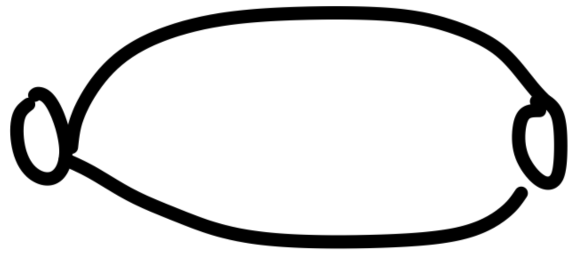
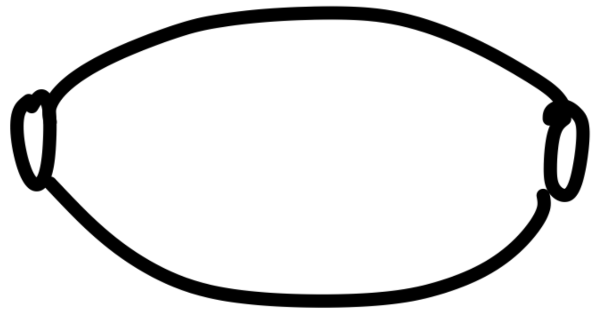
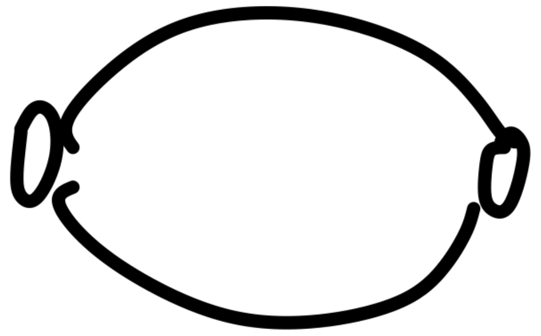
Phys529B: Topics of Quantum Theory

Lecture 8: interacting fermions and Non-Fermi liquid

instructor: Fei Zhou

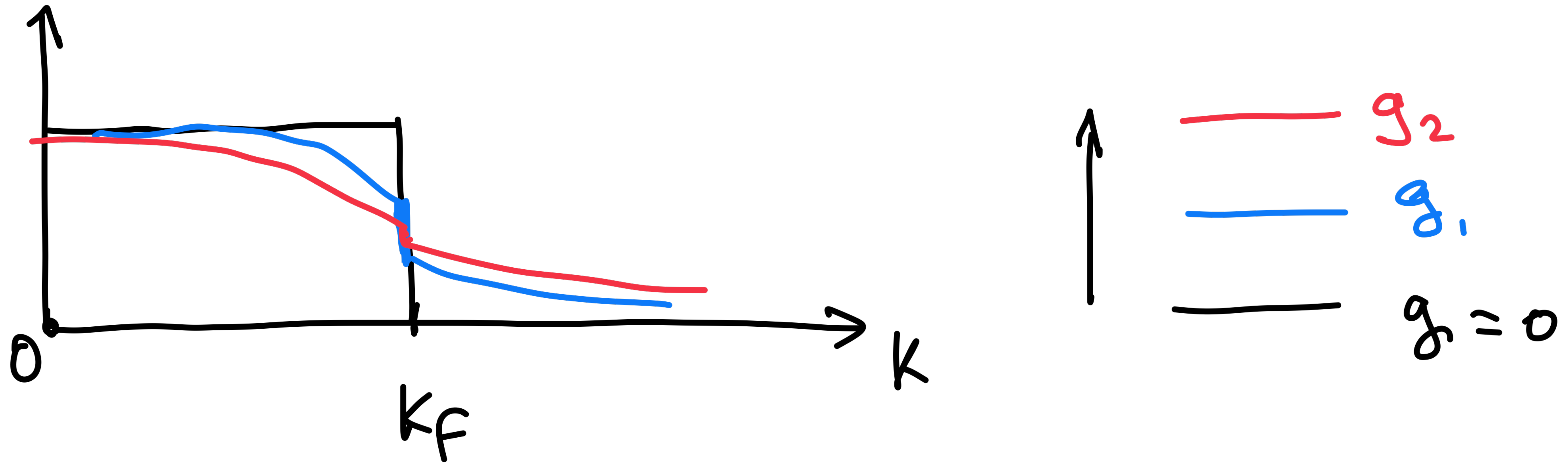
F.L. $G_B =$  $\Big/ \Big(1 - g$  $\Big)$

$1 - g$  $= 1 - g A(k_F)\left\{ 1 + \frac{\Omega}{v_F Q} \ln \frac{\Omega + v_F Q}{\Omega - v_F Q} \right\} = 0$

Real if $\Omega > v_F Q$

$$g \to 0, \quad \Omega - v_F Q \triangleq 2 v_F Q \, e^{-\frac{1}{g A(k_F)}}$$

- Sum rule: $$-\frac{1}{\pi}\int d\omega \mathscr{I}m G_R(\mathbf{k},\omega) = 1$$



Luttinger theorem (phase space invariance)

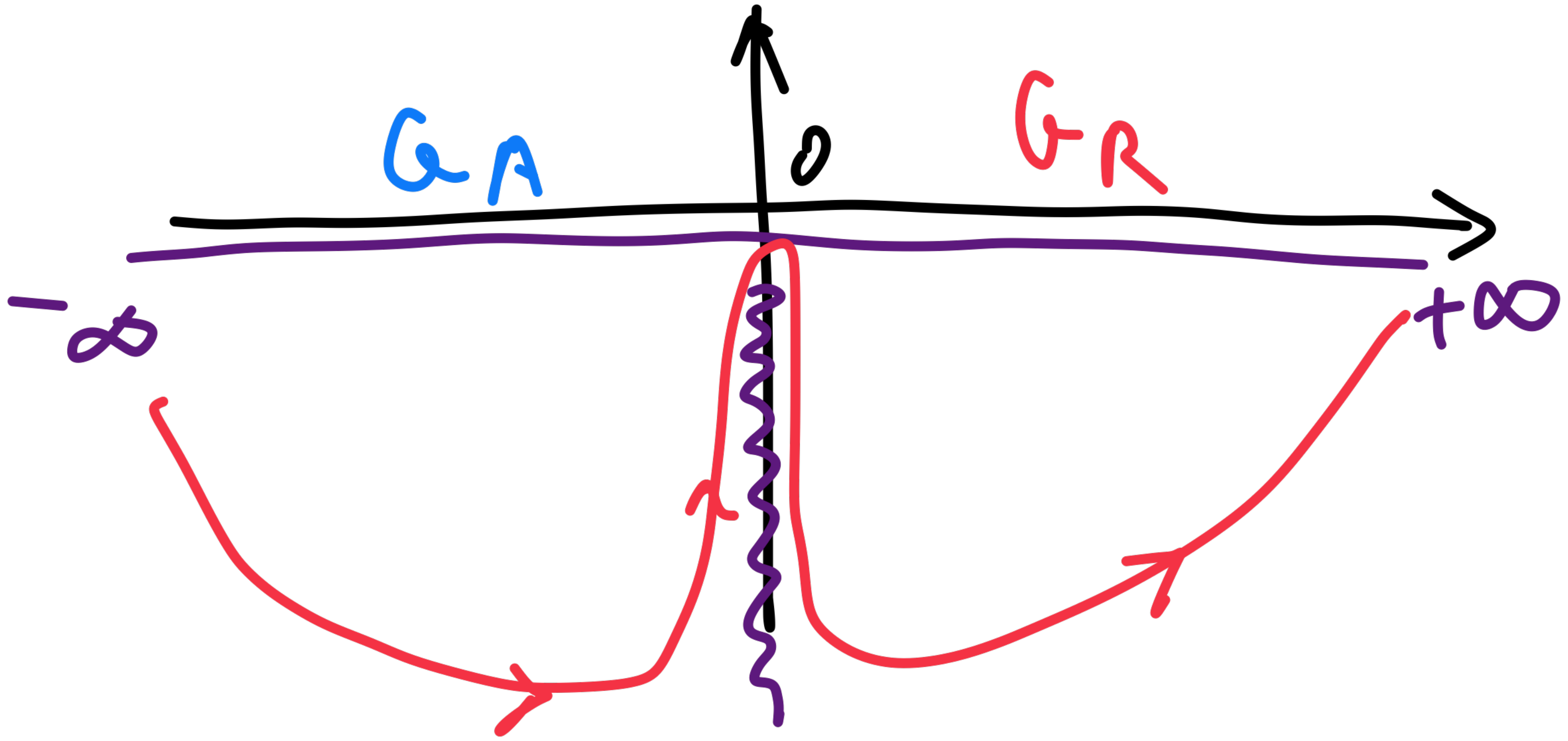
density

$$\int \frac{d^3p}{(2\pi)^3} = n_0 = \frac{1}{6\pi^2} k_F^3$$

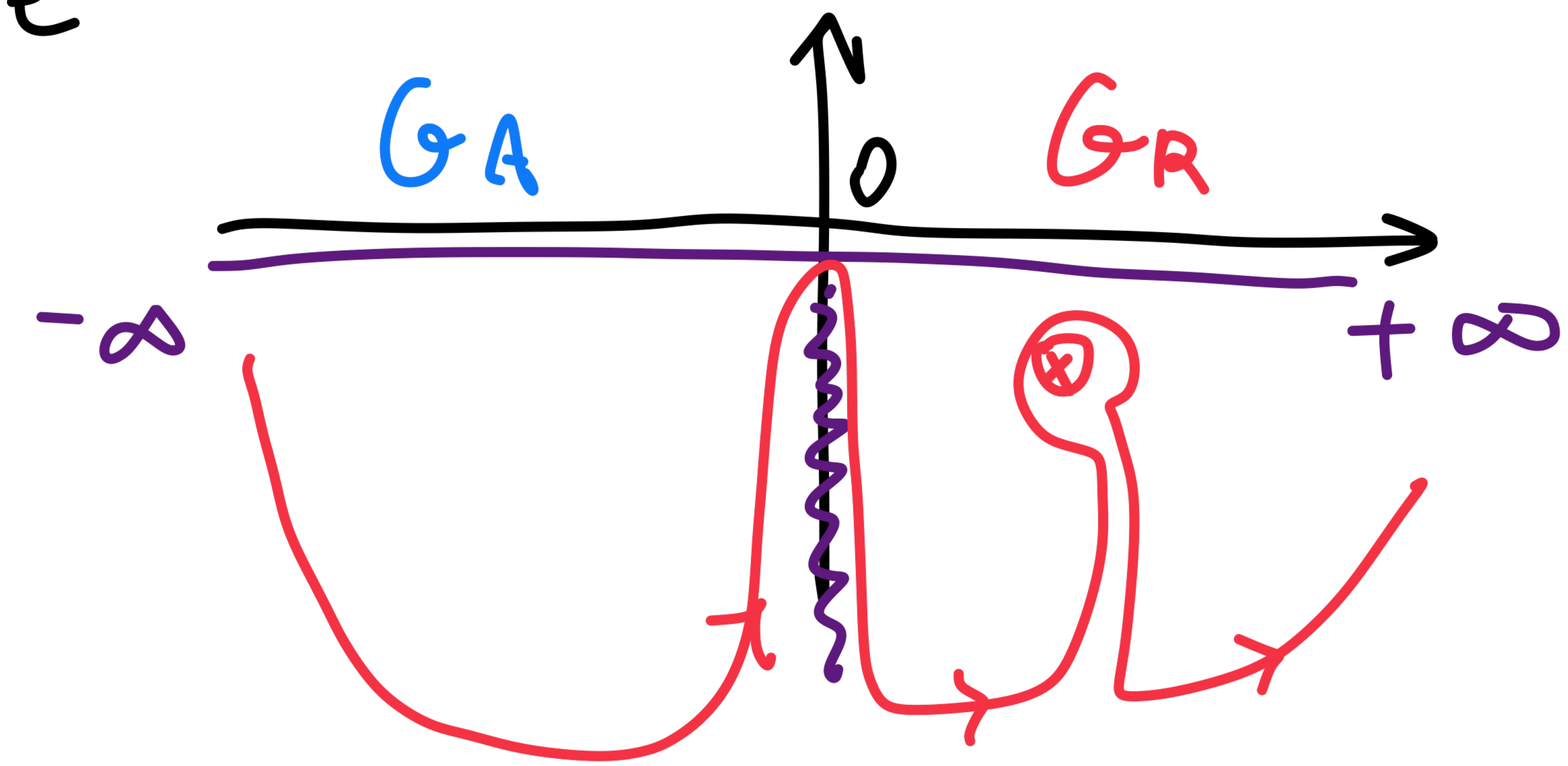
$$G(\vec{p}, \omega=0) > 0$$

$$1 - n_k = -i \int \frac{d\omega}{2\pi} G(k,\omega)\, e^{-i\omega 0^+}$$

$k < k_F$



$k > k_F$



$\mathrm{Res}\, G_R(k, \omega > 0)$ at poles with $\omega > 0$

$$\delta n = n(k = k_F + 0^-) - n(k = k_F + 0^+) = Z(k_F + 0^+, \omega > 0) - Z(k_F + 0^-, \omega > 0)$$

