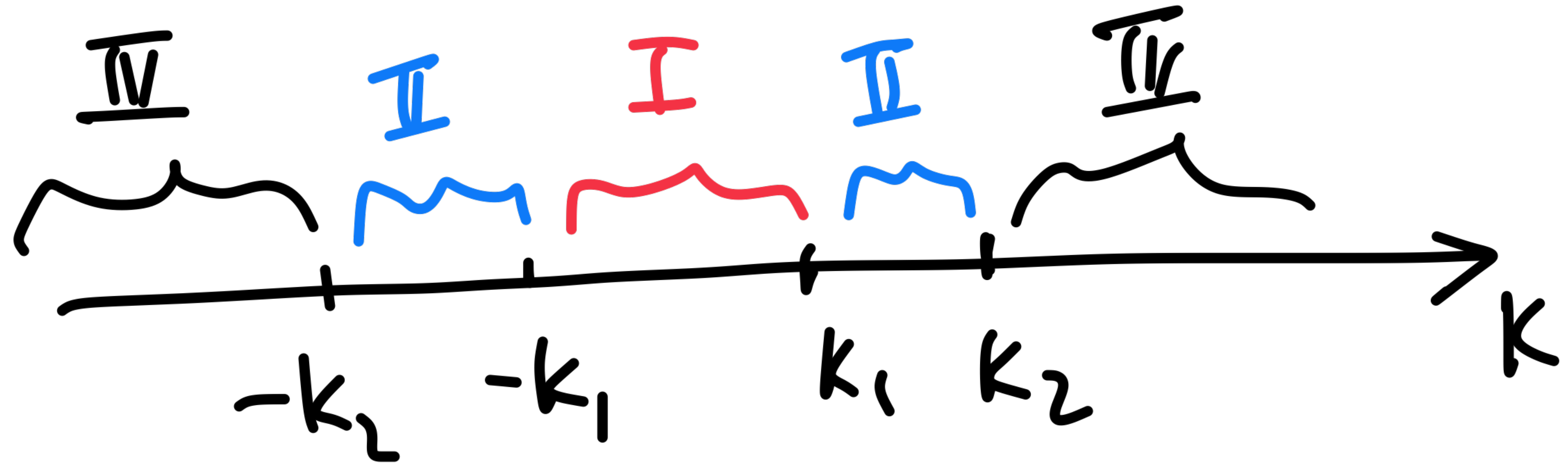
IV: $n_k = 0$

II: $n_k = 1$

I: $n_k = 2$

$U < 2t$





III: $n_k = 0$

II: $n_k = 1$

I: $n_k = 2$

$$\sum_k n_k \,\Theta(k_2 - k) = N_L(k_2)$$

