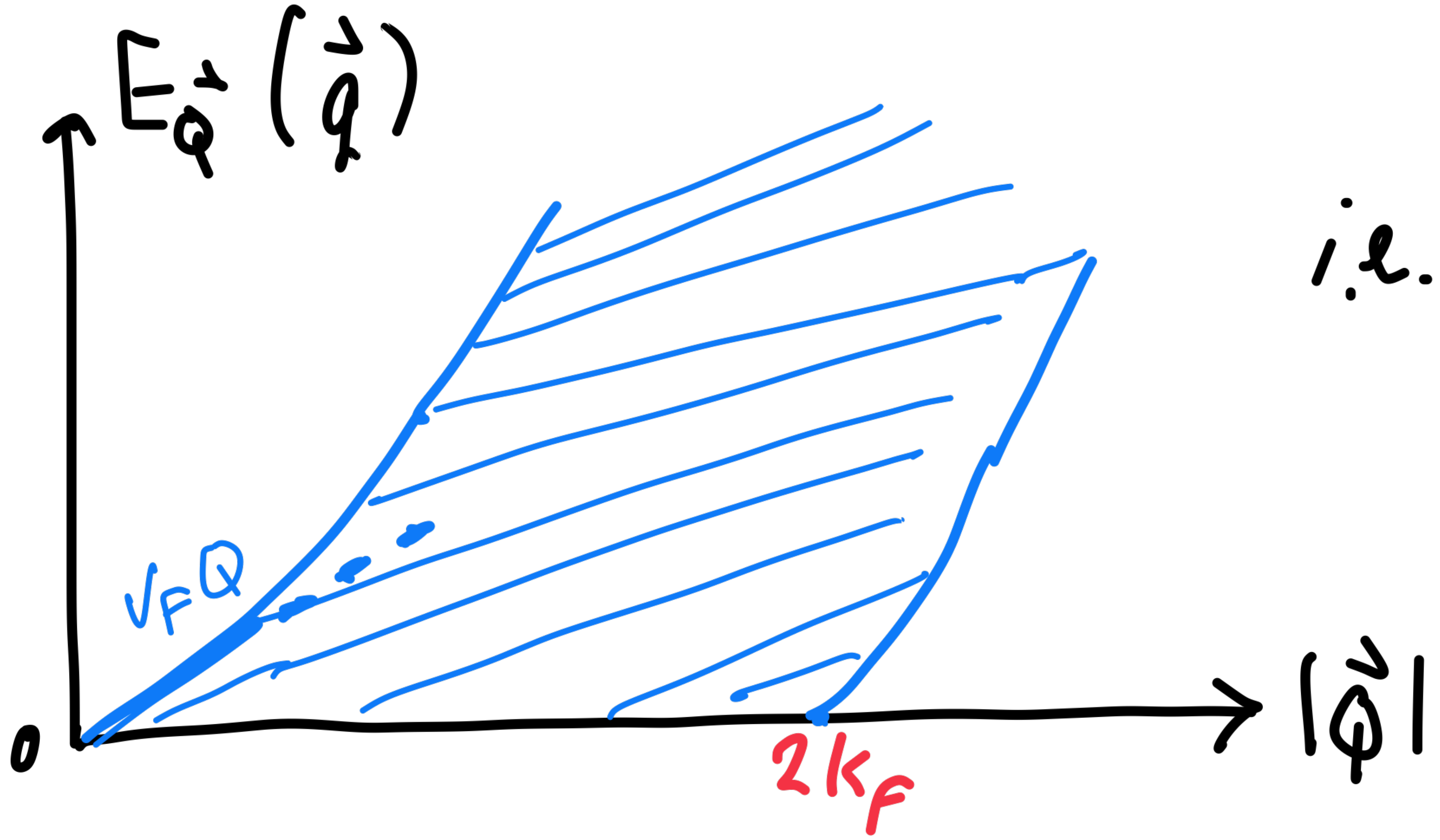
Phys529B: Topics of Quantum Theory

Lecture 6: basic introduction to interacting fermions

instructor: Fei Zhou

- Fermi Liquid theory (nice discussions in AGD, chapter 1 and 4)
- 1) there is a finite step in the occupation number at exactly K_F. This defines a Fermi surface. $n_{k_F-0} - n_{k_F+0} = Z$
- 2) quasi-particles are of finite life time and become well defined once near Fermi surface,I.e. in the low energy sector. $\frac{1}{\tau_k} = \gamma_k \ll |\xi_k|$
- 3) apart from mass renormalization, wave function renormalization Z occurs at Fermi surface.
- 4) there are low energy emergent bosonic particles.
- 5) for a fixed k, time ordered 'G" is not analytical in either lower or upper frequency planes. However, retarded (advanced) green functions are analytical in lower (upper) plane for any k.(a proof in Lehmann Rep.)

"F. G."