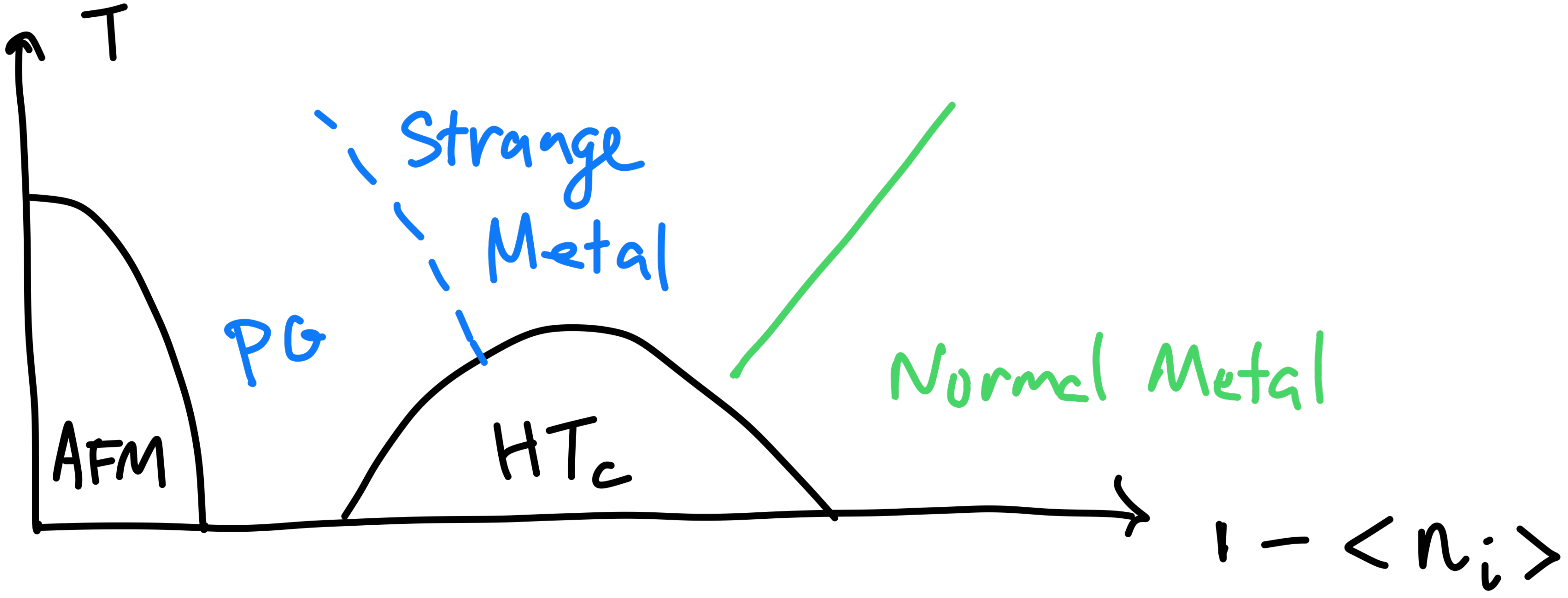
For F.L., $Z(k = k_F + 0^-) = 0$, $\quad \delta n = Z(k = k_F + 0^+)$.

- NFL phenomenology
- 1) Z vanishes at Fermi surfaces and no quasi-particles.
- 2) Z is finite across Fermi surface but Z on two sides of the Fermi surface cancels. I.e. $Z(k,\omega>0)$ is continuous across a Fermi surface near k=k_F. For given momentum and orbital, quasi-particle and hole co-exist.

Hubbard Model $\qquad n_i = \sum_{\sigma=\pm} a^\dagger_{i\sigma} a_{i\sigma}$

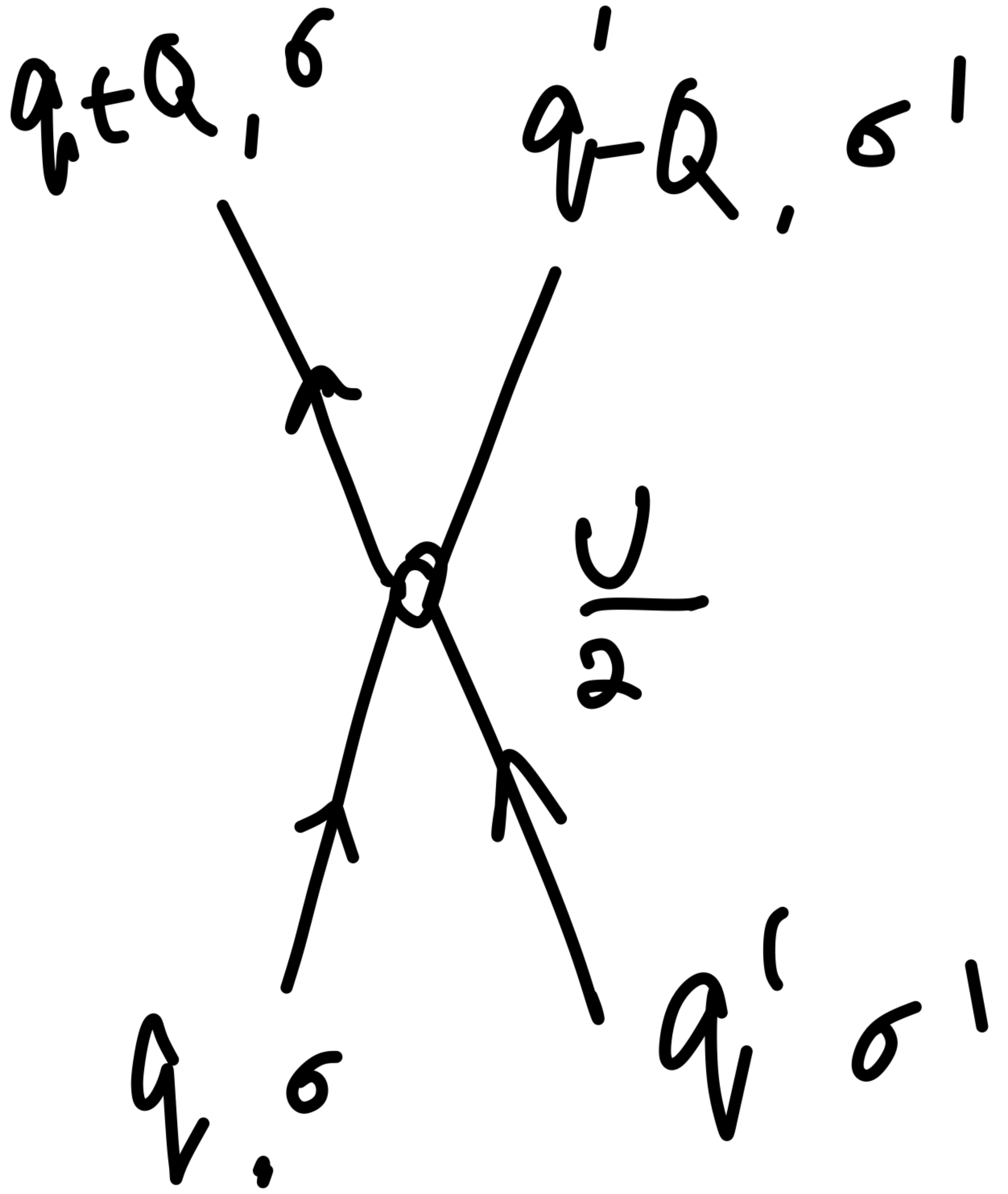
$$H = \underbrace{-t \sum_{\langle ij\rangle,\sigma} a^\dagger_{i\sigma} a_{j\sigma} + h.c.}_{K} + \underbrace{\frac{U}{2} \sum_i n_i (n_i - 1)}_{U},$$

$\langle n_i \rangle = 1$ is half-filling. $\langle n_i \rangle = 2$ Completely filled.

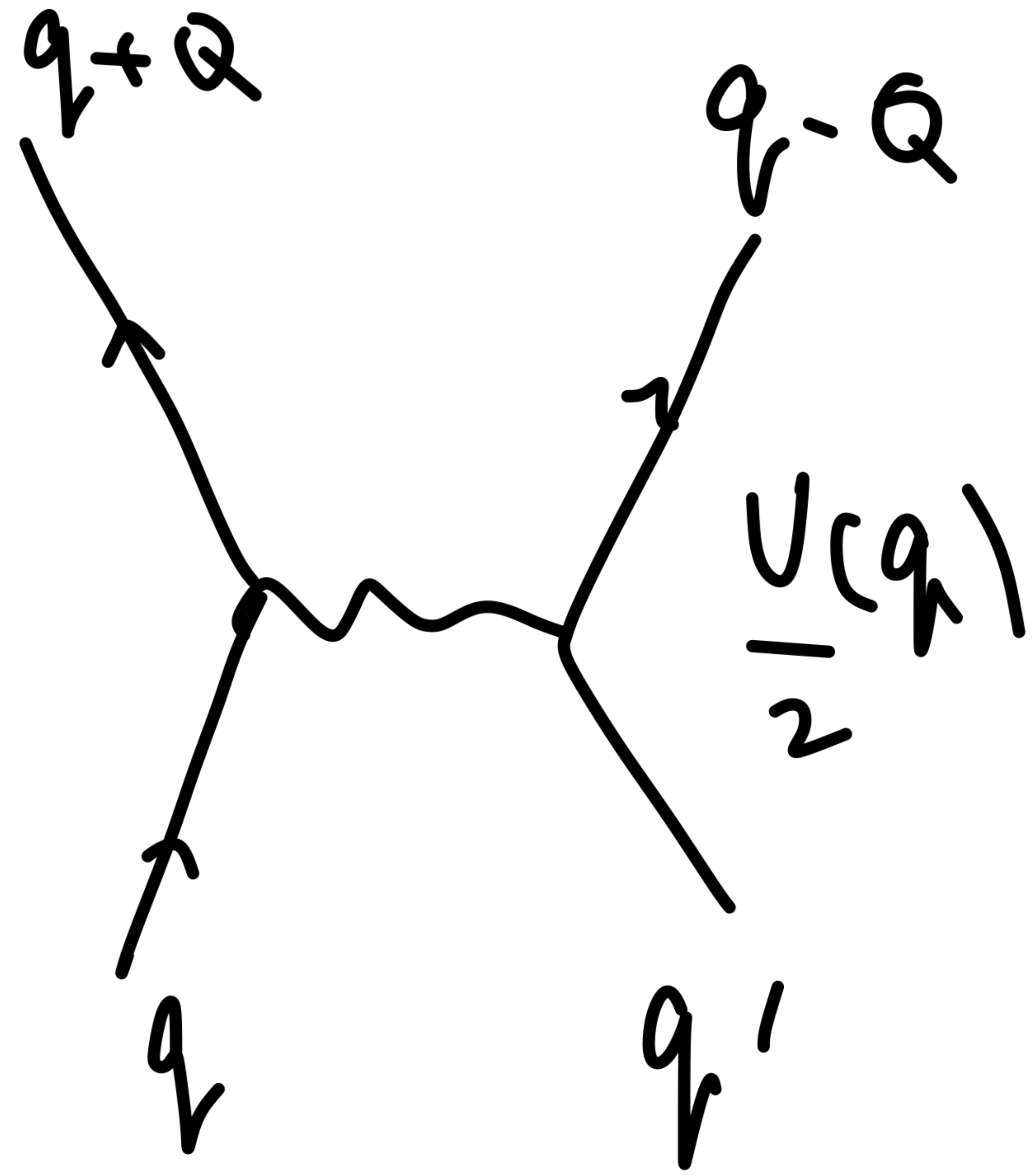


k - Space

$$H - \mu \hat{N} = \sum_k a_k^\dagger a_k \left(\epsilon_k - \frac{U}{2} - \mu \right) + \frac{U}{2\Omega} \sum a^\dagger_{q+Q} \, a_q \, a^\dagger_{q'-Q} \, a_{q'}$$



