Phys529B: Topics of Quantum Theory

Lecture 7: interacting fermions and Non-Fermi liquid

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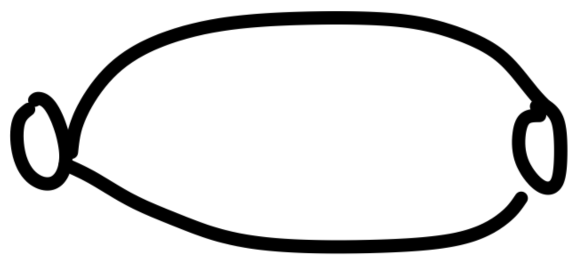
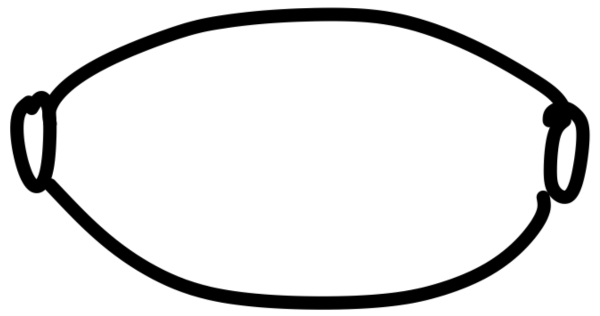
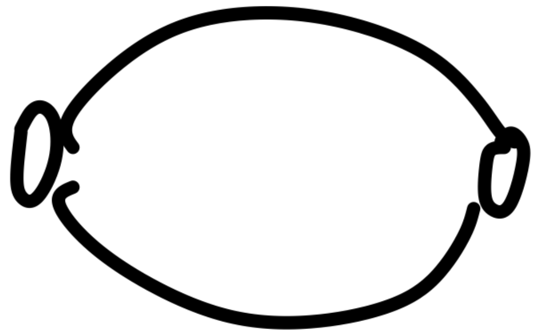
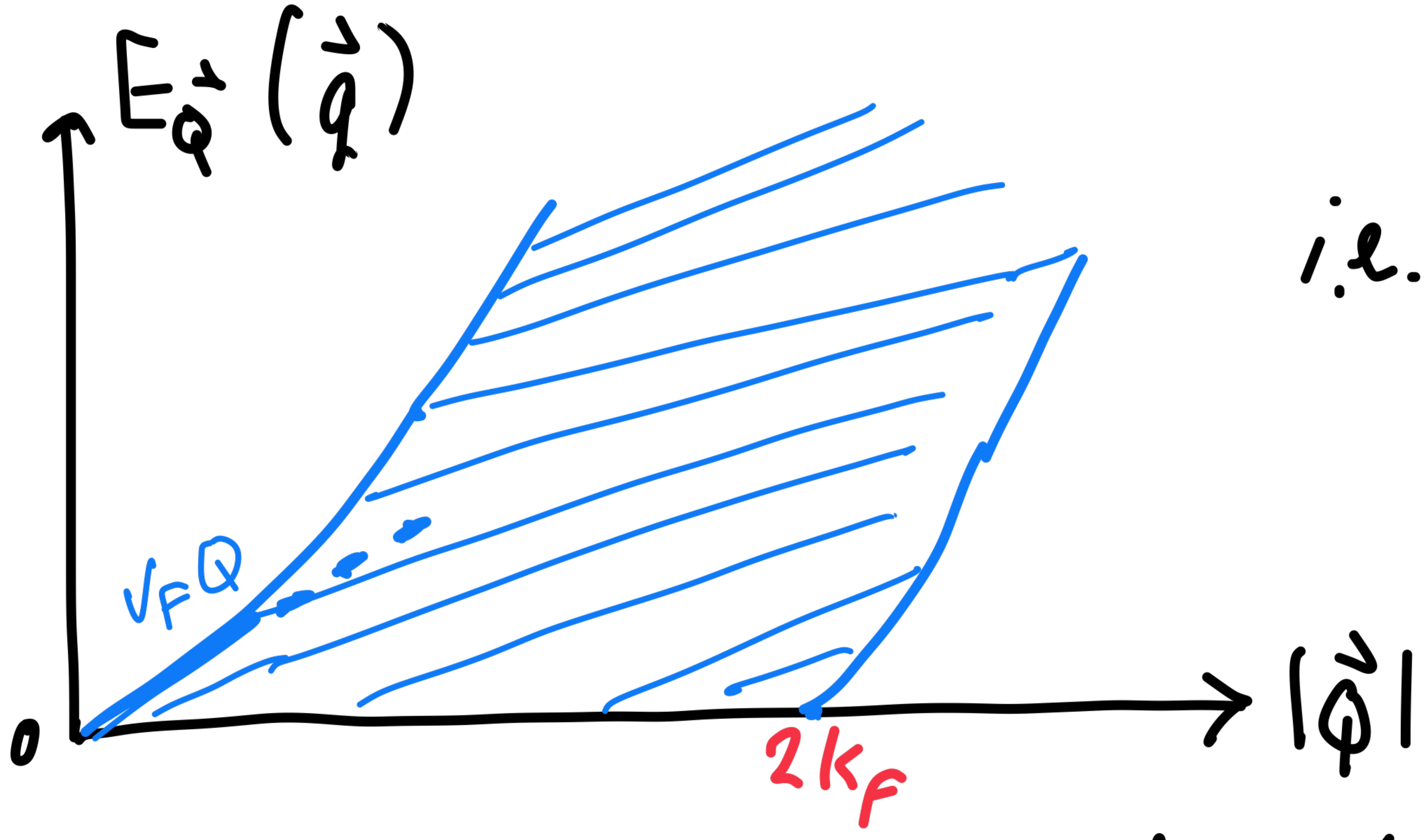
F.L. $\quad G_B =$  / $\Big(1 - g$  $\Big)$