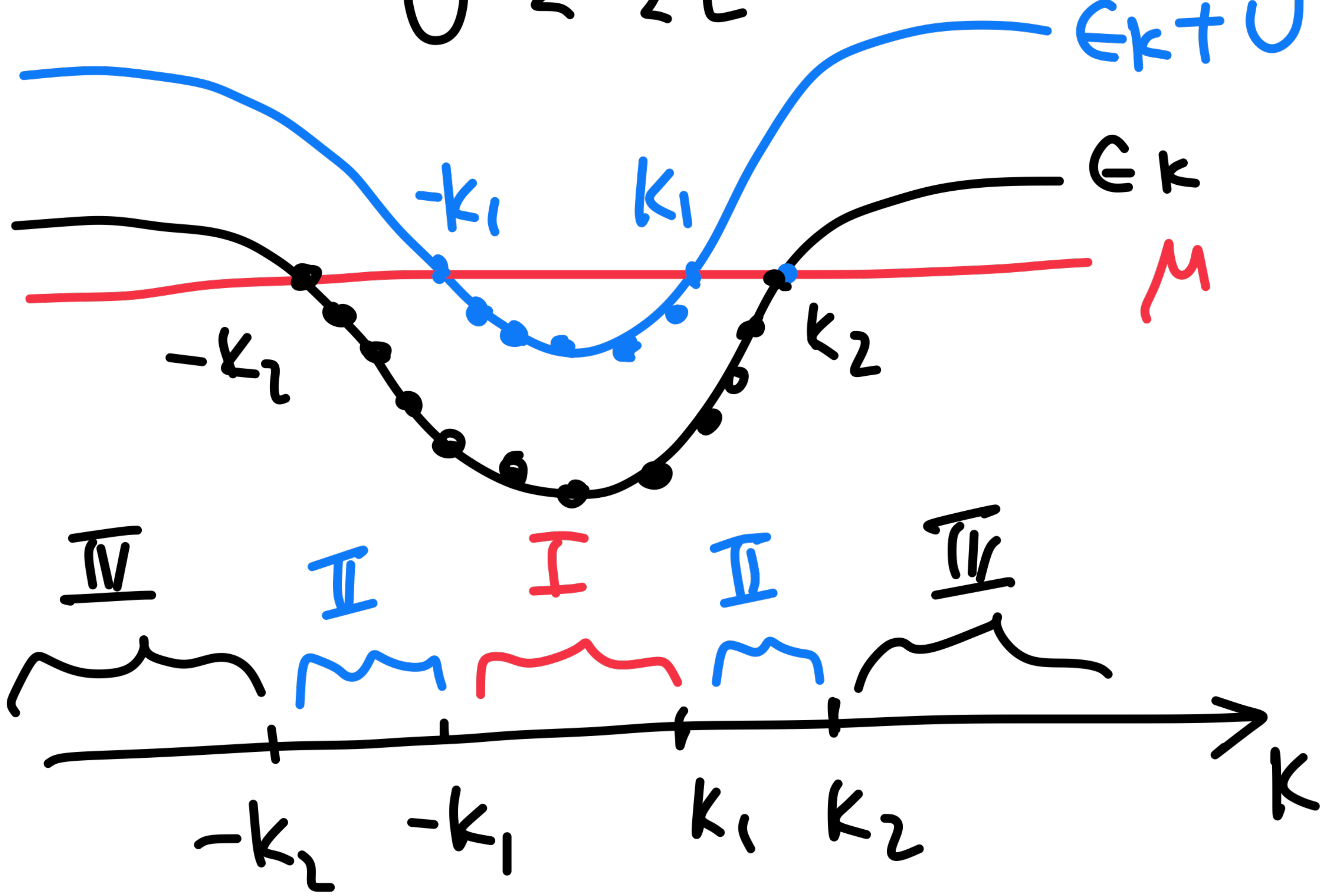
$$\sum_k n_k \,\Theta(k_1 - k) = N_u(k_1)$$

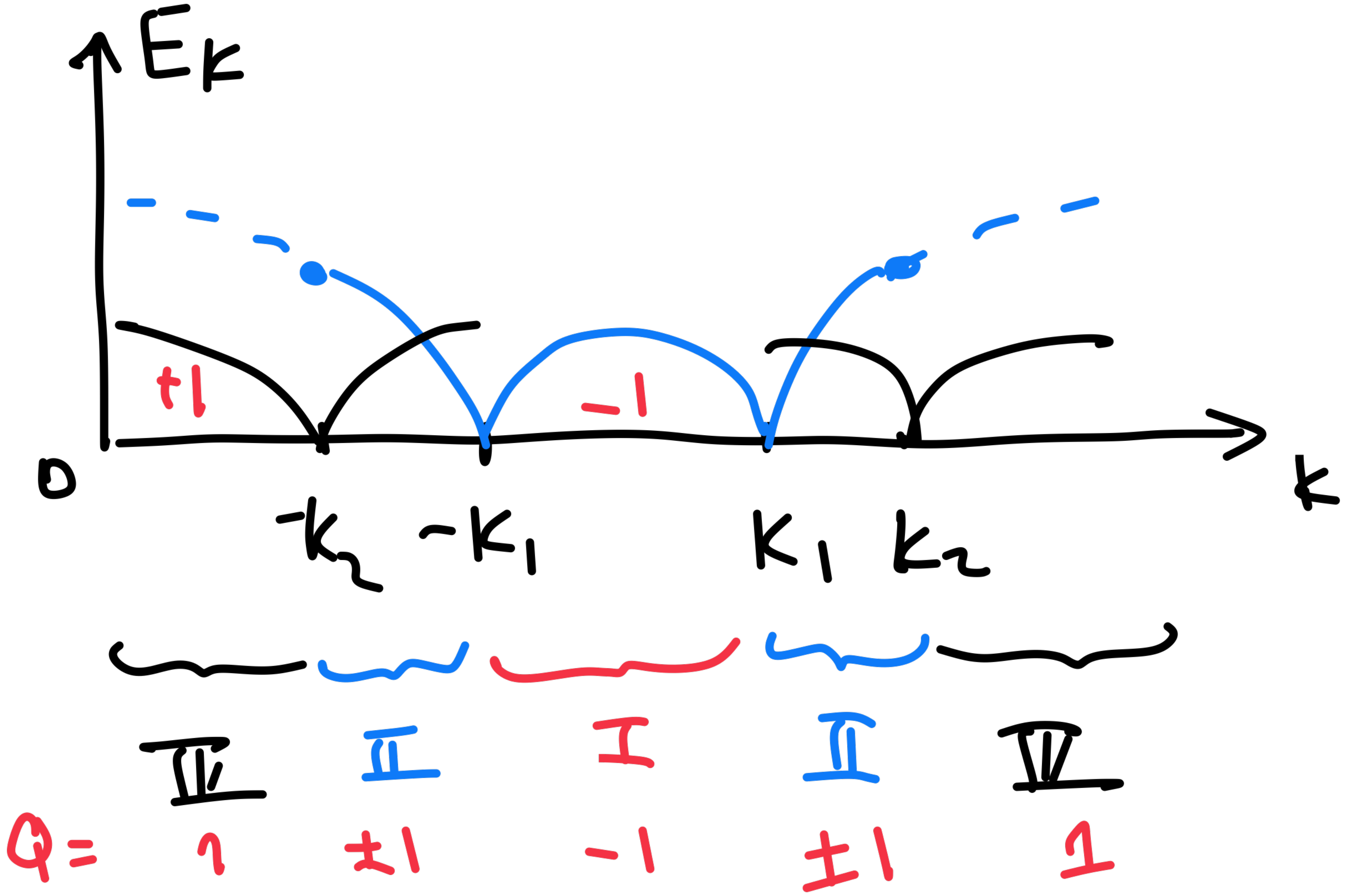
$$N_L(k_2) > \underbrace{\frac{N_L(k_2) + N_u(k_1)}{2}}_{=} > N_u(k_1)$$