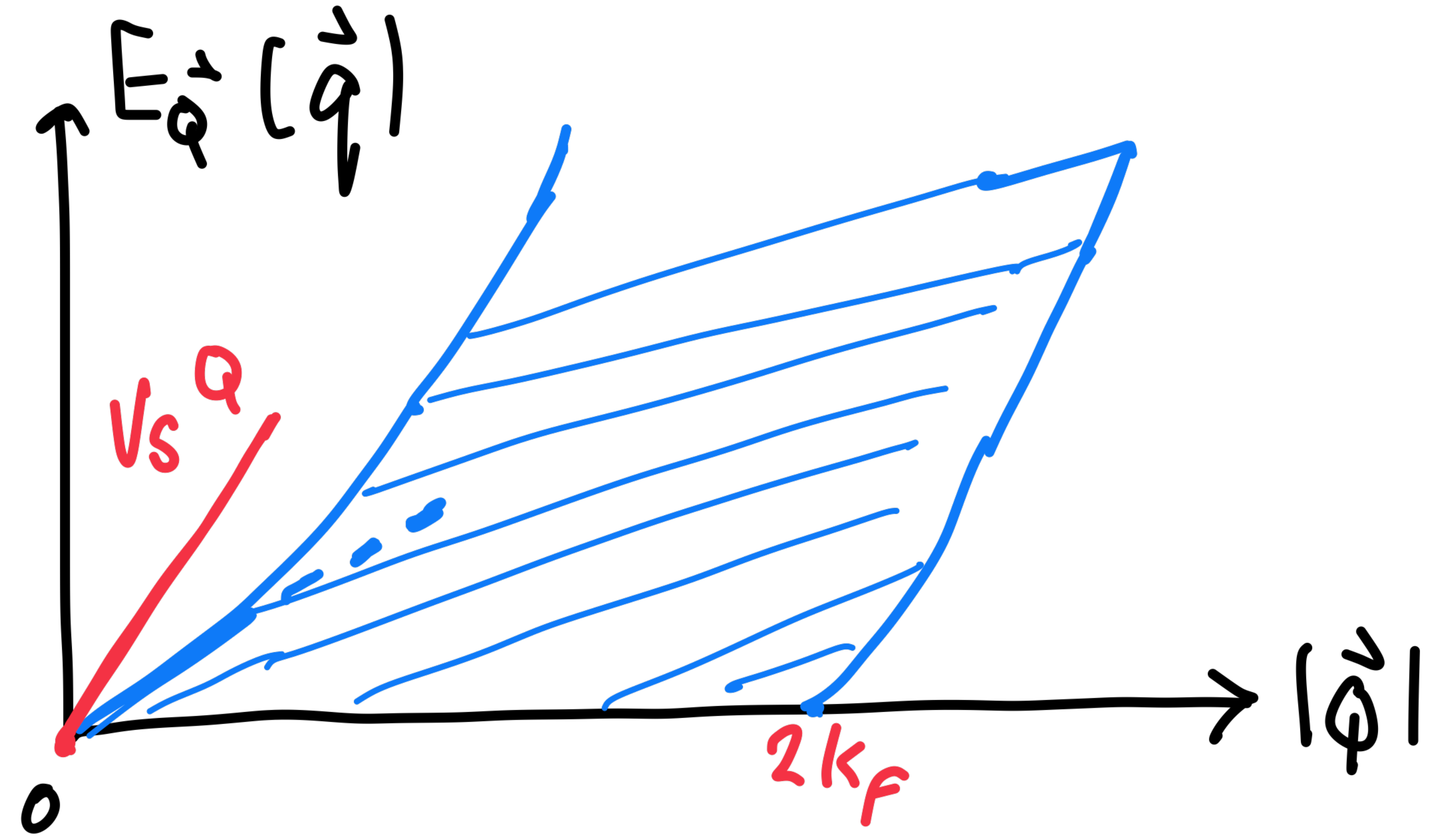


Continuum for bosonic states

i.e. No bosonic "particle"s in F. G.

In F.G., $G_B(\vec{Q}, \Omega)$ doesn't have isolated poles.

"F. L."