$1 - g$  $= 1 - g\, A(k_F) \left\{ 1 + \frac{\Omega}{v_F Q} \ln \frac{\Omega + v_F Q}{\Omega - v_F Q} \right\} = 0$

Real if $\Omega > v_F Q$

$g \to 0, \quad \Omega - v_F Q \;\hat{=}\; 2 v_F Q\, e^{-\frac{1}{g A(k_F)}}$

"F. G."

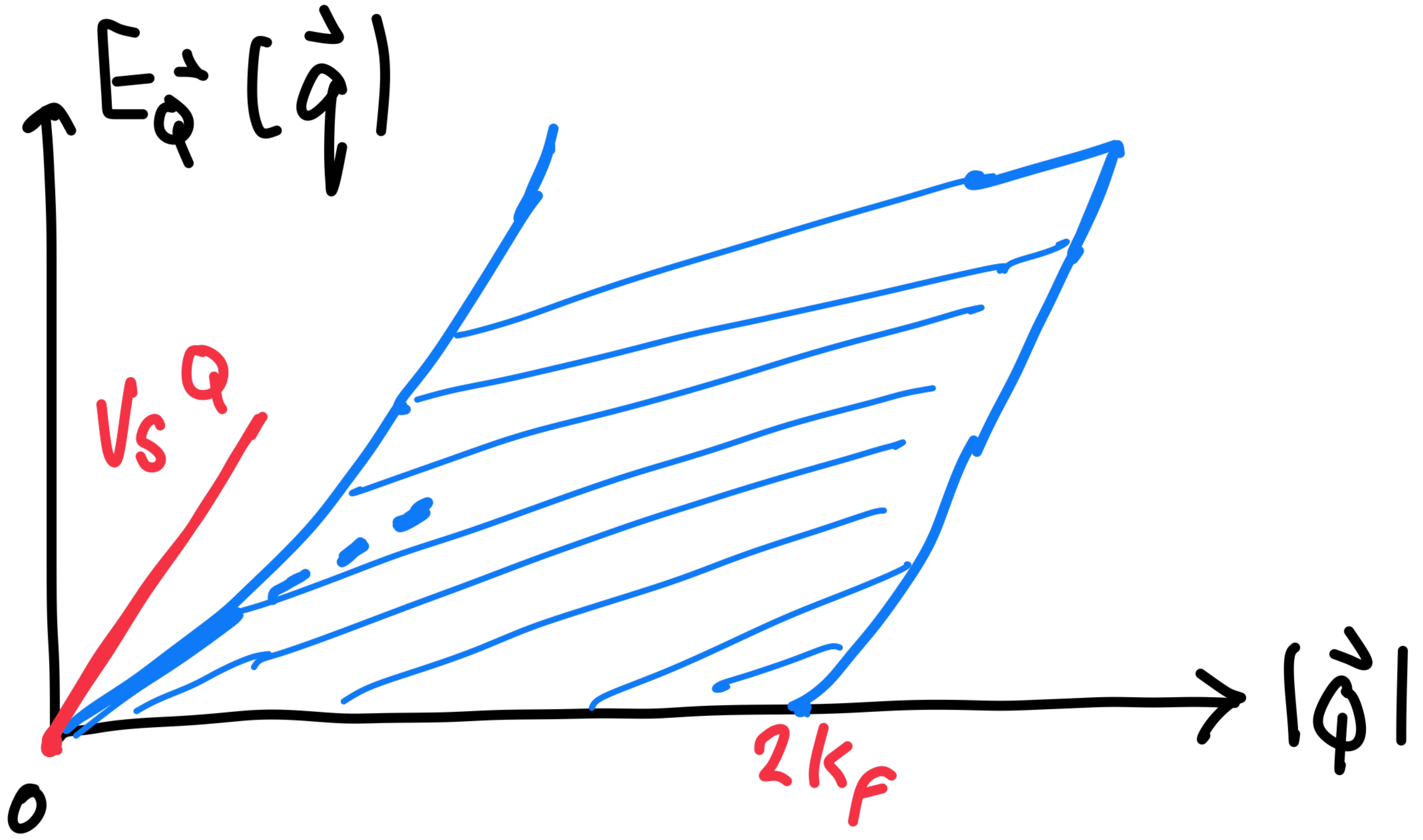


Continuum for bosonic states

i.e. No bosonic "particle"s in F. G.