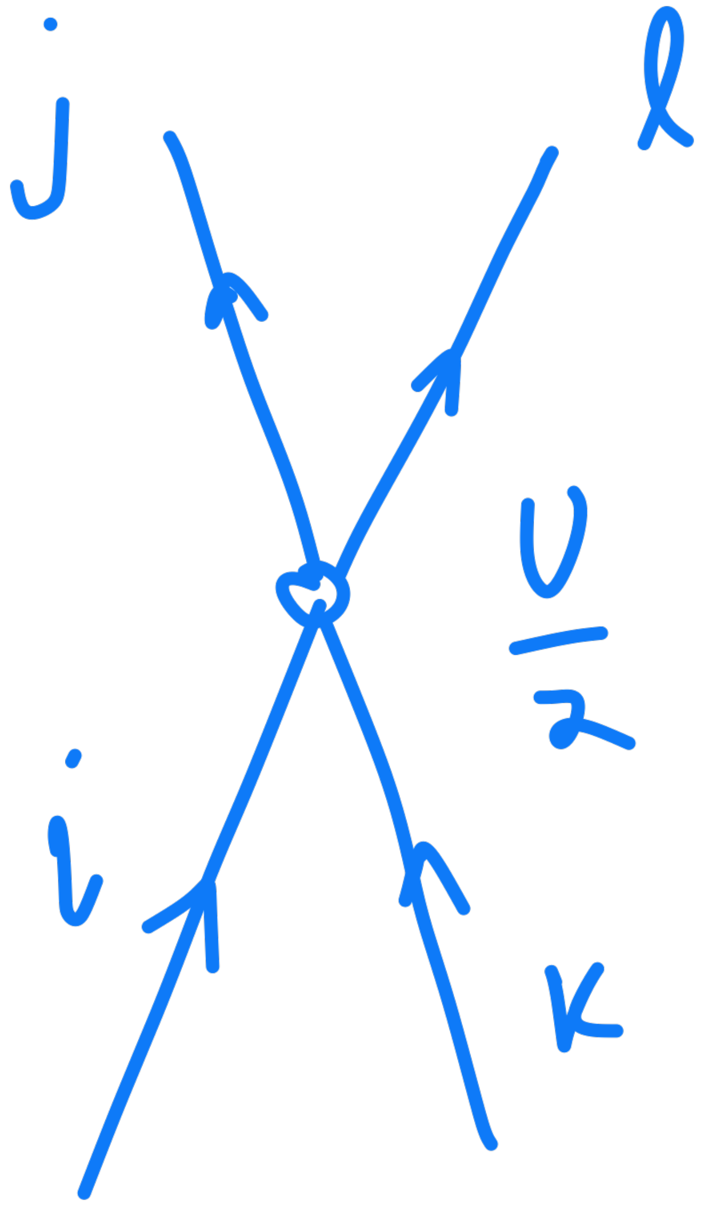
" "
On-site interaction



"Non-Local / long Range"

HK Model (Hatsugai-Kohmoto)

$$H = -t\sum_{\langle ij\rangle} a_i^\dagger a_j + h.c. + \frac{U}{2}\sum_{ijkl} a_i^\dagger a_j \cdot a_k^\dagger a_l \cdot \delta_{i+k,\,j+l}$$



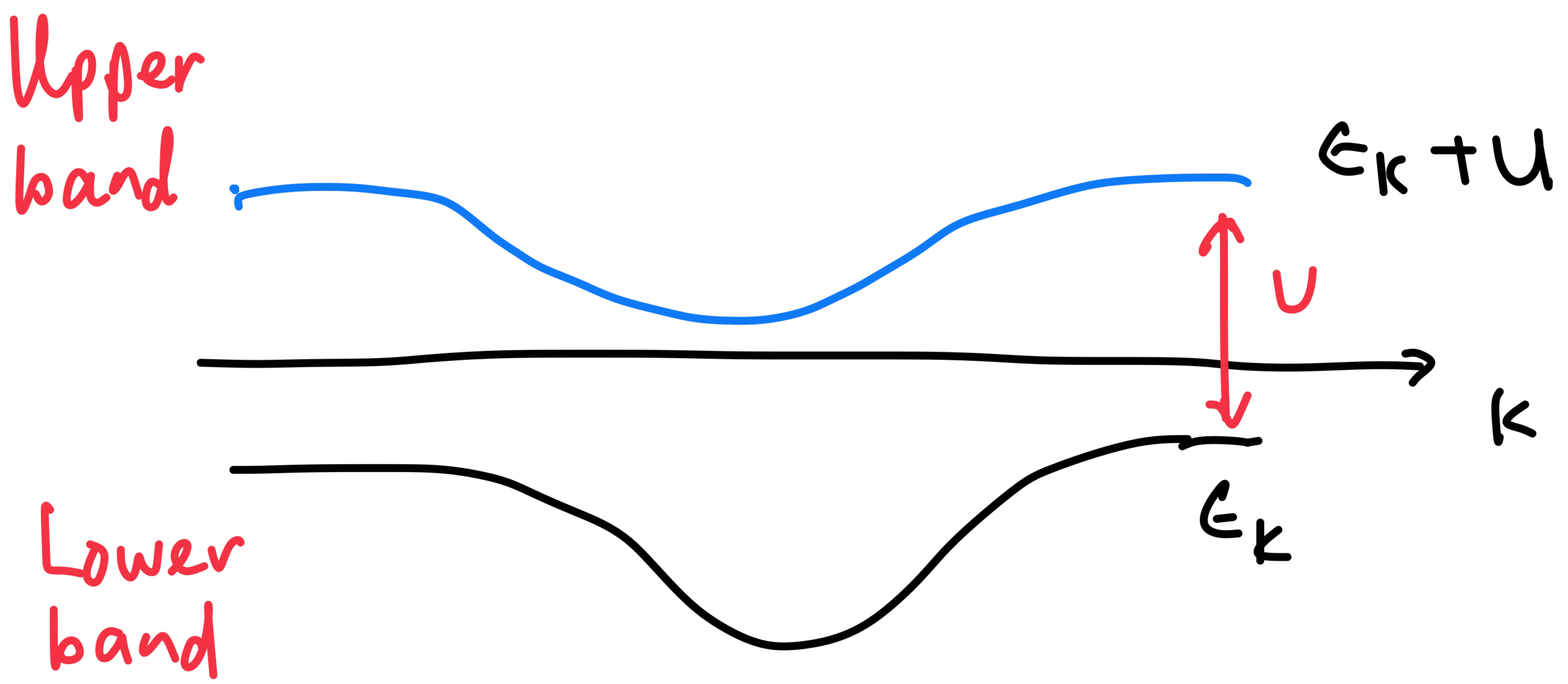
$$H = \sum_k a_k^\dagger a_k (\epsilon_k - \mu) + \sum_k \frac{U}{2} \overbrace{n_k}^{\text{local in k-space}} n_k$$

$n_k(n_k - 1)$

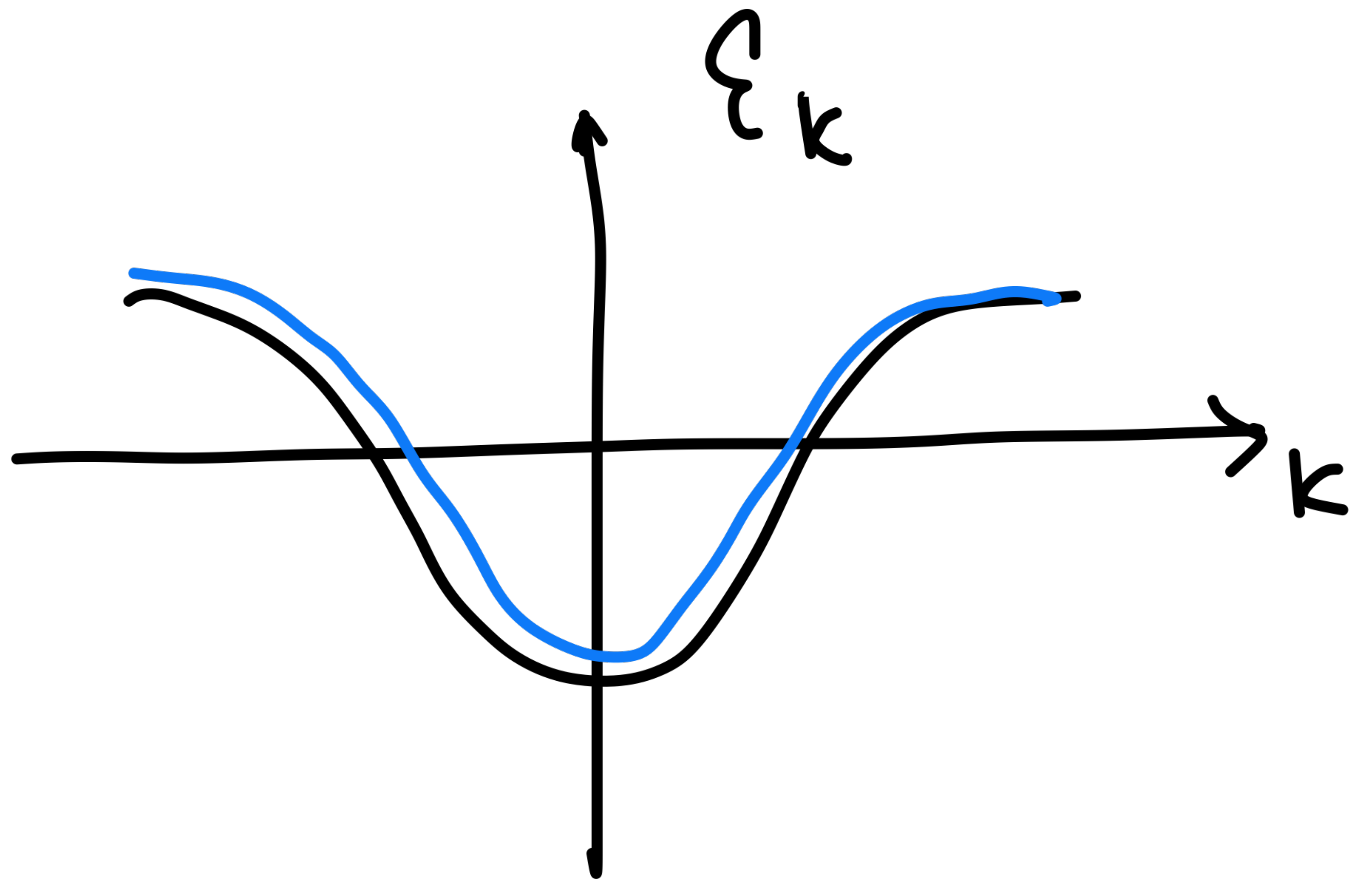
$$= \sum_k H_k, \qquad [H_k, H_{k'}] = 0$$

$$H = \sum_k a_k^\dagger a_k (\epsilon_k - \mu) + \sum_k \frac{U}{2} n_k (n_k - 1), \qquad [n_k, H] = 0$$

$\langle n_k \rangle = 1$, half-filling $\qquad \epsilon_k = -t(\cos k_x + \cos k_y + \cos k_z)$