$$N_L(k_2) > N(k_F) > N_u(k_1)$$

$$k_2 > k_F > k_1$$

$U < 2t$





$$G_\sigma(k,\omega) = \frac{Z_p\,(Q=1)}{\omega-(\epsilon_k+U-\mu)+i\delta} + \frac{Z_h\,(Q=-1)}{\omega-(\epsilon_k-\mu)-i\delta} \quad \text{for } k \in \text{II}$$

$Z_p + Z_h = 1$ and $Z_p = Z_h = \frac{1}{2}$ ← fractionalization

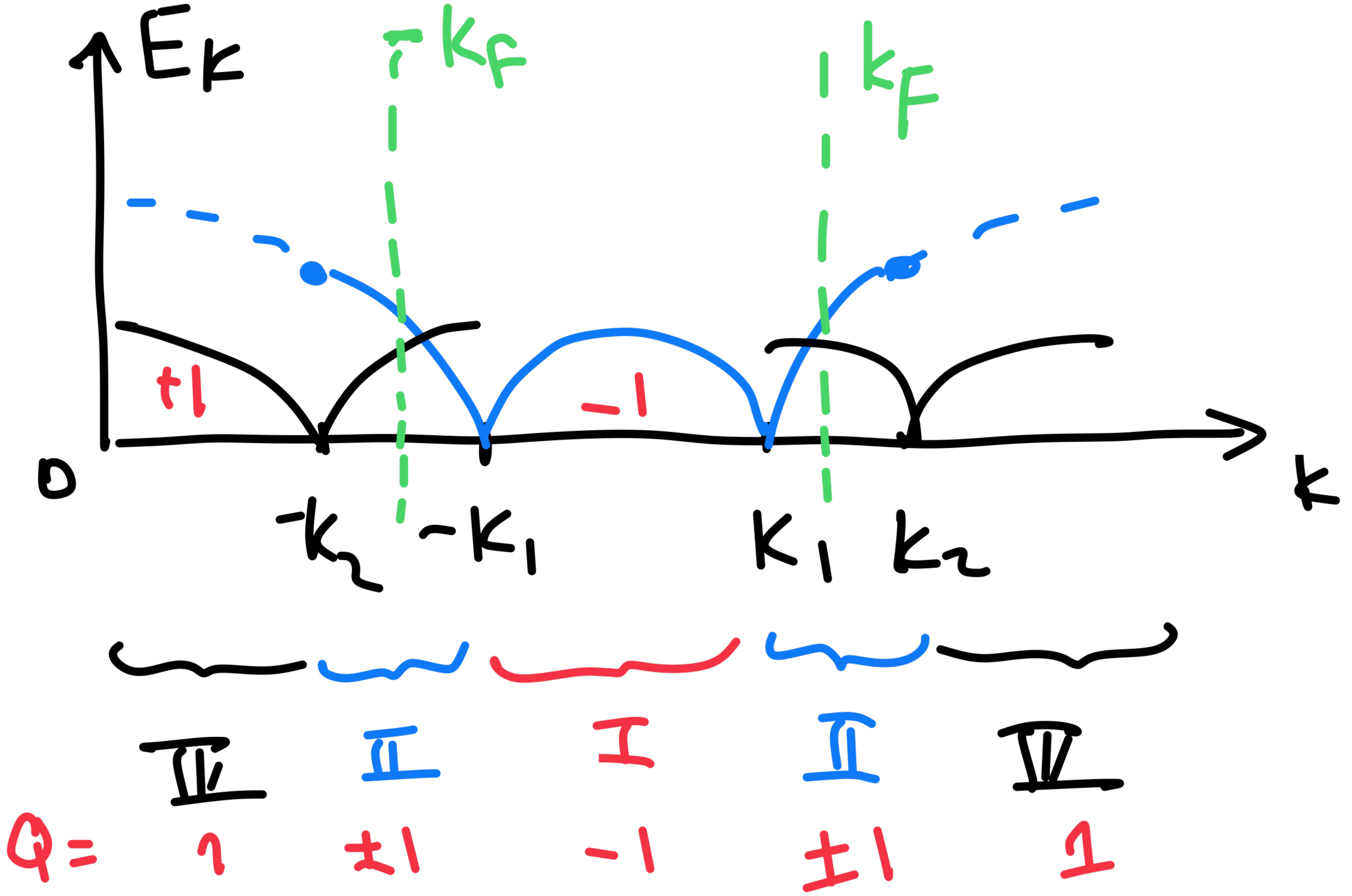
Ref: $G_\sigma(k,\omega) = \dfrac{Z}{\omega-\xi_k+i\eta_k}$ + Reg.