In F. G., $G_B(\vec{Q}, \Omega)$ doesn't have isolated poles.

"F. L"



In F. L., $G_B(\vec{Q}, \Omega)$ has simple isolated poles, i.e. emergent bosonic fields.

("zero sound")

$v_s = v_s(v_F, \tilde{g}) > v_F$

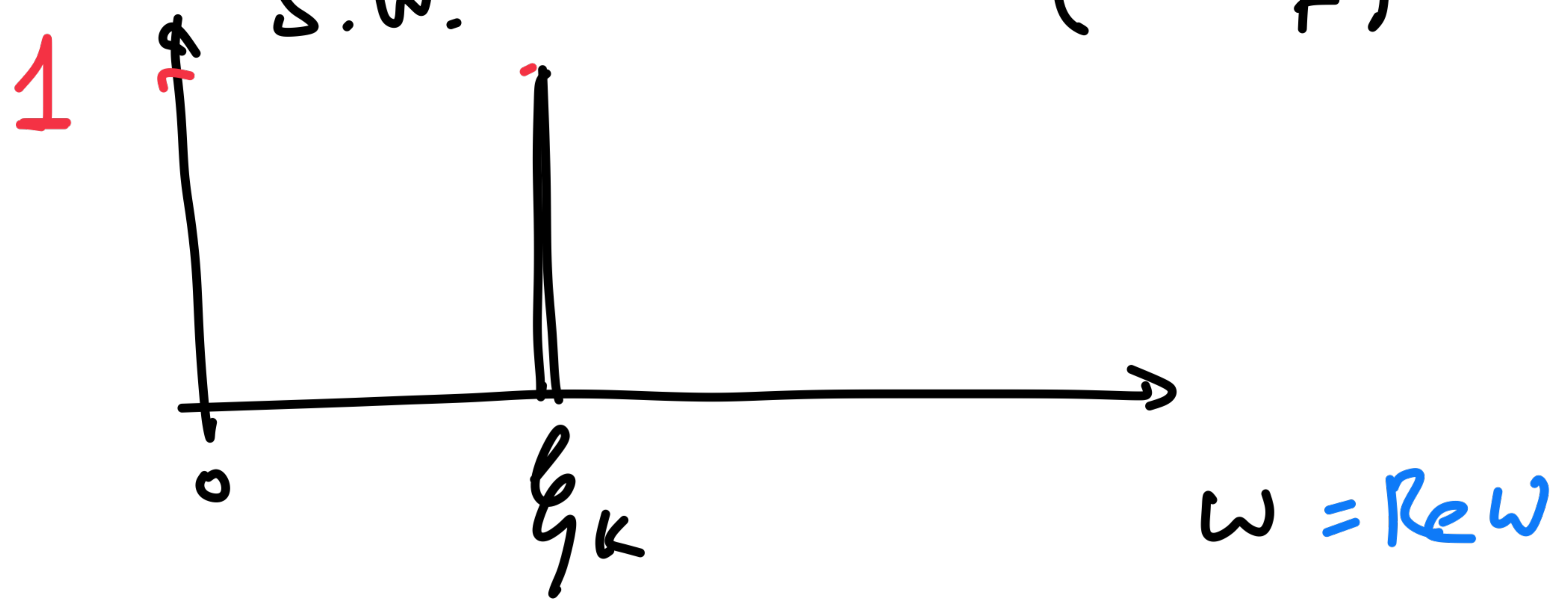
- Some formal aspects of interacting fermions (AGD, chapter 1 and 4)
- 1) sum rules of the spectral weight (imaginary part of the retarded G)
- 2) Luttinger theorem (Luttinger, Phys. rev. 119, 1153b(1960).)
- 3) Analytical properties of Green’s functions

$$-\frac{1}{\pi}\,\mathrm{Im}\,G_R(k,\omega) = \text{Spectral weight (S.W.)}$$

F.G.