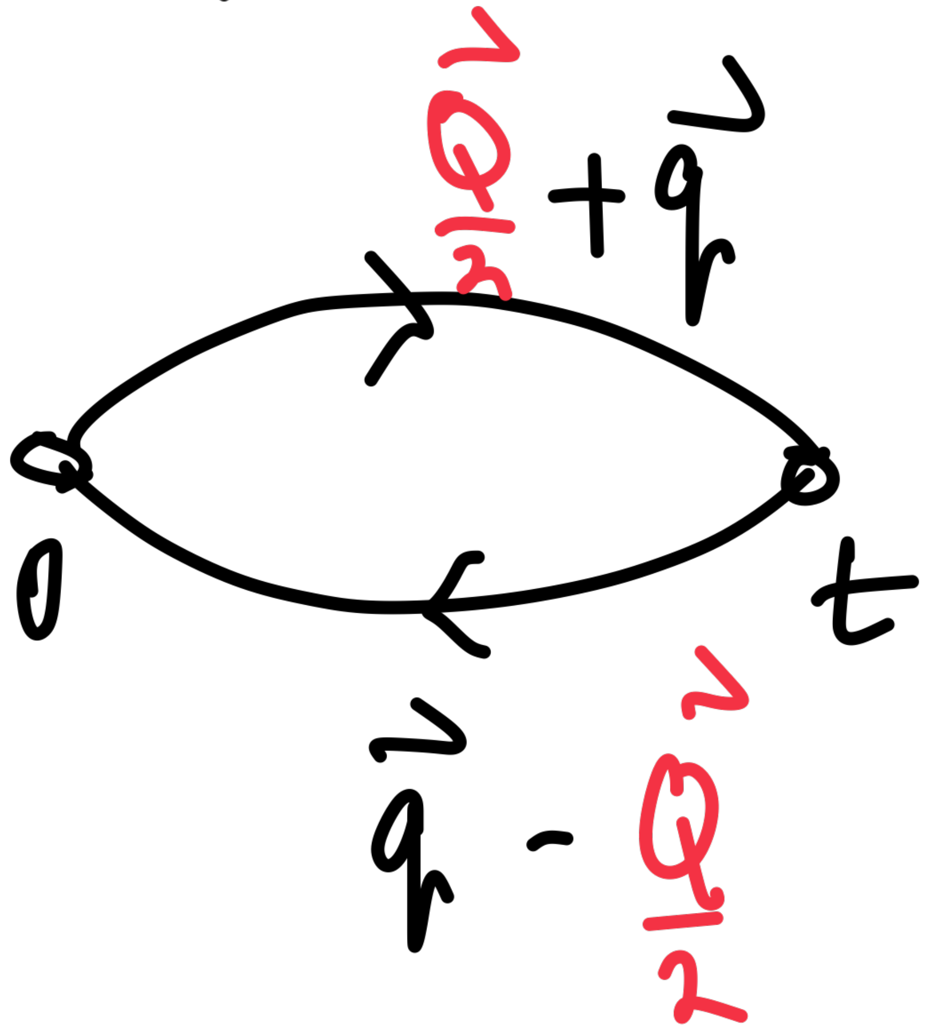


In F.L., $G_B(\vec{Q}, \Omega)$ has simple isolated poles, i.e. emergent bosonic fields.

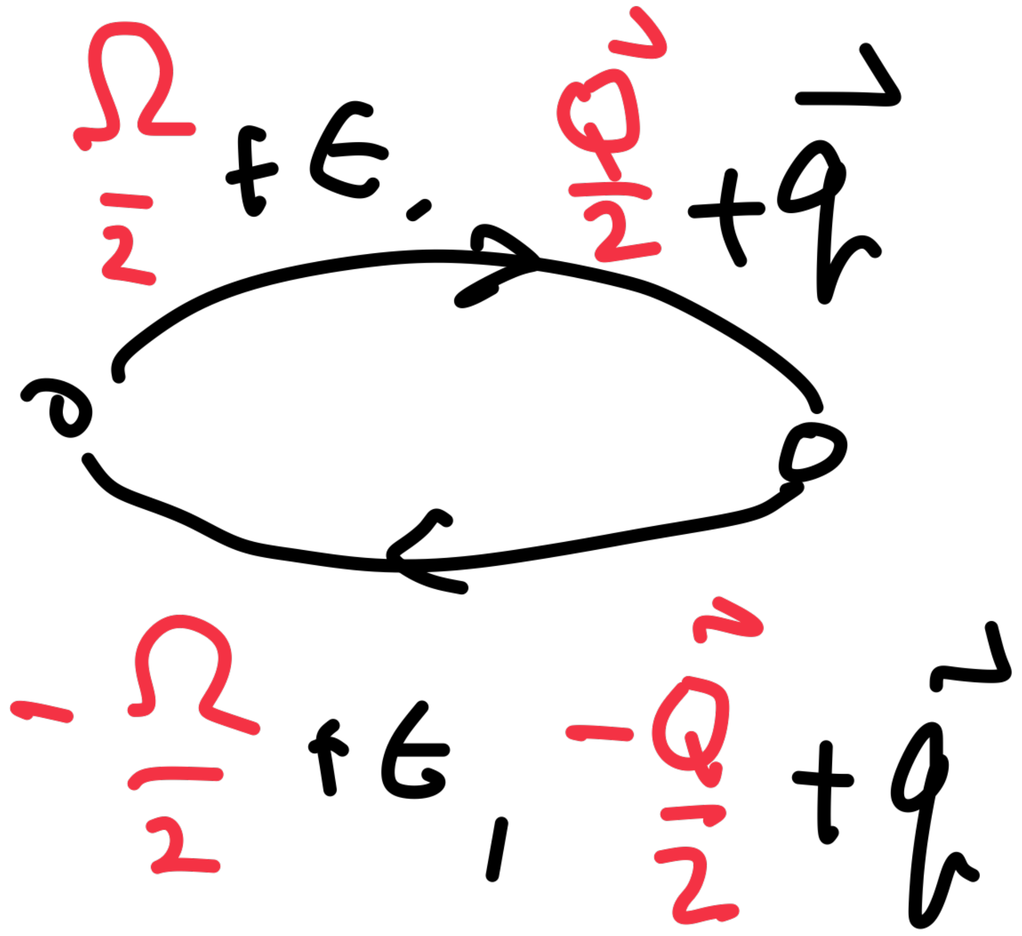
("zero sound")