$$n(k=k_F+0^-) - n(k=k_F+0^+)$$

$$= Z_p(k=k_F+0^+) - Z_p(k=k_F+0^-)$$

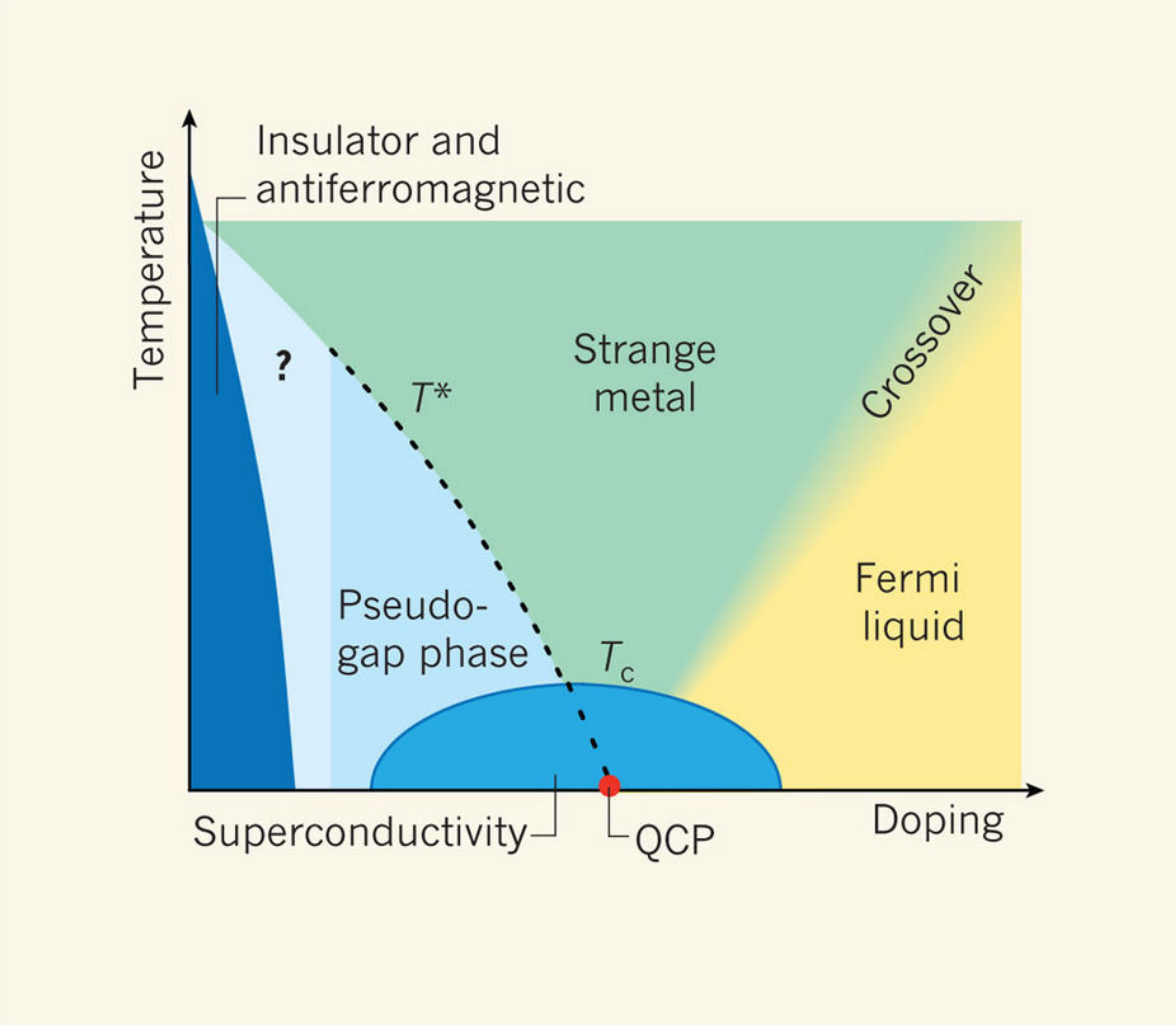
$$= \frac{1}{2} - \frac{1}{2} = 0$$

NFL.



$$G_\sigma(k,\omega) = \frac{Z_p(Q=1)}{\omega - (\epsilon_k + U - \mu) + i\delta} + \frac{Z_h(Q=-1)}{\omega - (\epsilon_k - \mu) - i\delta} \quad \text{for } k \in \mathrm{II}$$

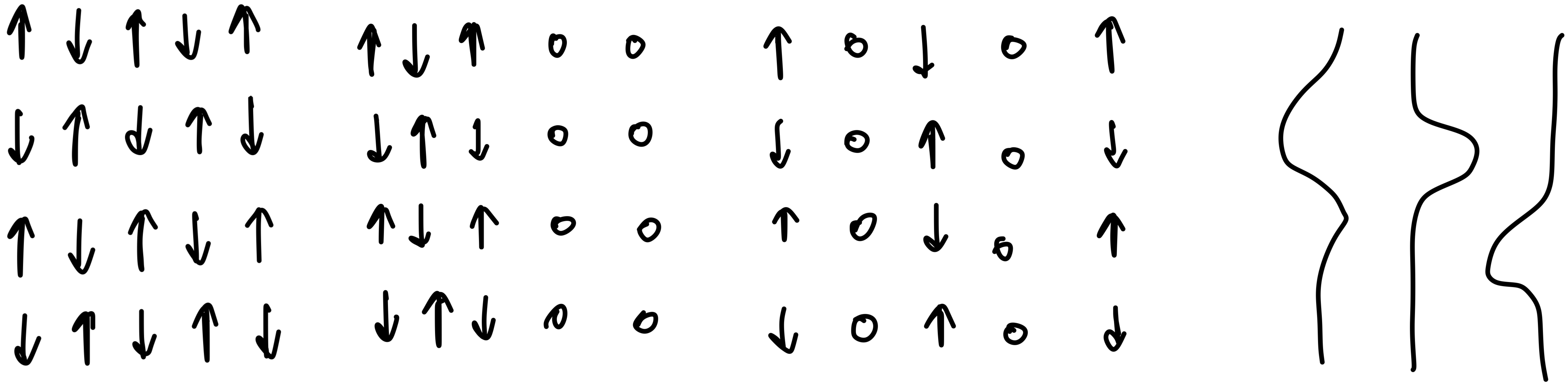
$Z_p + Z_h = 1$ and $Z_p = Z_h = \frac{1}{2}$ ← fractionalization


Temperature
Insulator and
antiferromagnetic
?
T*
Strange
metal
Crossover
Pseudo-
gap phase
Tc
Fermi
liquid
Superconductivity
QCP
Doping

- Other Non-Fermi liquid models inspired by
- 1) Stripe physics and Nematic Liquid Model
- E. Fradkin and S. A. Kivelson, Phys. Rev. B 59, 8065 (1999); S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature 393, 550 (1998); V. Oganesyan. S. Kivelson, and E. Fradkin, Phys. Rev. B 64, 195109 (2001).
- 2) Metal near QPC or critical metals (See Sachdev et al.).
- Fitzpatrick, Karchru, Kaplan, Raghu, PRB 89, 165114(2014).