$$S.W. = 1\cdot\delta(\omega-\xi_k)$$

$(k > k_F)$

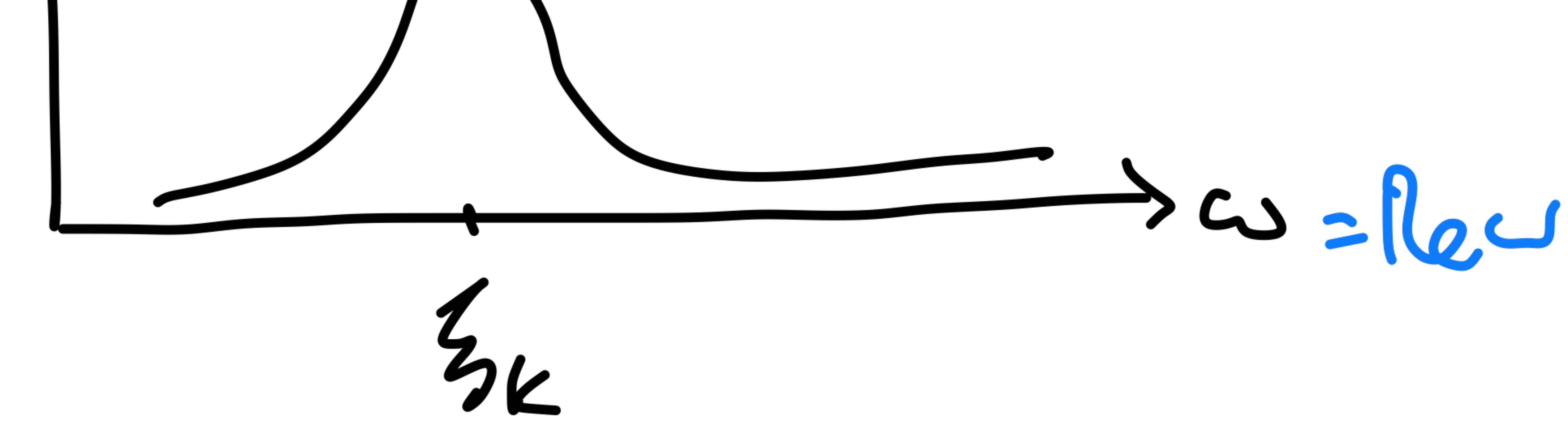


F.L.

$$S.W. = \frac{1}{\pi}\,\frac{\gamma_k\, Z}{(\omega-\xi_k)^2+\gamma_k^2} + \dots$$