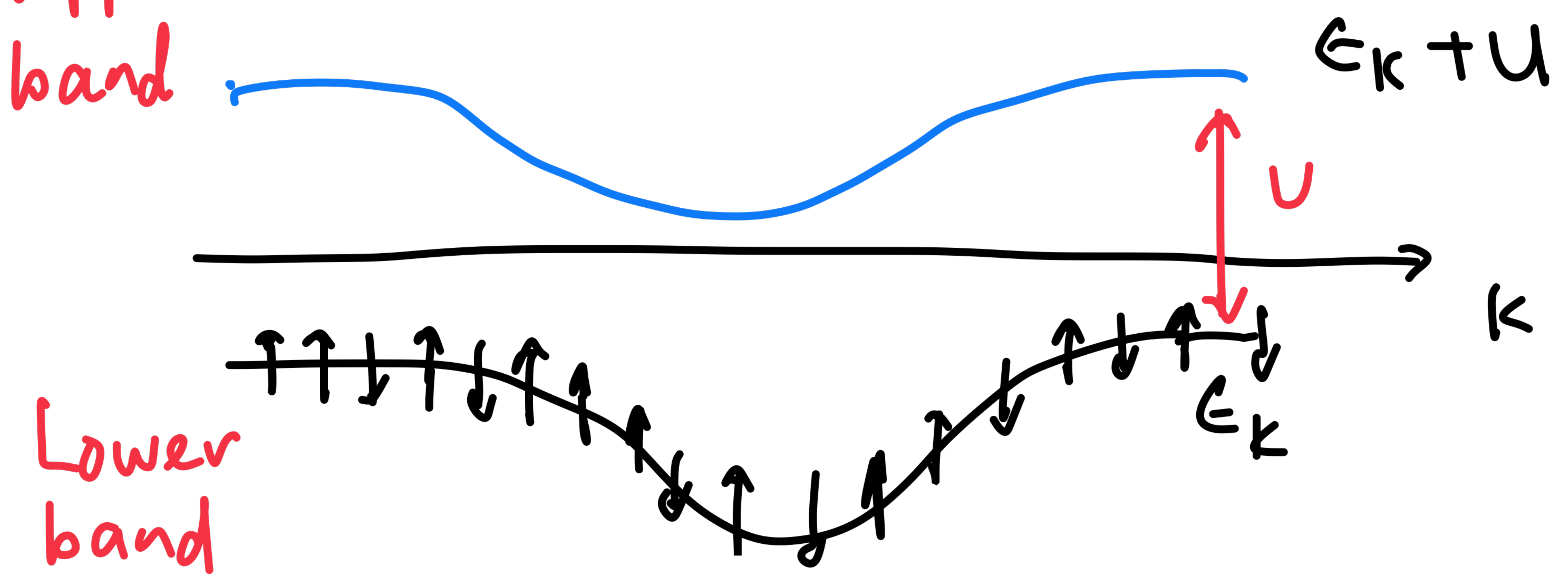


$U > 2t$



$U = 0$

$\langle n_k \rangle = 1$



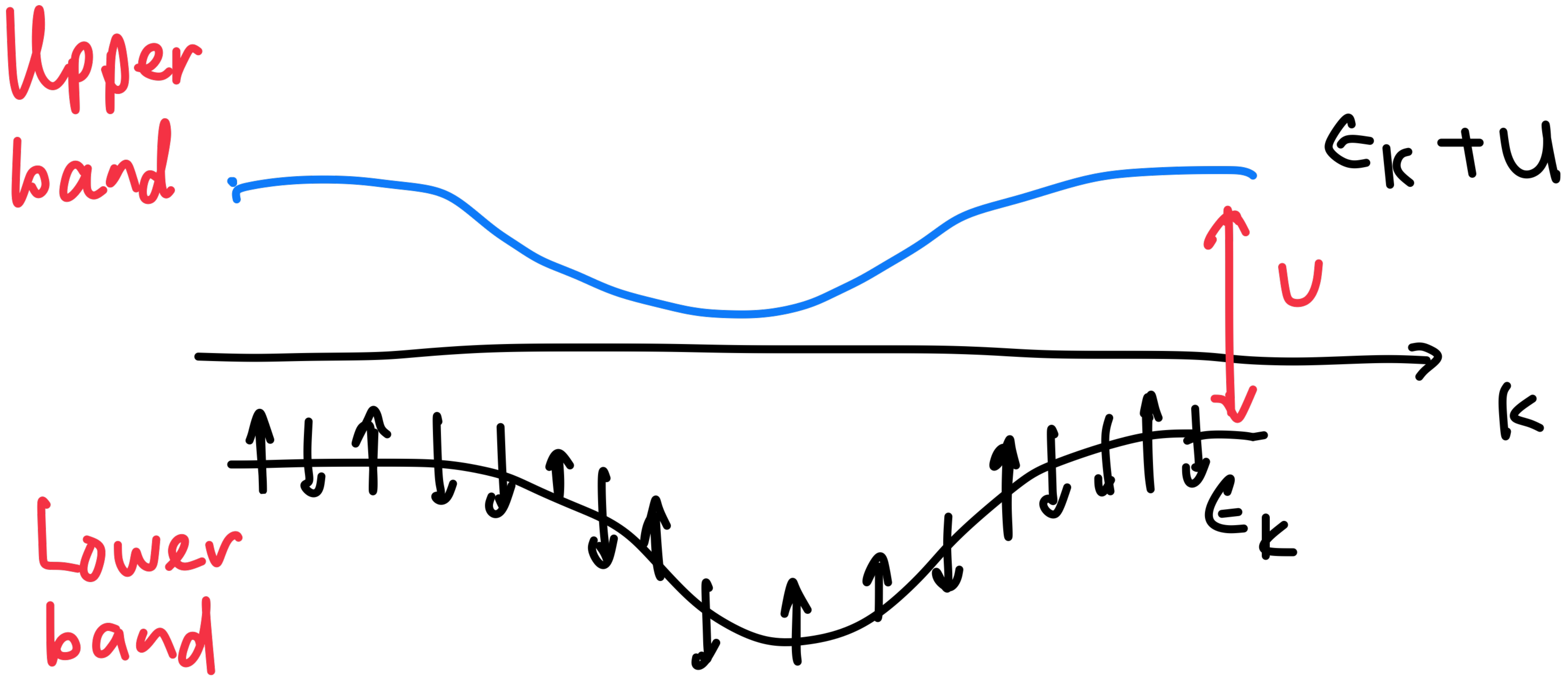
each k-state : $\underbrace{|\uparrow\rangle \text{ or } |\downarrow\rangle}_{2\text{-fold}}$

Ground State : 2^N-fold

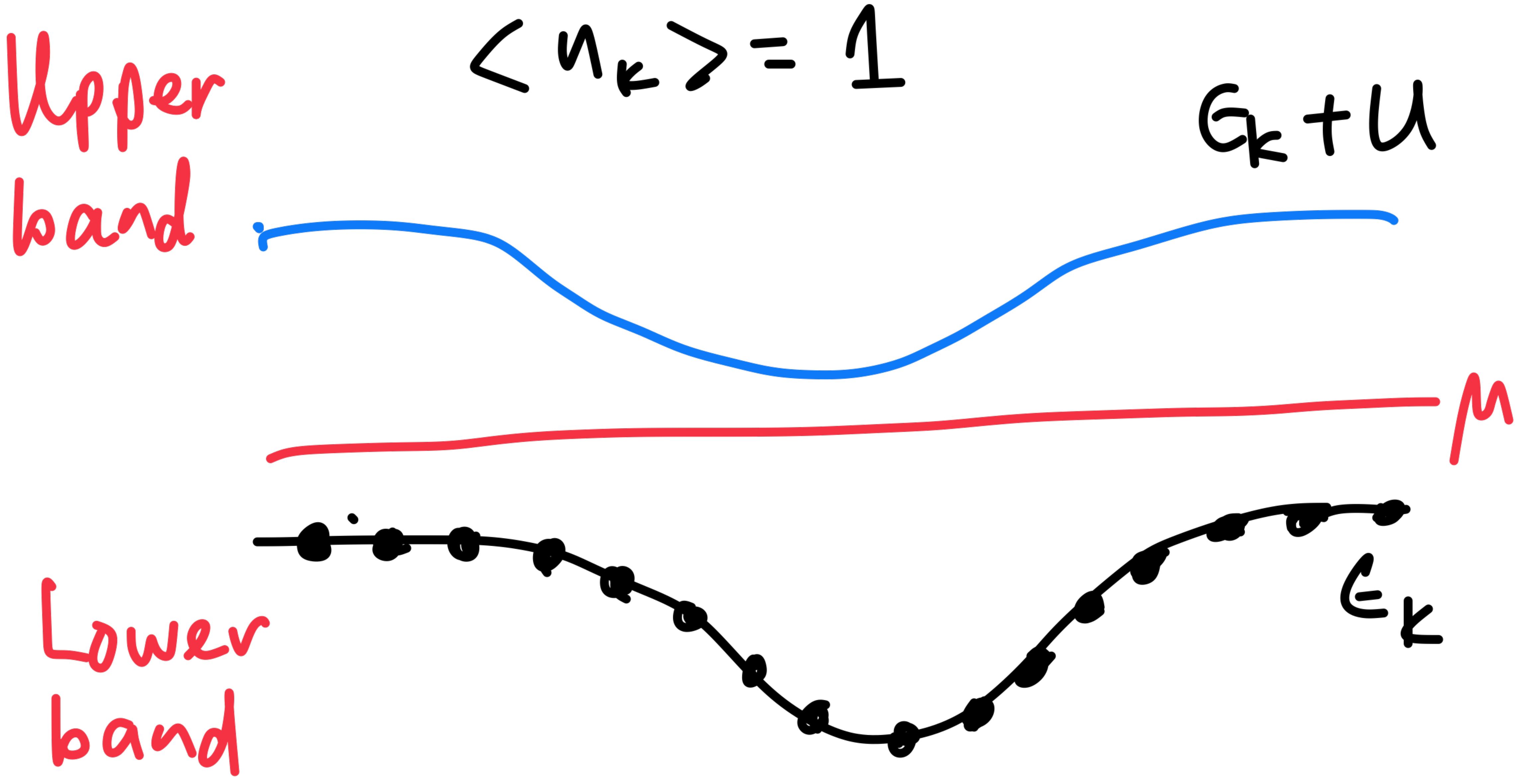
N = Number of fermions

$\langle S_z \rangle = 0$ for any k.