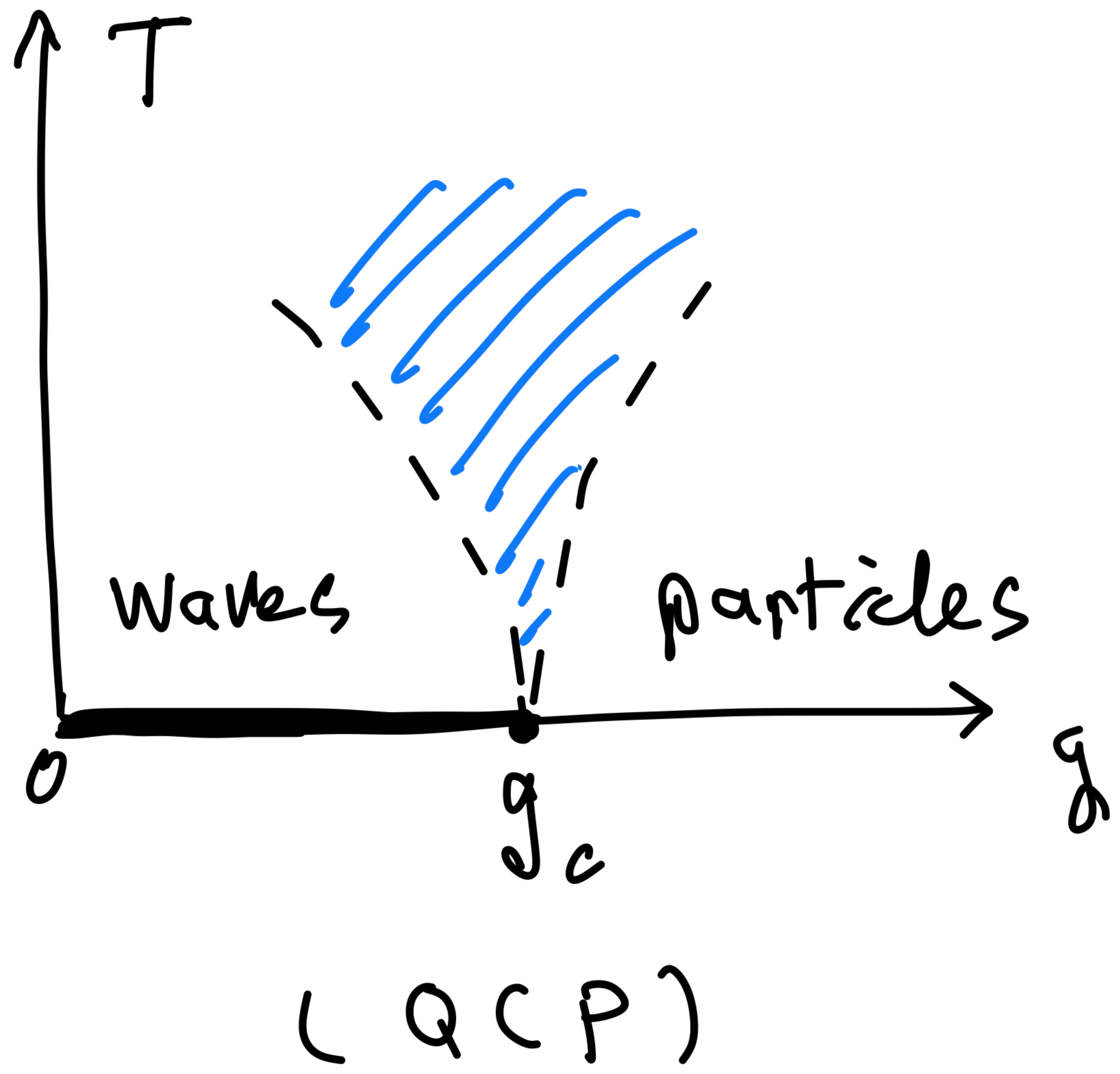
I

- Mott Insulator - Phase separation-Strip phase - electronic Nematic liquid



fermi Surfaces interacts with nematic modes.