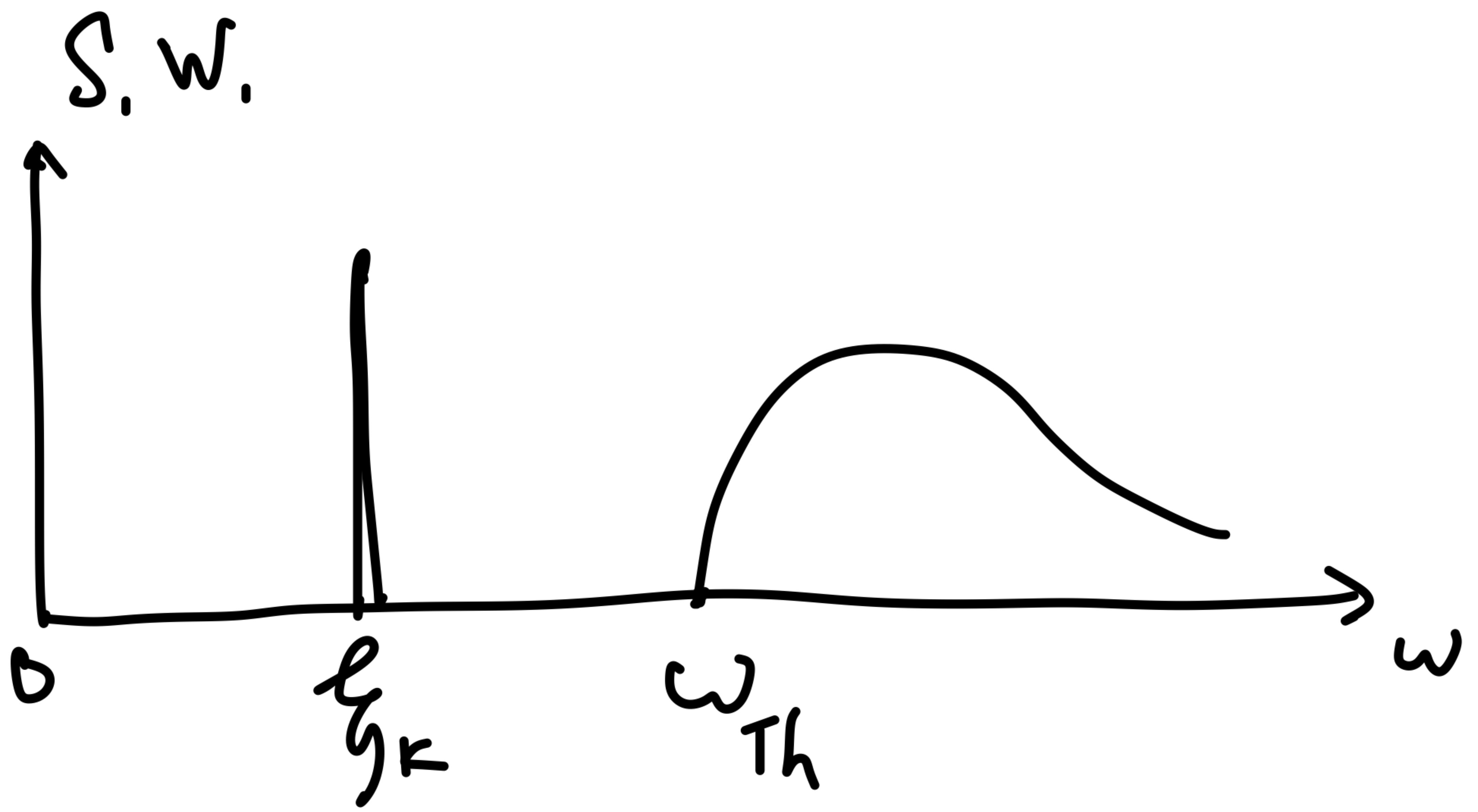
$Z < 1$

$(k > k_F)$



General interacting fermions

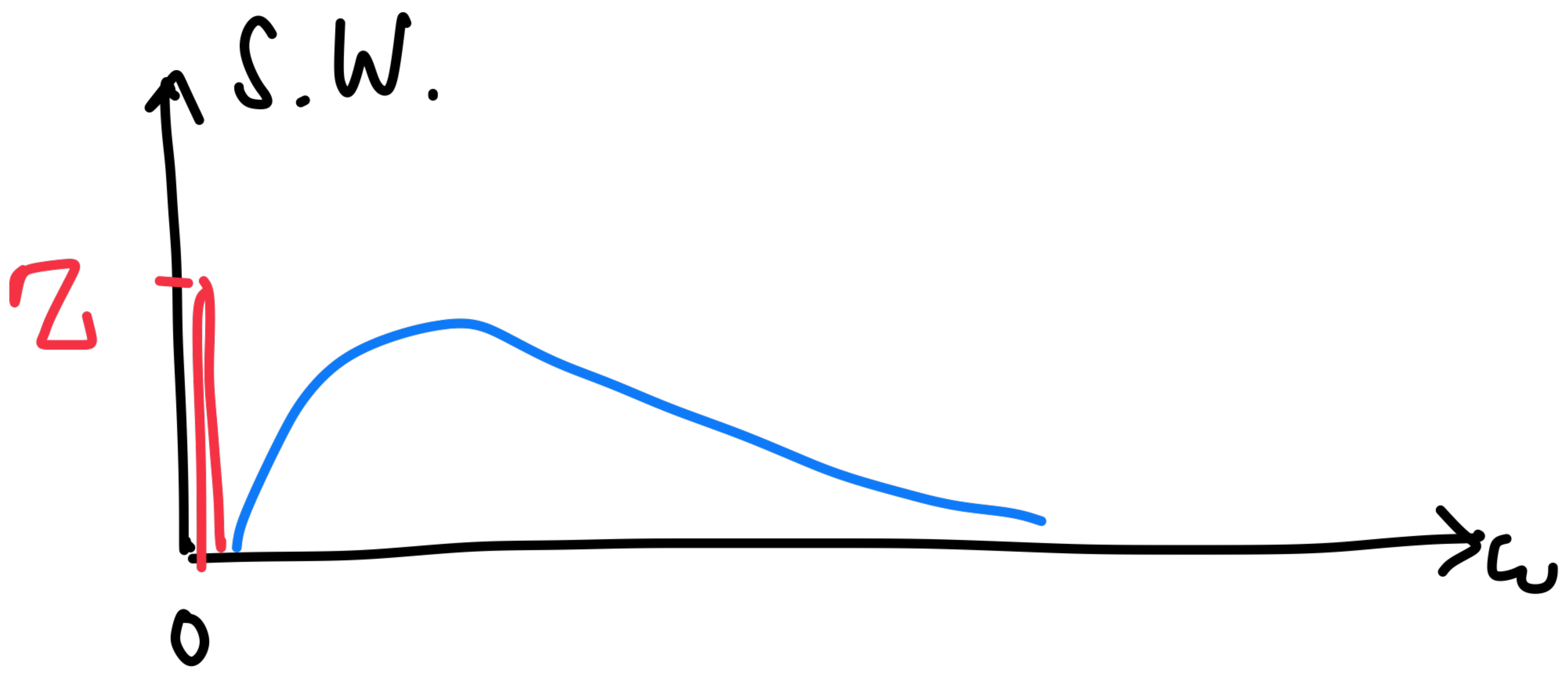
$$S.W. = Z\,\delta(\omega - \xi_k) + \text{Reg.}$$