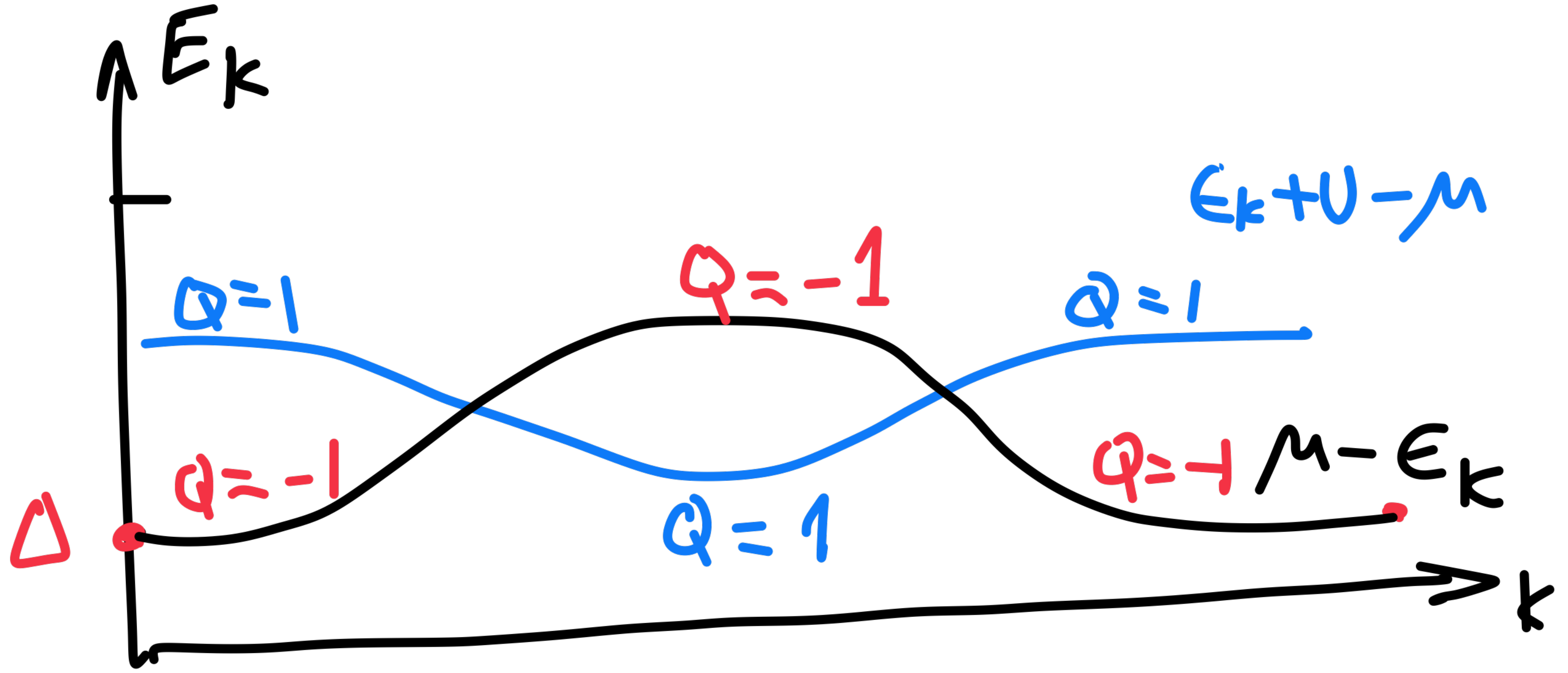
Ground state Non-deg, when $U = 0$



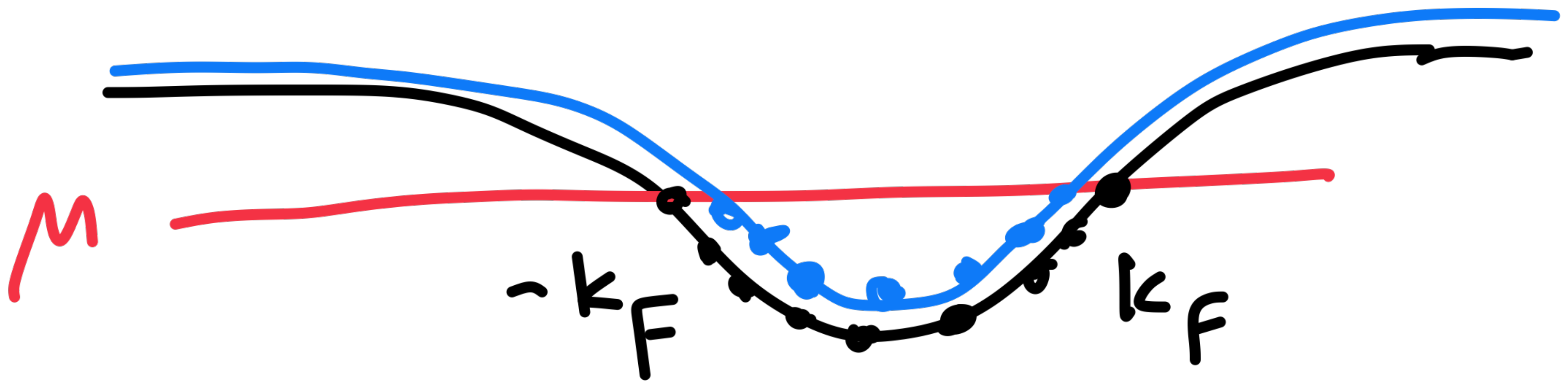
$U > 2t$



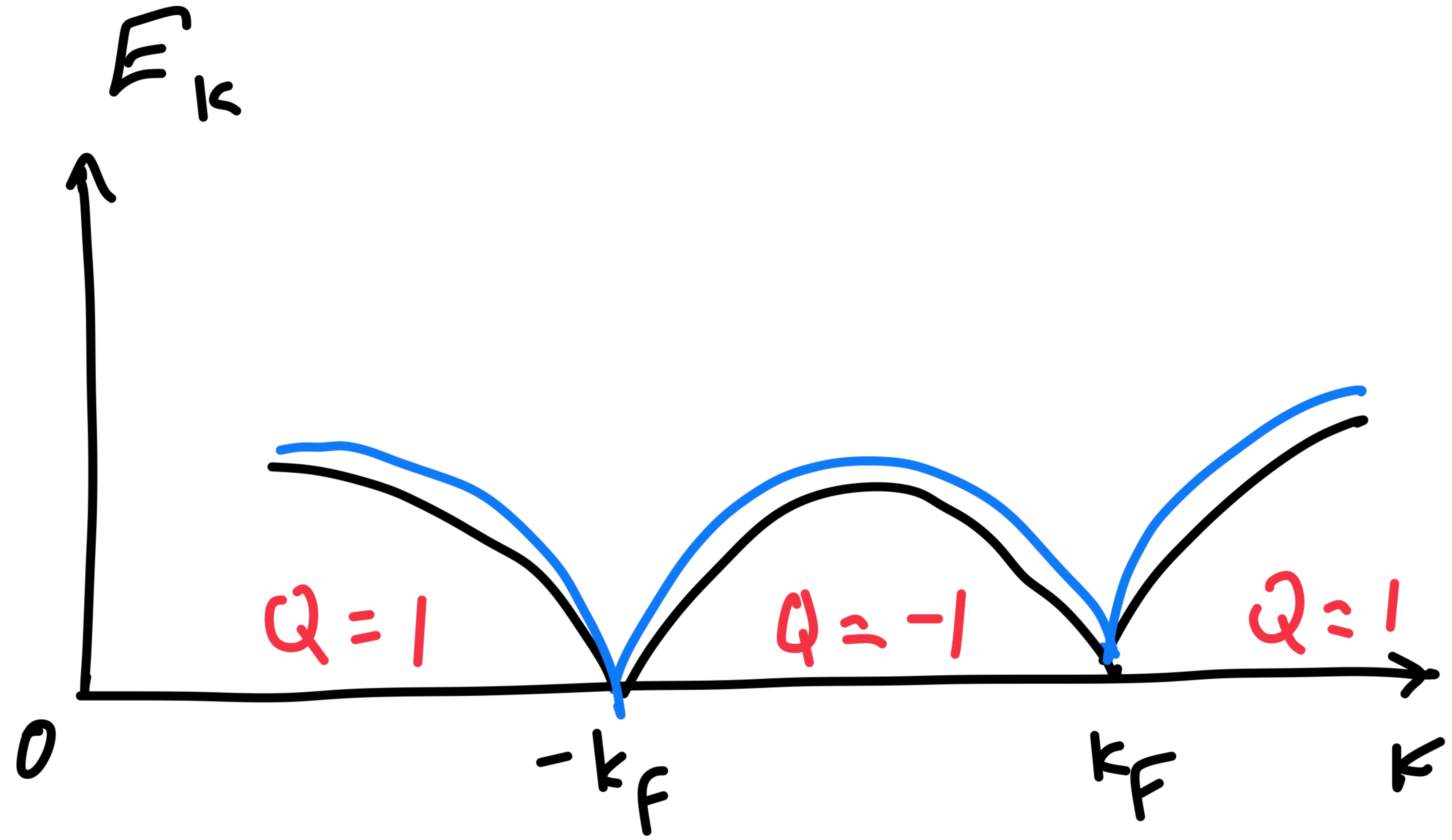
$U > 2t$

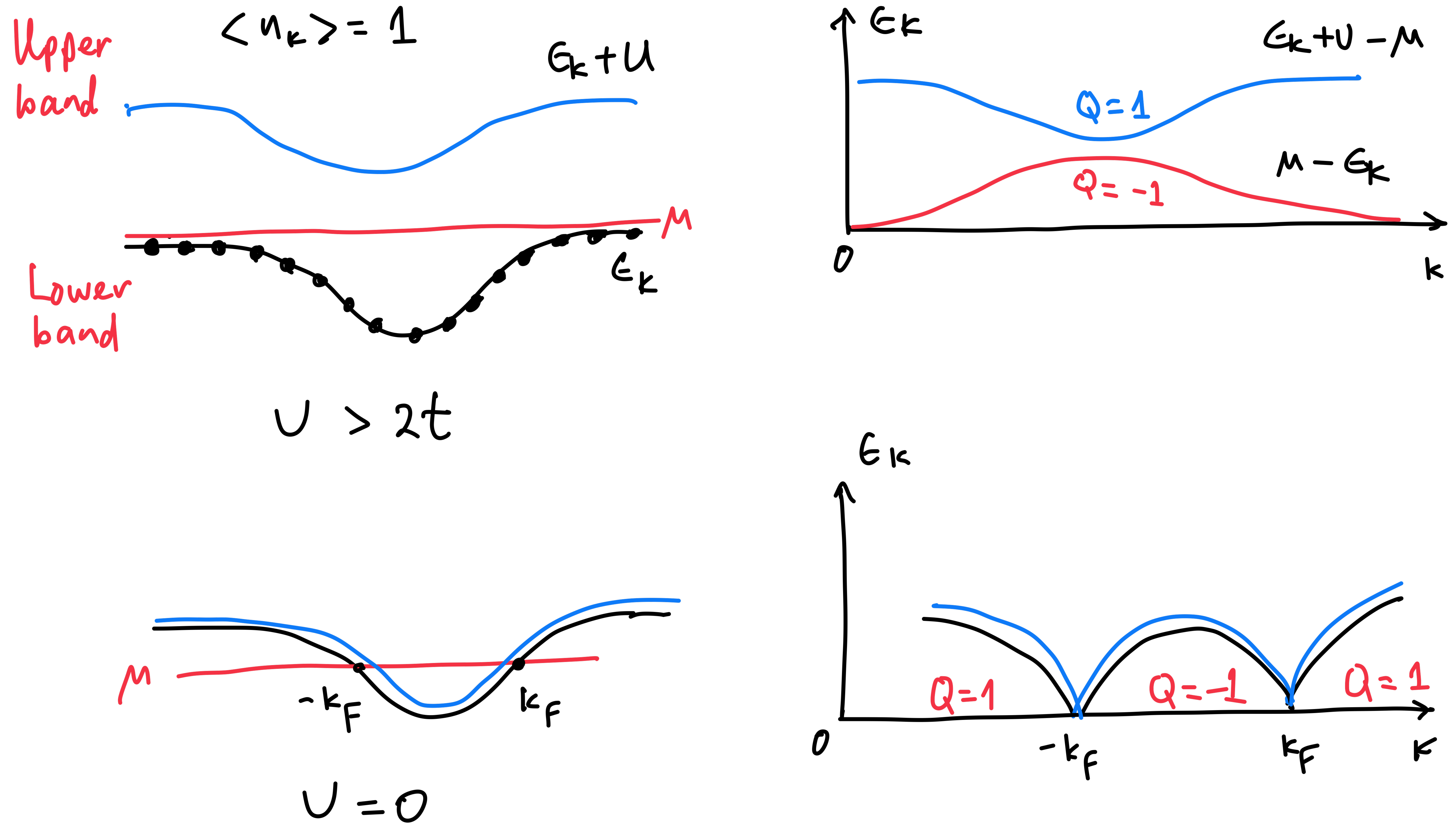


Gaped / Mott Insulator



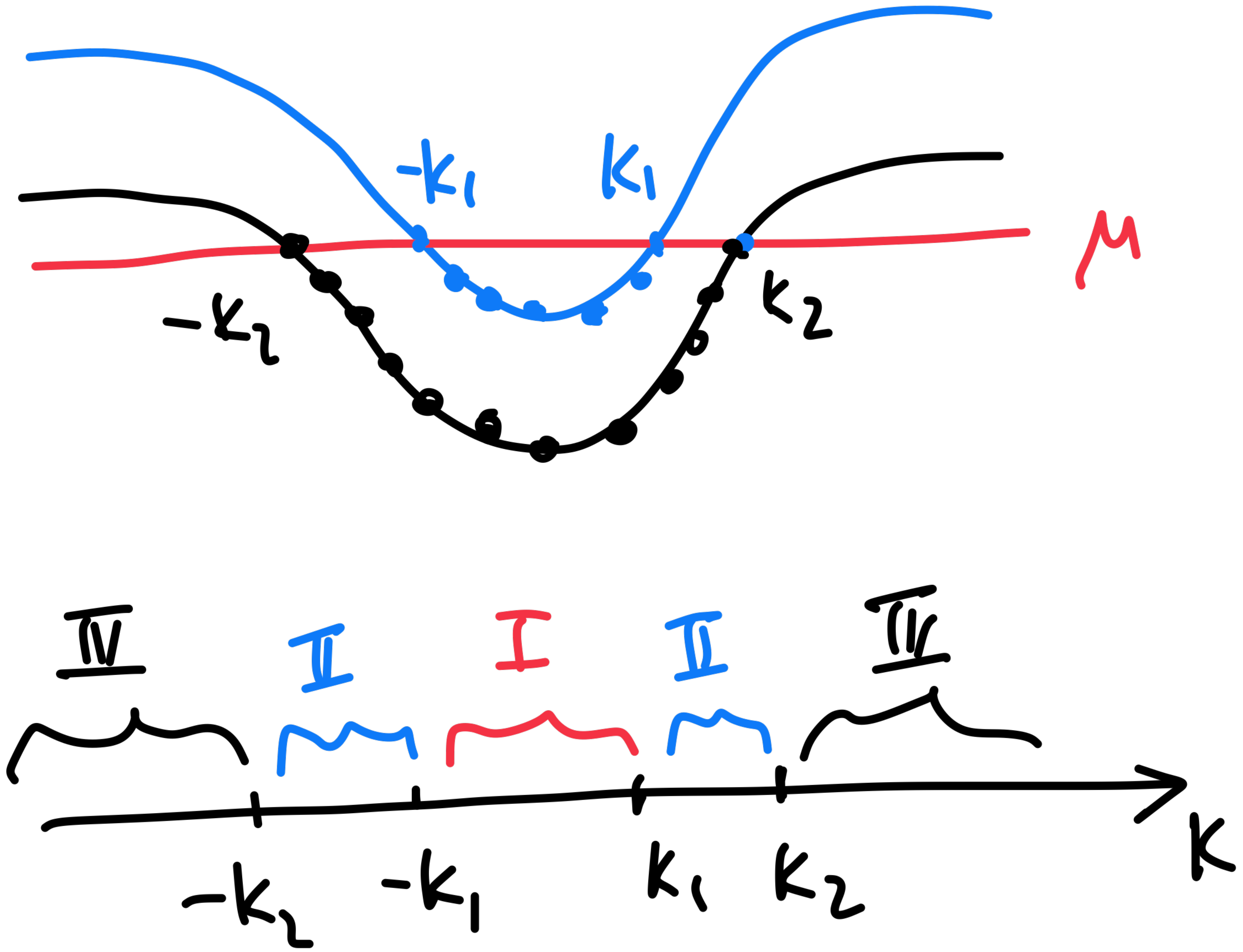
$U = 0$

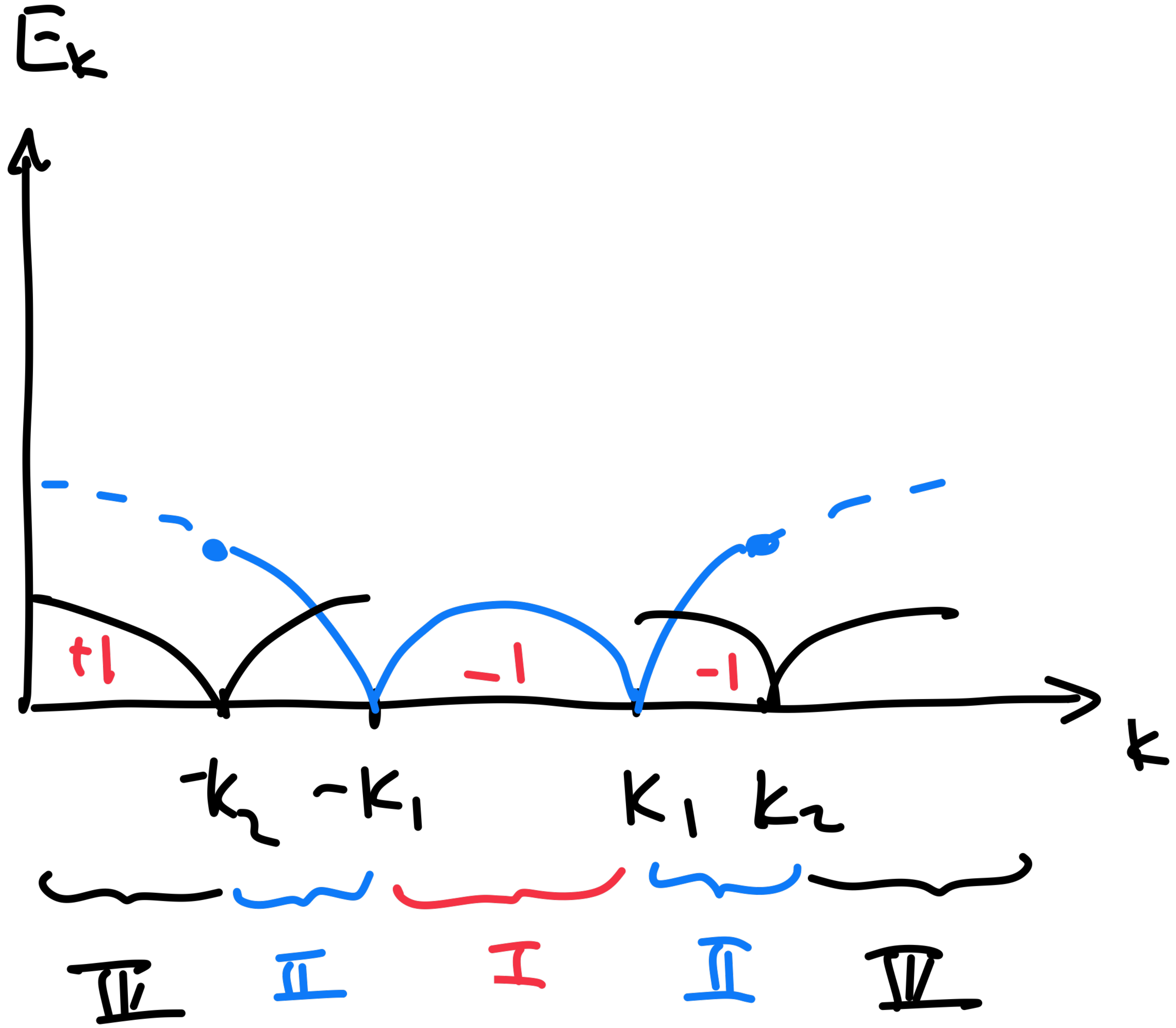