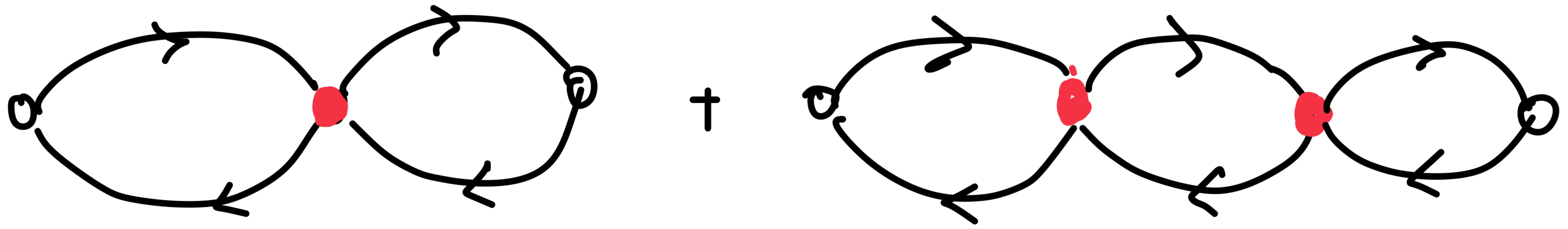
$V_S = V_S(V_F, \tilde{g}) > V_F$

$$G_B(\vec{Q}, t) = \langle 0 | -i\, T\, \phi_{\vec{Q}}(t)\, \phi^\dagger_{\vec{Q}}(0) | 0 \rangle, \quad \phi^\dagger_{\vec{Q}} = \sum_{\vec{q}} \psi^\dagger_{\frac{\vec{Q}}{2}+\vec{q}}\, \psi_{\vec{q}-\frac{\vec{Q}}{2}}$$



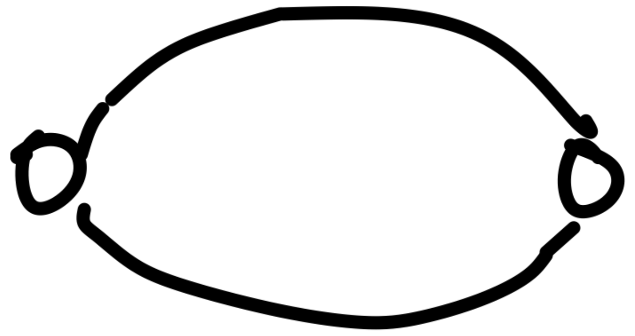
$\longrightarrow G_B(\vec{Q}, \Omega)$

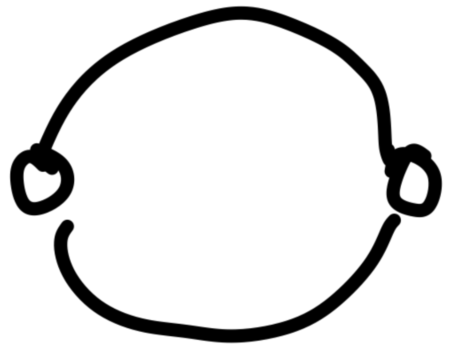


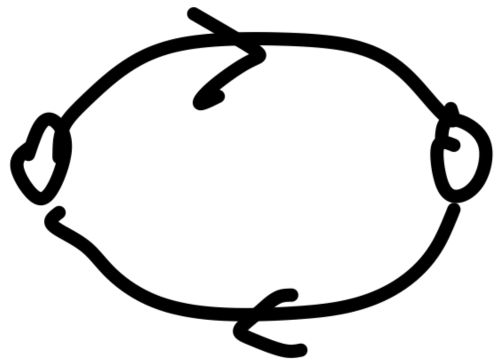
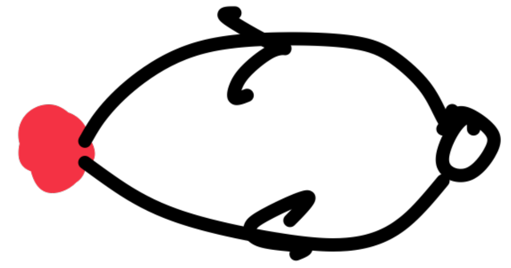
$+$  $+ \cdots$