F.L. $k \to k_F$

$$S.W. = +\frac{1}{\pi}\frac{\gamma_k Z}{(\omega - \xi_k)^2 + \gamma_k} + \ldots$$

$$\xrightarrow{\gamma_k \to 0} Z\,\delta(\omega - \xi_k) + \ldots$$



- Sum rule: $$-\frac{1}{\pi}\int d\omega \mathscr{I}m G_R(\mathbf{k},\omega) = 1$$

Sum Rules for interacting fermions (Lehmann Rep)

$$G(\vec{k},t) = \langle g.s.| -i T \psi_k(t)\, \psi_k^{+}(0) |g.s.\rangle$$

$$G(\vec{k},\omega) = \int_0^\infty dE \left\{ \frac{A(\vec{k},E)}{\omega - E + i\delta} + \frac{B(-\vec{k},E)}{\omega + E - i\delta} \right\}$$

$$A(\vec{k},E) = \sum_n \langle g.s.| \psi_k(0) |n\rangle \langle n| \psi_k^{+}(0) |g.s.\rangle\, \delta(E-\mathcal{E}_n)$$

$$B(\vec{k},E) = \sum_n \langle g.s.| \psi_k^{+}(0) |n\rangle \langle n| \psi_k(0) |g.s.\rangle\, \delta(E-\mathcal{E}_n)$$

$$\mathrm{Re}\, G(\vec{k},\omega) = P\int_0^{\infty} dE \left\{ \frac{A(\vec{k},E)}{\omega - E} + \frac{B(-\vec{k},E)}{\omega + E} \right\}$$

$$\mathrm{Im}\, G(\vec{k},\omega) = -\pi A(\vec{k},\omega)\,\theta(\omega) + \pi B(-\vec{k},-\omega)\,\theta(-\omega)$$

$$G_R = G(\omega)\,\theta(\omega) + G^*(\omega)\,\theta(-\omega), \qquad G_A = G_R^*$$

$$\mathrm{Im}\, G_R = -\pi\left(A(\vec{k},\omega)\,\theta(\omega) + B(-\vec{k},-\omega)\,\theta(-\omega)\right)$$

$$-\frac{1}{\pi}\int_{-\infty}^{+\infty} \mathrm{Im}\, G_R\, d\omega = \int_0^{\infty} A(\vec{k},\omega)\, d\omega + \int_0^{\infty} B(-\vec{k},\omega)\, d\omega = 1$$

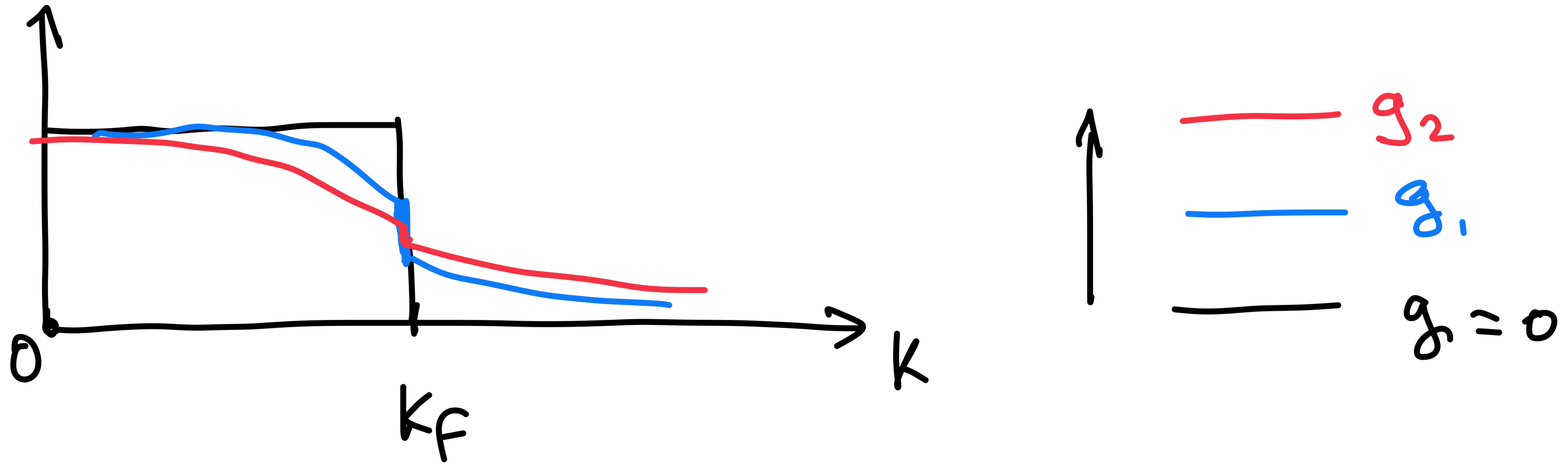
$$\mathrm{Re}\,G_R(\vec{k},\omega) = \mathcal{P}\int_0^{\infty} dE \left\{ \frac{A(\vec{k},E)}{\omega - E} + \frac{B(-\vec{k},E)}{\omega + E} \right\}$$

$$= -\frac{\mathcal{P}}{\pi}\int_{-\infty}^{\infty} dE \left\{ \frac{\mathrm{Im}\,G_R(\vec{k},E)\,\Theta(E)}{\omega - E} + \frac{\mathrm{Im}\,G_R(\vec{k},E)\,\Theta(-E)}{\omega - E} \right\}$$

$$= -\frac{\mathcal{P}}{\pi}\int_{-\infty}^{+\infty} dE \left\{ \frac{\mathrm{Im}\,G_R(\vec{k},E)}{\omega - E} \right\}$$