Upper band
$\langle n_k \rangle = 1$
$\epsilon_k + U$
μ
ϵ_k
Lower band
$U > 2t$
ϵ_k
$\epsilon_k + U - \mu$
$Q = 1$
$\mu - \epsilon_k$
$Q = -1$
0
k
μ
$-k_F$
k_F
$U = 0$
ϵ_k
$Q=1$
$Q=-1$
$Q=1$
0
$-k_F$
k_F
k

$U < 2t$





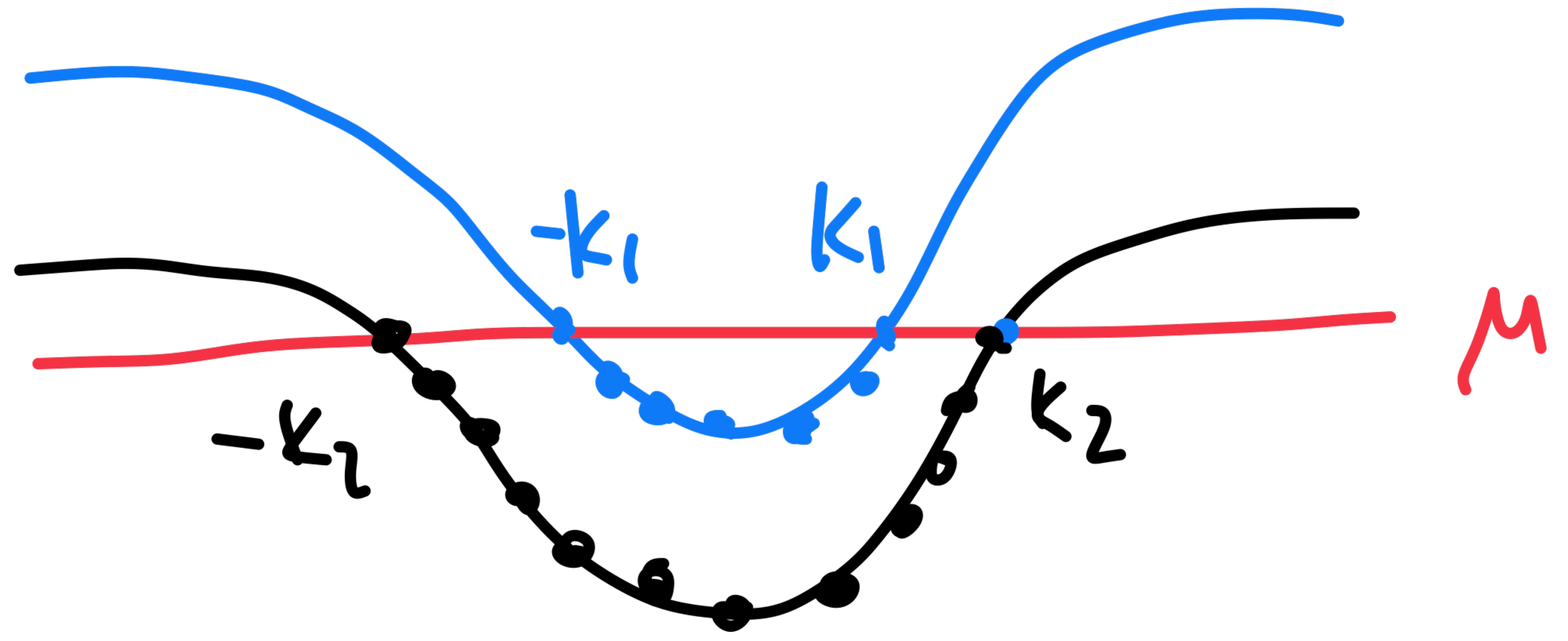
$Q = \pm 1, \quad \pm 1, \quad -1, \quad \pm 1 \quad +1$