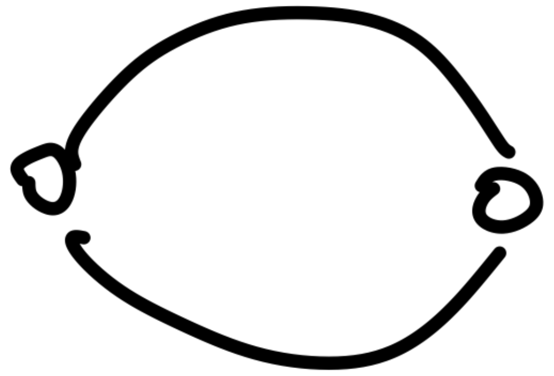
● : interaction "g"

$G_B(\Omega, \vec{Q}) =$



$1 - g$ 

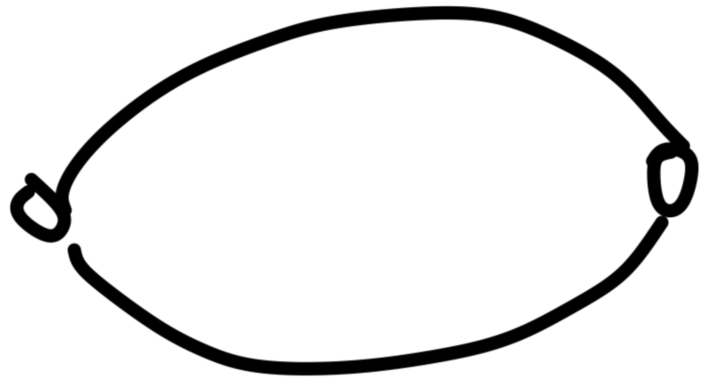
, g  $=$ 



$$-i\int \frac{d\epsilon}{2\pi}\frac{d\vec{q}}{(2\pi)^3}\, G\left(\frac{\Omega}{2}+\epsilon, \frac{\vec{Q}}{2}+\vec{q}\right) G\left(-\frac{\Omega}{2}+\epsilon, -\frac{\vec{Q}}{2}+\vec{q}\right)$$

$$= \int \frac{d\vec{q}}{(2\pi)^3}\, \frac{1}{\Omega - v\vec{q}\cdot\vec{Q} + i\delta}\, \theta\left(\xi_{\frac{\vec{Q}}{2}+\vec{q}}\right) \theta\left(\xi_{-\frac{\vec{Q}}{2}+\vec{q}}\right)$$