(<u>Satisfies the Kramer-Kronig Relation</u>)

$\longrightarrow$ $G_R(\vec{k},\omega)$ always analytical in the upper-half plane.



Luttinger theorem (phase space invariance)

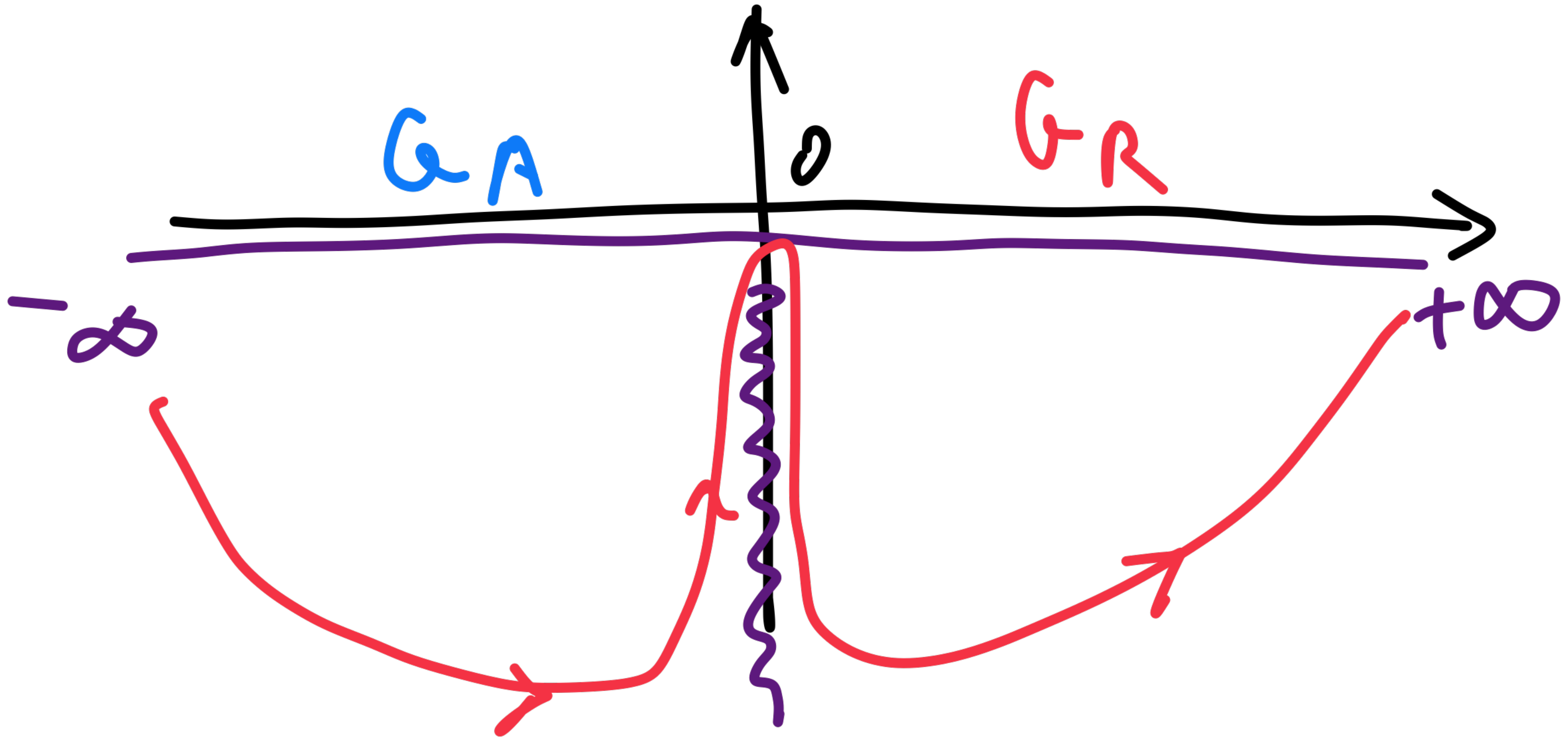
density

$$\int \frac{d^3p}{(2\pi)^3} = n_0 = \frac{1}{6\pi^2} k_F^3$$

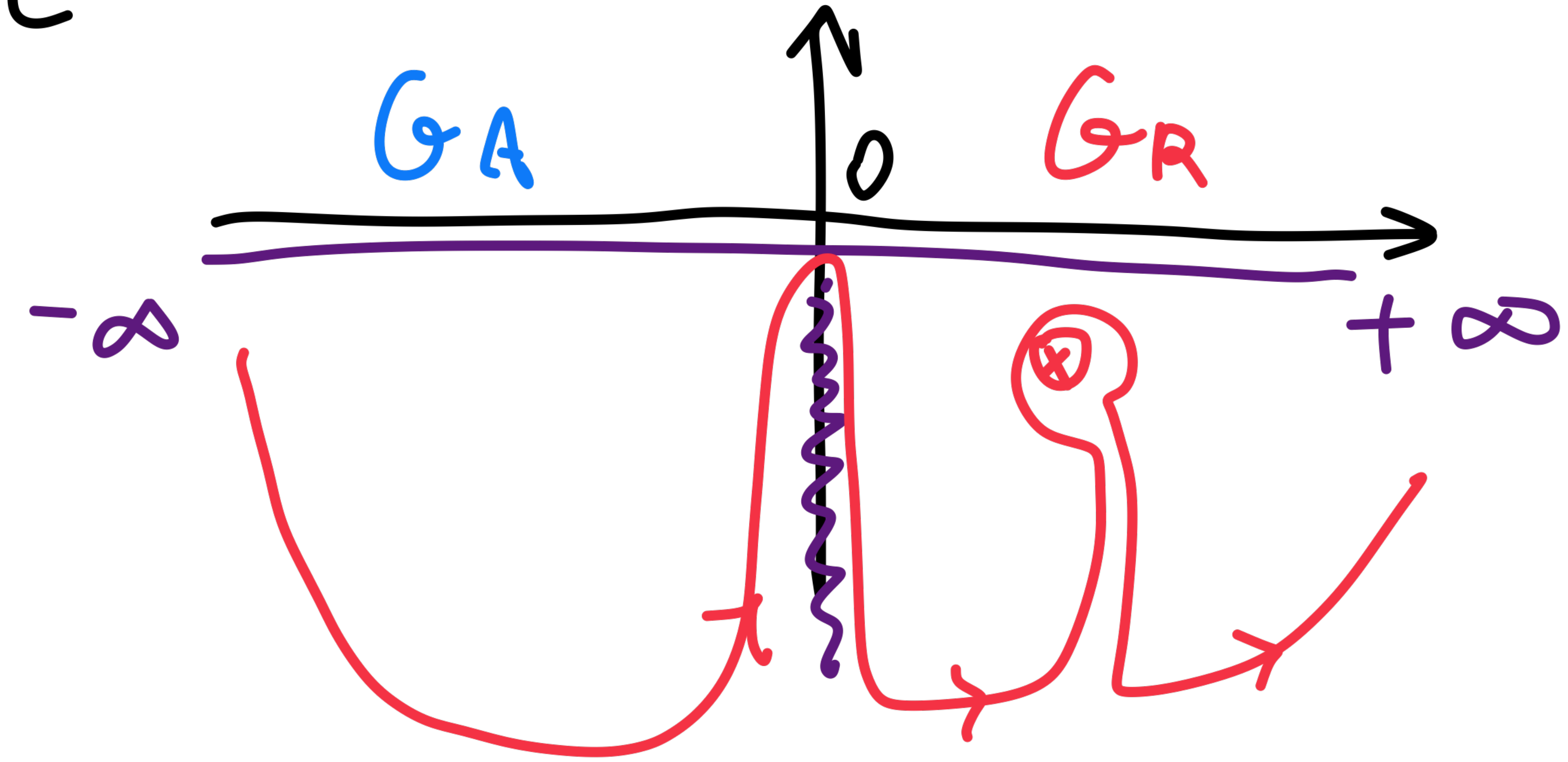
$$G(\vec{p}, \omega=0) > 0$$

$$1 - n_k = -i\int \frac{d\omega}{2\pi}\, G(k,\omega)\, e^{-i\omega 0^+}$$

$k < k_F$



$k > k_F$



$\text{Res}_R\, G(k, \omega>0)$ at poles with $\omega > 0$

$$\delta n = n(k = k_F + 0^-) - n(k = k_F + 0^+) = Z(k_F + 0^+, \omega>0) - Z(k_F + 0^-, \omega>0)$$

For F.L., $Z(k = k_F + 0^-) = 0$, $\quad \delta n = Z(k = k_F + 0^+)$.

- NFL phenomenology
- 1) Z vanishes at Fermi surfaces and no quasi-particles.
- 2) Z is finite across Fermi surface but Z on two sides of the Fermi surface cancels. I.e. $Z(k, \omega > 0)$ is continuous across a Fermi surface near k=k_F. For given momentum and orbital, quasi-particle and hole co-exist.