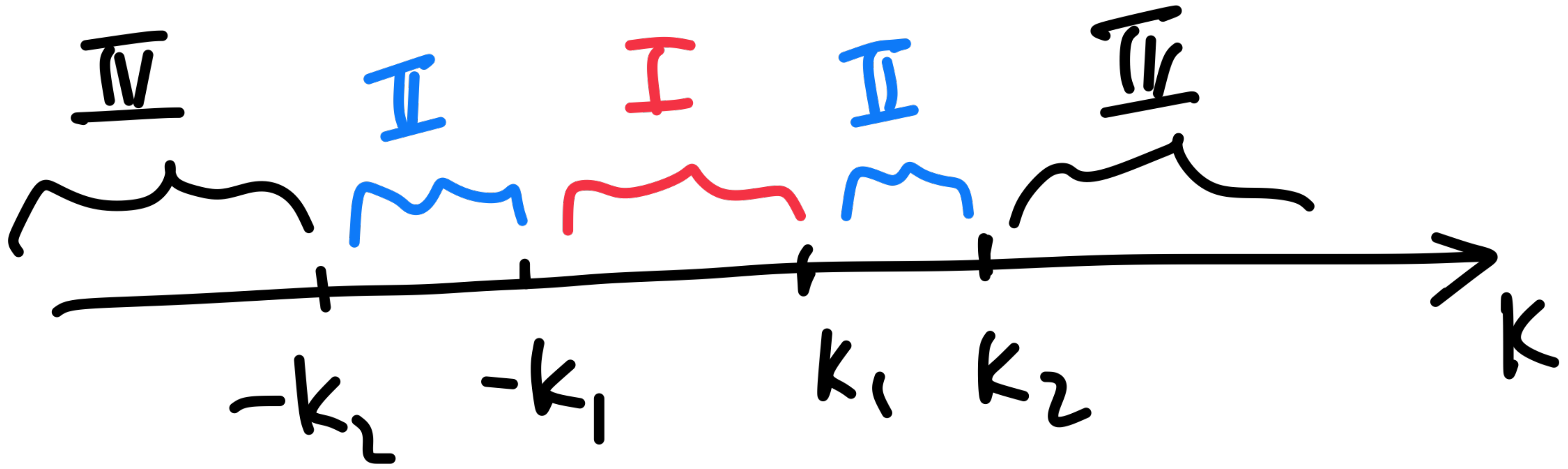
IV : $n_k = 0$

II : $n_k = 1$

I : $n_k = 2$

$U < 2t$





III: $n_k = 0$

II: $n_k = 1$

I: $n_k = 2$

$$\sum_k n_k \, \theta(k_2 - k) = N_L(k_2)$$

$$\sum_k n_k \, \theta(k_1 - k) = N_u(k_1)$$

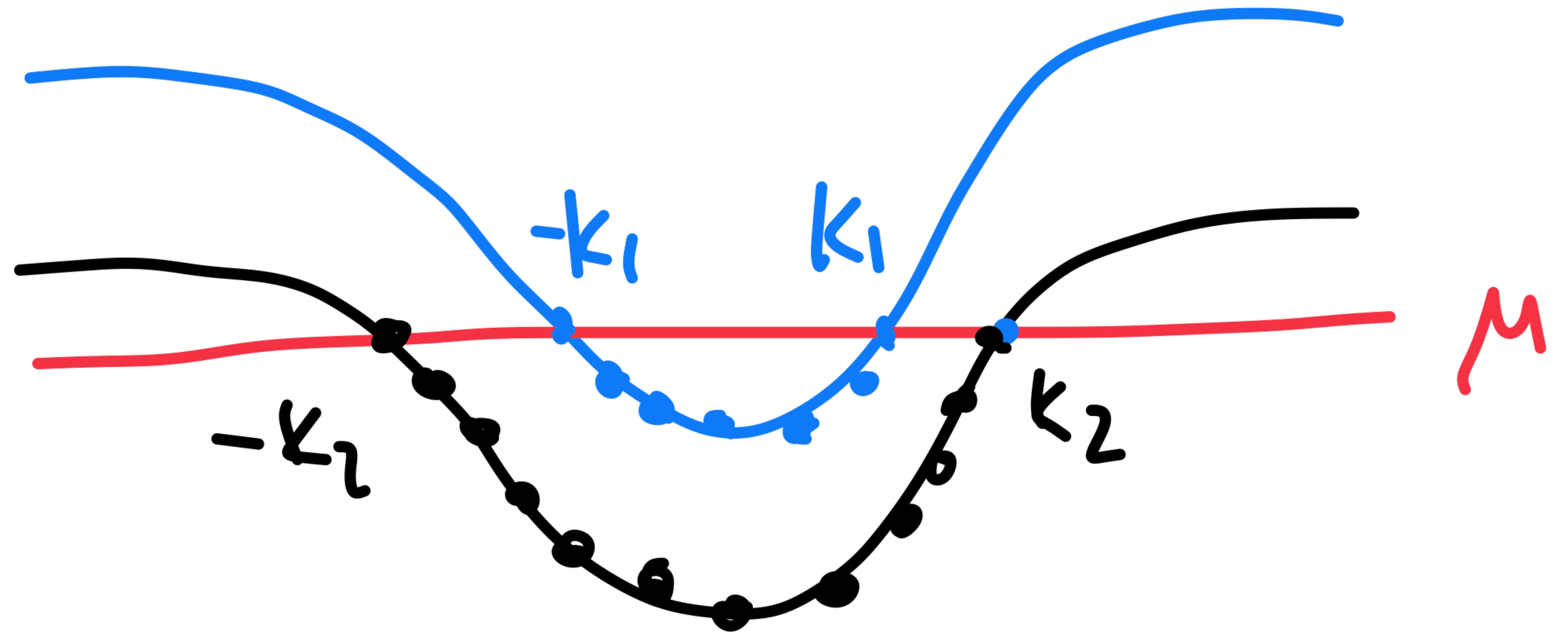
$$N_L(k_2) > \frac{N_L(k_2) + N_u(k_1)}{2} > N_u(k_1)$$

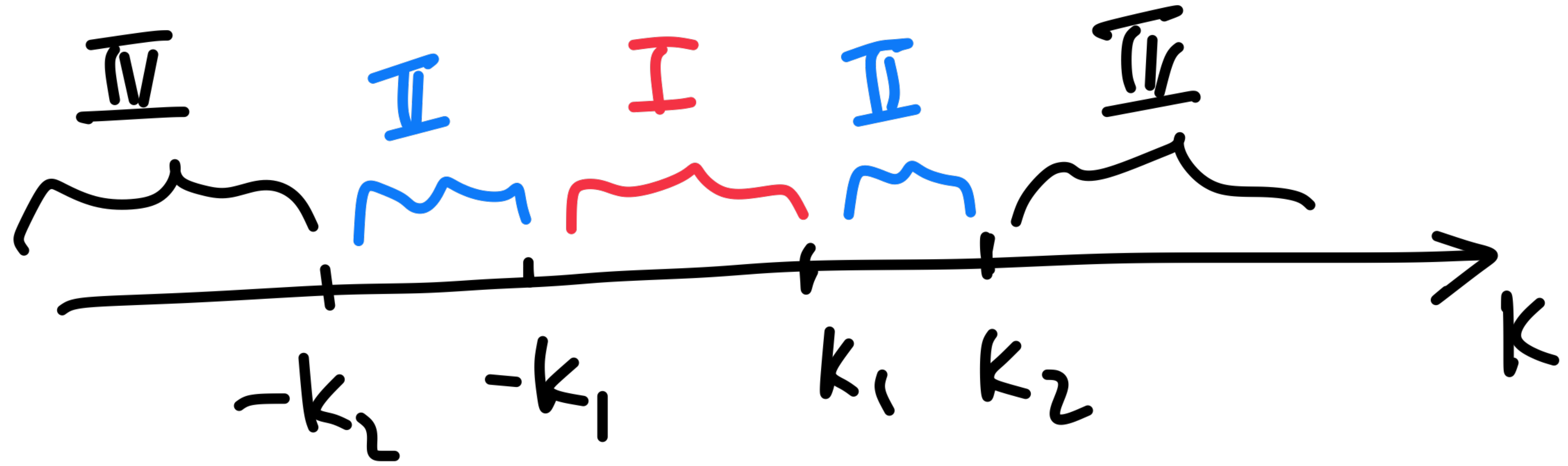
$$\frac{N_L(k_2) + N_u(k_1)}{2} = N(k_F)$$