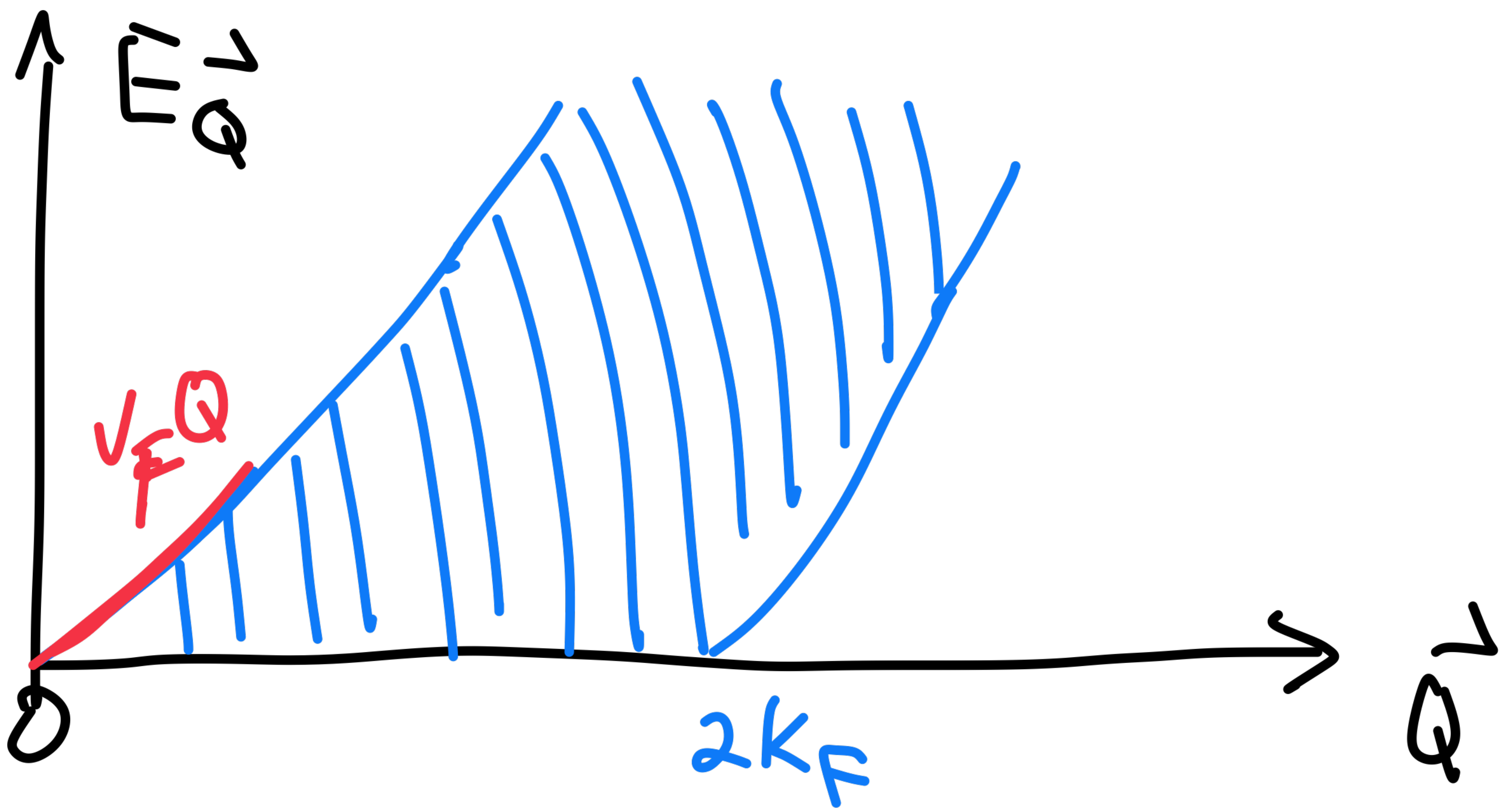
$+ \cdots$

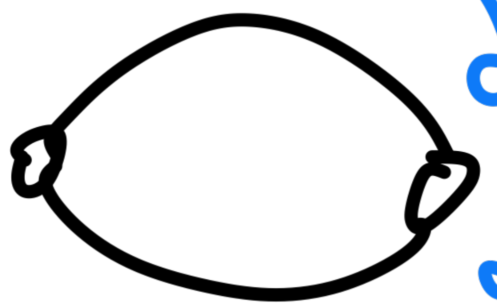


$$= A(k_F)\left[1 + \frac{\Omega}{v_F Q}\ln\left|\frac{\Omega + v_F Q}{\Omega - v_F Q}\right|\right]$$

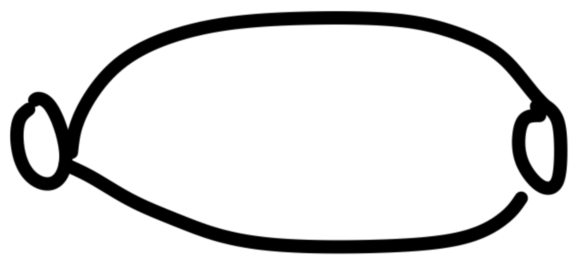
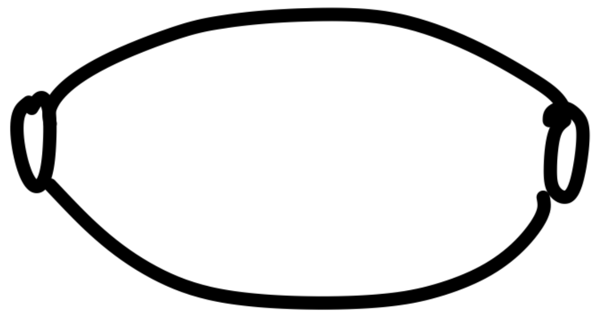
$$+ B(k_F)\left[i\pi\,\Theta(v_F Q-\Omega)\,\Theta(\Omega) - i\pi\,\Theta(-\Omega)\,\Theta(v_F Q+\Omega)\right]\Omega$$

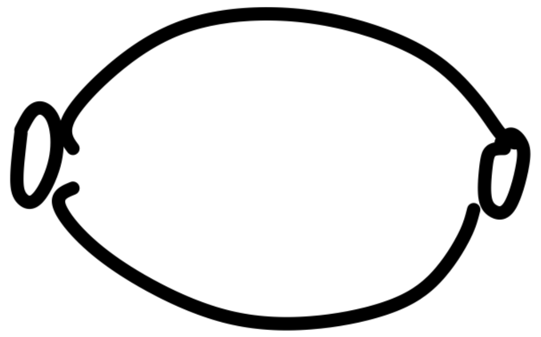
$v_F Q > \Omega > 0$ $\qquad$ $0 > \Omega > -v_F Q$



$G_B(\Omega, \vec{Q}) =$ 

for F. G.