II A.



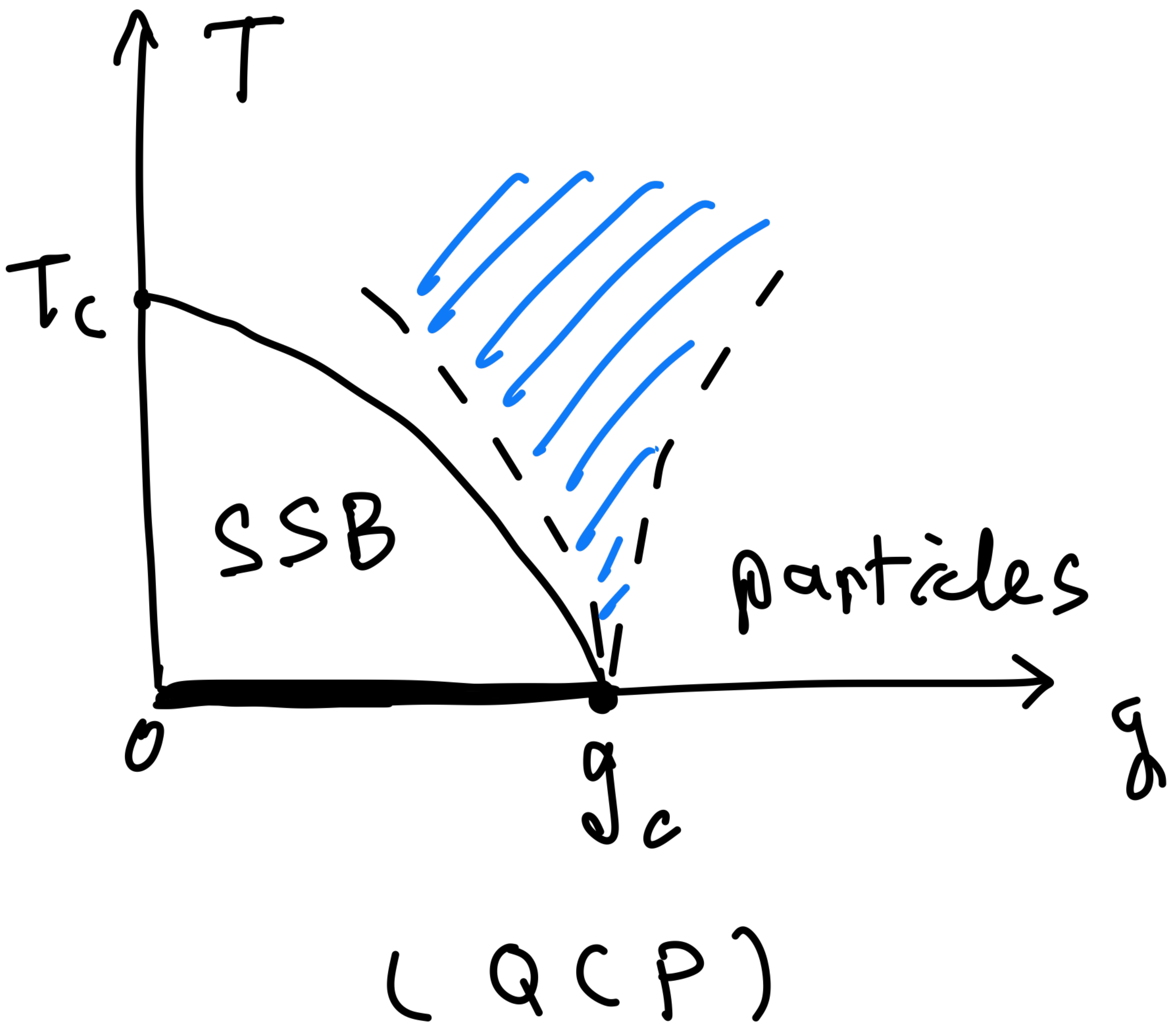
Near a QCP

Fermi Surface +

Wilson-Fisher bosons

(Scale / Conformal symmetry)

II A.



Near a QCP

Fermi Surface +

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(Scale / Conformal symmetry)