$$N_L(k_2) > N(k_F) > N_u(k_1)$$

$$k_2 > k_F > k_1$$

$U < 2t$





IV : $n_k = 0$

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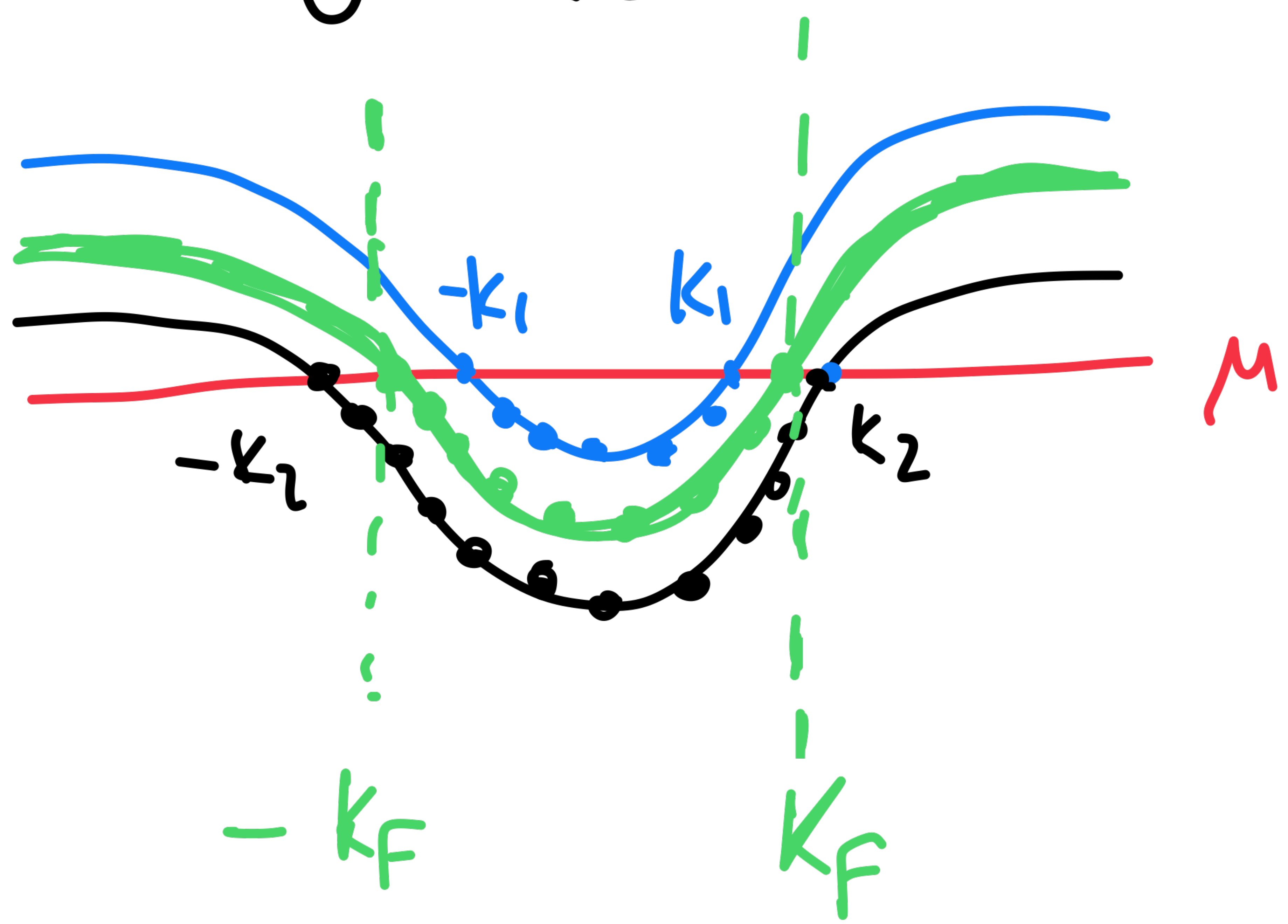
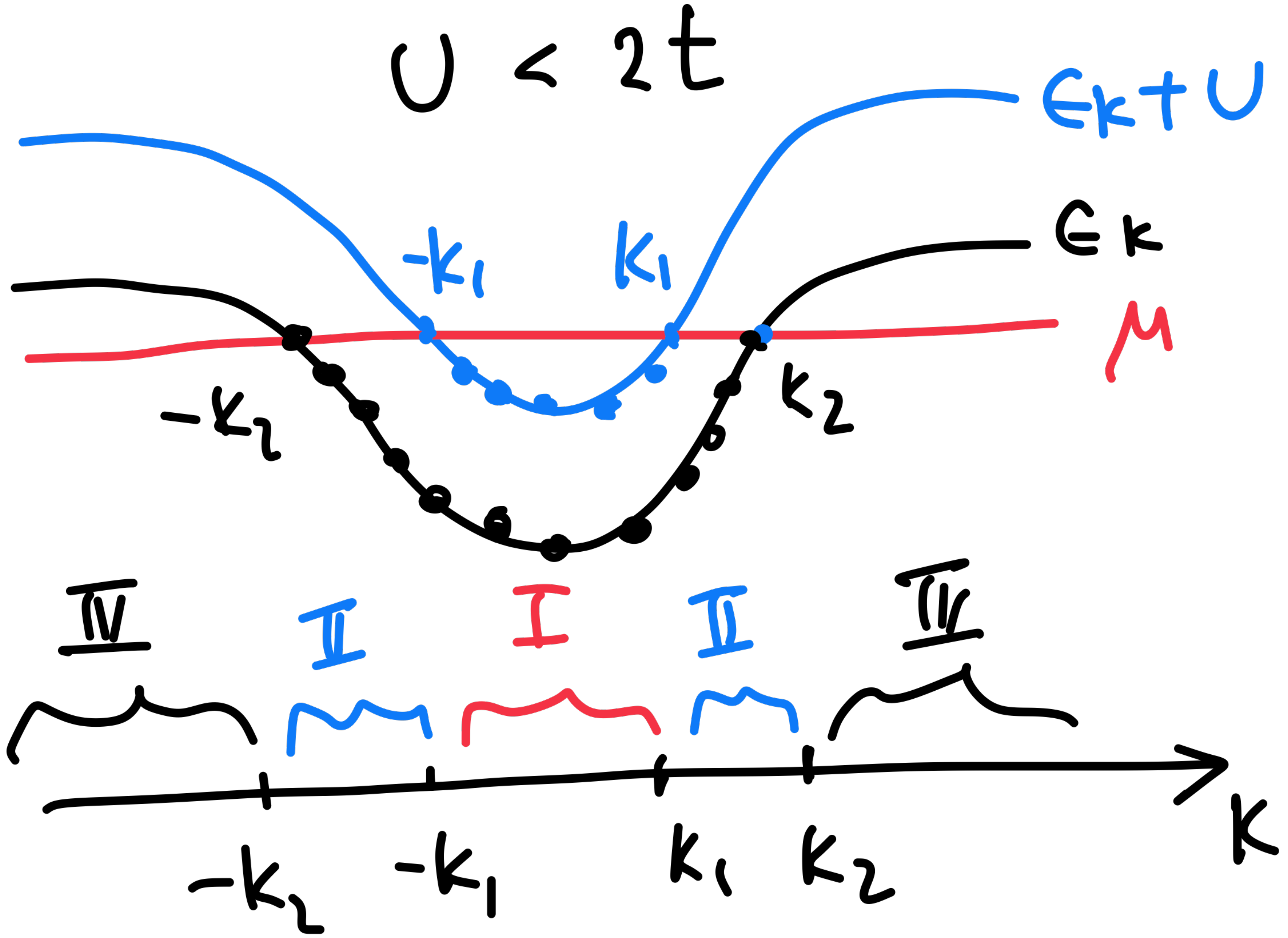
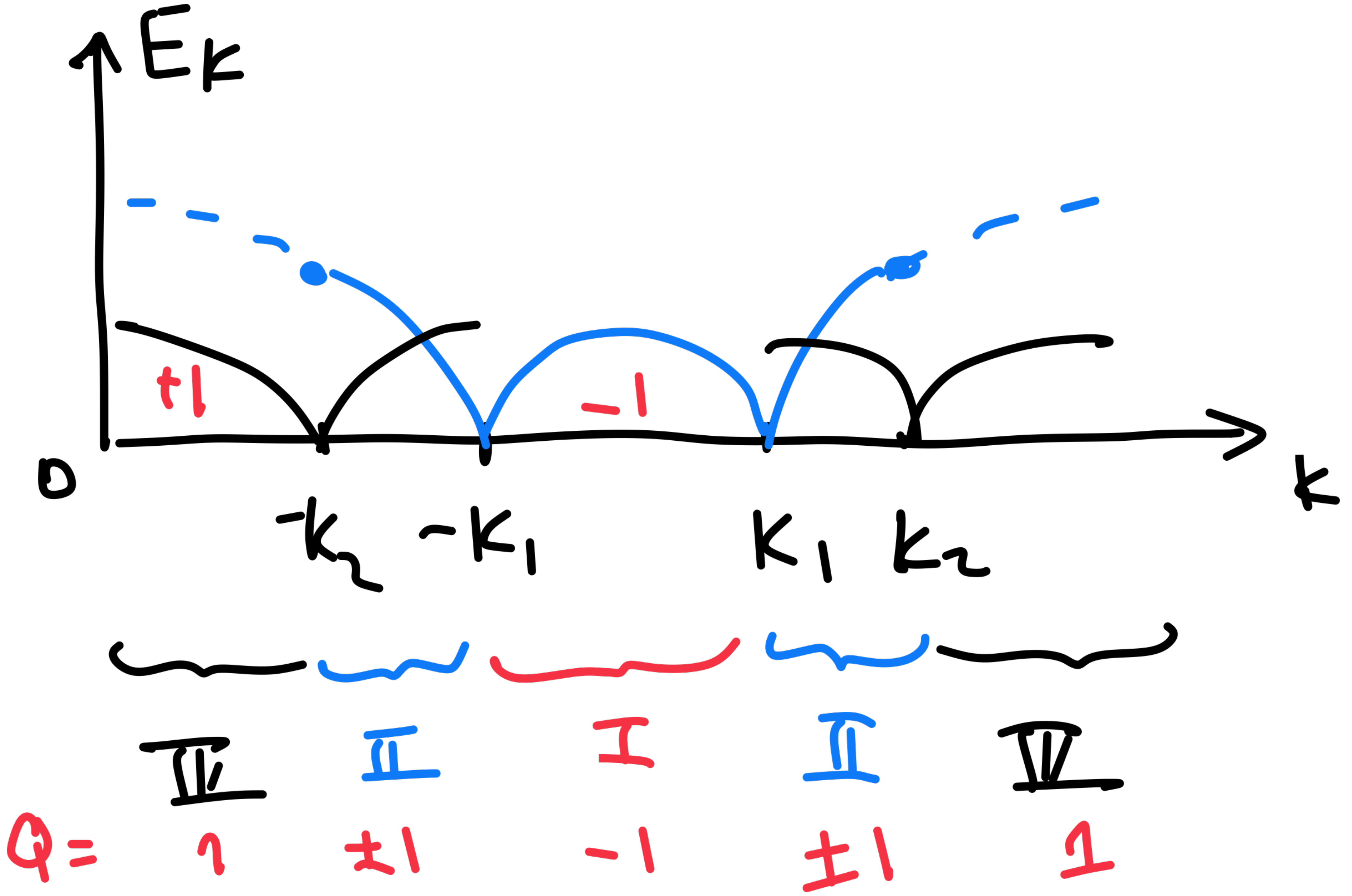
$U < 2t$



More formal def.

$G(\vec{k}, \omega=0) > 0$ for $k < k_F$





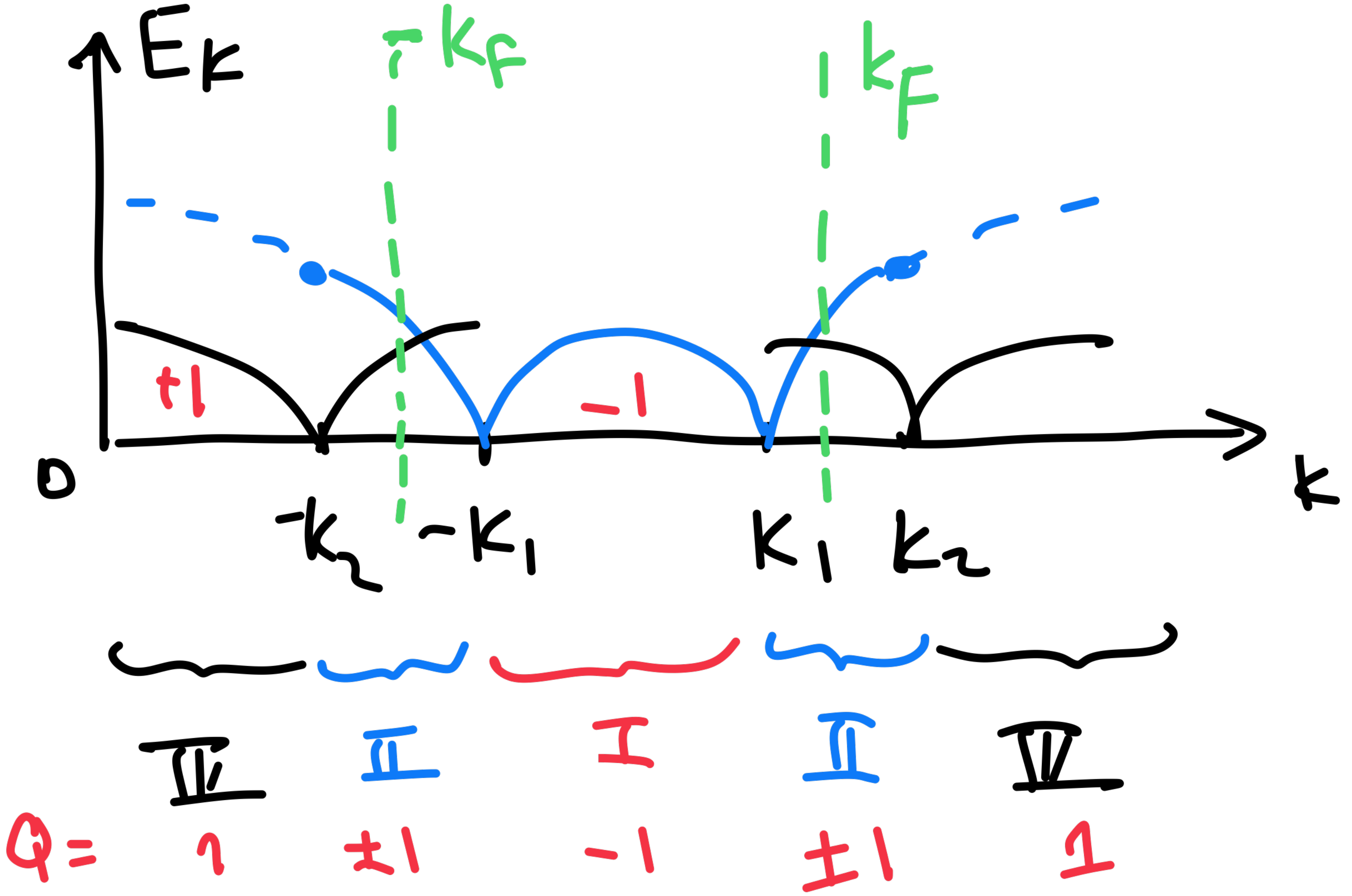
$$G_\sigma(k,\omega) = \frac{Z_p\,(Q=1)}{\omega - (\epsilon_k + U - \mu) + i\delta} + \frac{Z_h\,(Q=-1)}{\omega - (\epsilon_k - \mu) - i\delta} \quad \text{for } k \in \text{II}$$

$Z_p + Z_h = 1$ and $Z_p = Z_h = \frac{1}{2}$ ← fractionalization

Ref: $G_\sigma(k,\omega) = \dfrac{Z}{\omega - \xi_k + i\eta_k} + \text{Reg.}$

$$n(k=k_F+0^-)-n(k=k_F+0^+)$$
$$= Z_p(k=k_F+0^+) - Z_p(k=k_F+0^-)$$
$$= \frac{1}{2} - \frac{1}{2} = 0$$

NFL.



$$G_\sigma(k,\omega) = \frac{Z_p(Q=1)}{\omega-(\epsilon_k+U-\mu)+i\delta} + \frac{Z_h(Q=-1)}{\omega-(\epsilon_k-\mu)-i\delta} \quad \text{for } k \in \mathrm{II}$$

$Z_p + Z_h = 1$ and $Z_p = Z_h = \frac{1}{2}$ ← fractionalization