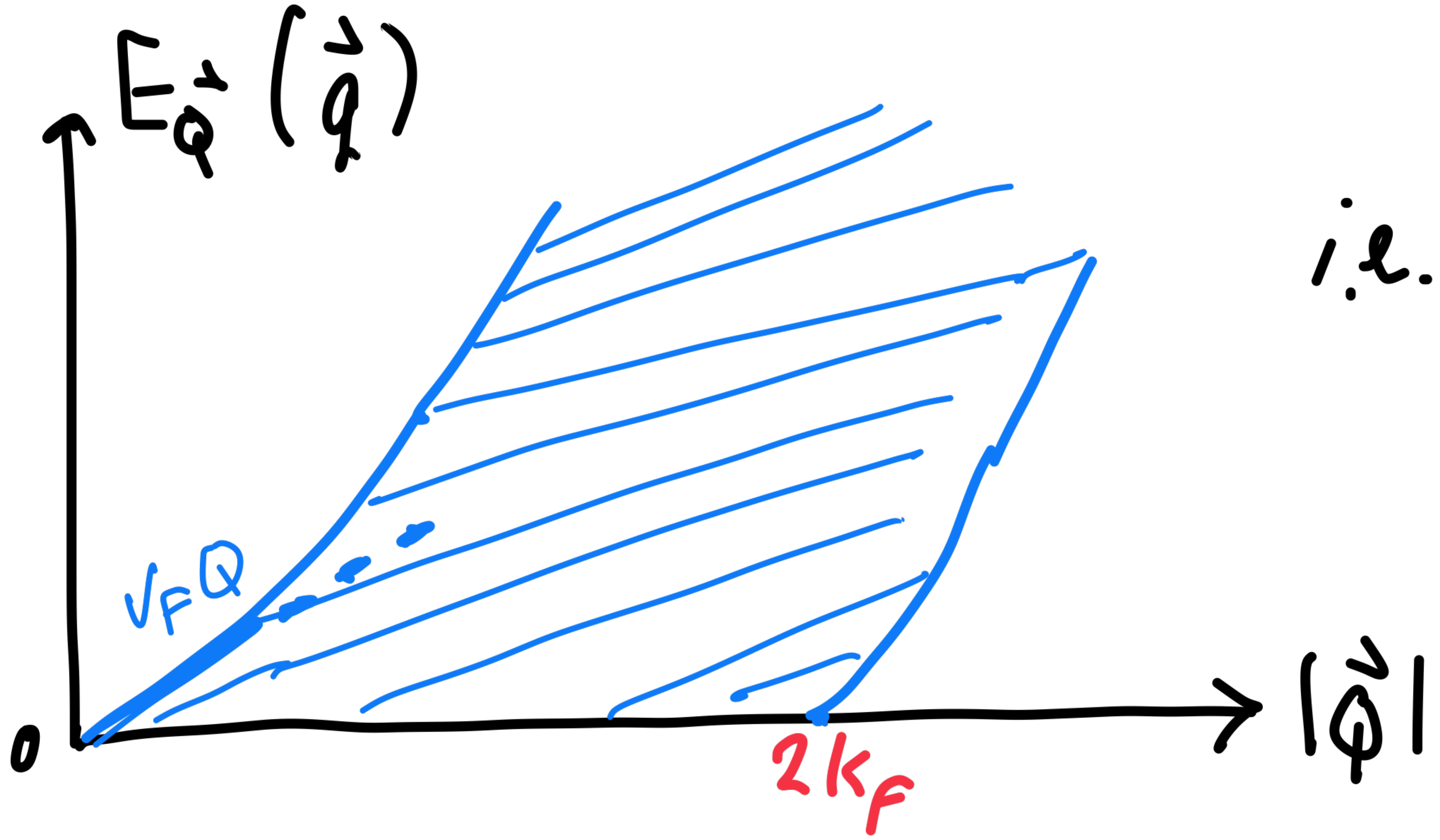
F.L. $G_B =$  / ($1 - g$ )

$1 - g$  $= 1 - g\, A(k_F) \left\{ 1 + \frac{\Omega}{v_F Q} \ln \frac{\Omega + v_F Q}{\Omega - v_F Q} \right\} = 0$

Real if $\Omega > v_F Q$

$$g \to 0, \quad \Omega - v_F Q \hat{=} 2 v_F Q \, e^{-\frac{1}{g A(k_F)}}$$

"F. G."

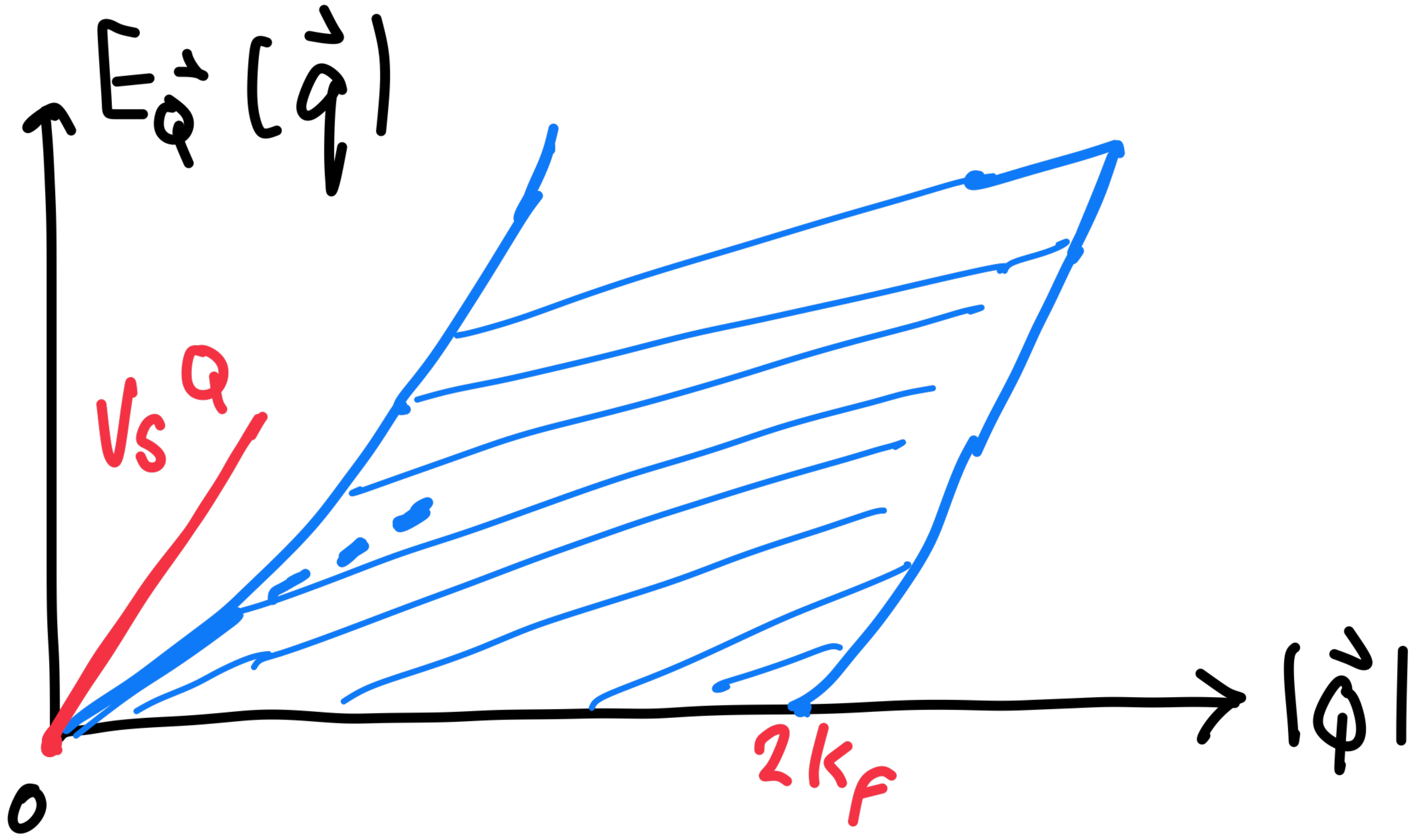


<u>Continuum for bosonic states</u>

i.e. No bosonic "particle"s in F. G.

In F. G., $G_B(\vec{Q}, \Omega)$ doesn't have isolated poles.

"F. L"



In F. L., $G_B(\vec{Q}, \Omega)$ has simple isolated poles, i.e. emergent bosonic fields.

("zero sound")

$v_s = v_s(v_F, \tilde{g}) > v_F$

- Some more formal aspects of interacting fermion dynamics
- 1) sum rules of the spectral weight (imaginary part of the retarded G)
- 2) Analytical properties of Green's functions

Sum Rules for interacting fermions (Lehmann Rep)

$$G(\vec{k},t) = \langle g.s.| -i T \psi_k(t) \psi_k^{+}(0) |g.s.\rangle$$

$$G(\vec{k},\omega) = \int_0^\infty dE \left\{ \frac{A(\vec{k},E)}{\omega - E + i\delta} + \frac{B(-\vec{k},E)}{\omega + E - i\delta} \right\}$$

$$A(\vec{k},E) = \sum_n \langle g.s.| \psi_k(0) |n\rangle\langle n| \psi_k^{+}(0) |g.s.\rangle \, \delta(E-\varepsilon_n)$$

$$B(-\vec{k},E) = \sum_n \langle g.s.| \psi_k^{+}(0) |n\rangle\langle n| \psi_k(0) |g.s.\rangle \, \delta(E-\varepsilon_n)$$