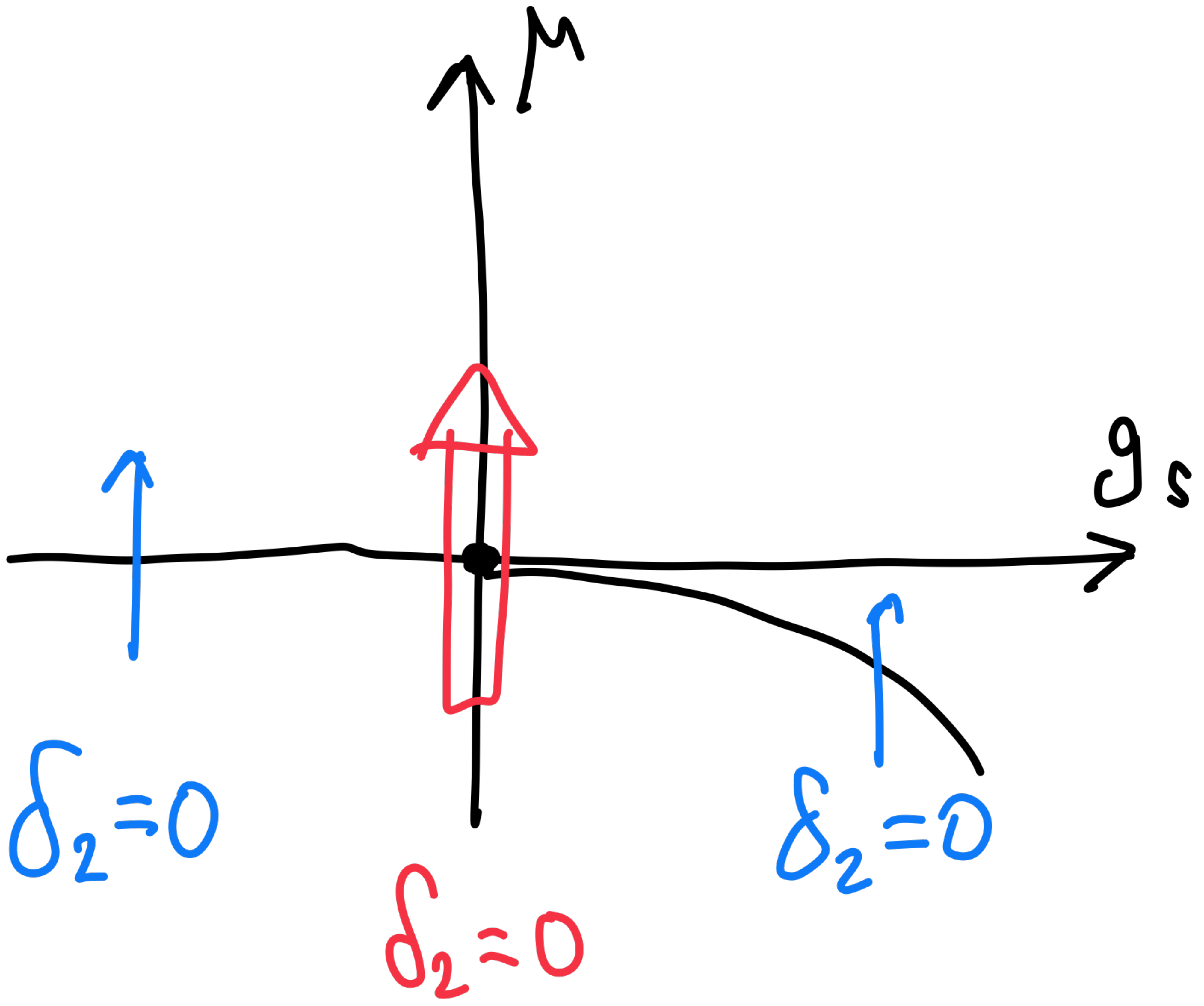
Phys529B: Topics of Quantum Theory

Lecture 22: introduction to 2D an 3D CFT states: relativistic ones

instructor: Fei Zhou

Characterizing $SO(2,1)$ CFT via Green's functions

$(\delta_2 = 0, \ \delta_4 = -1)$

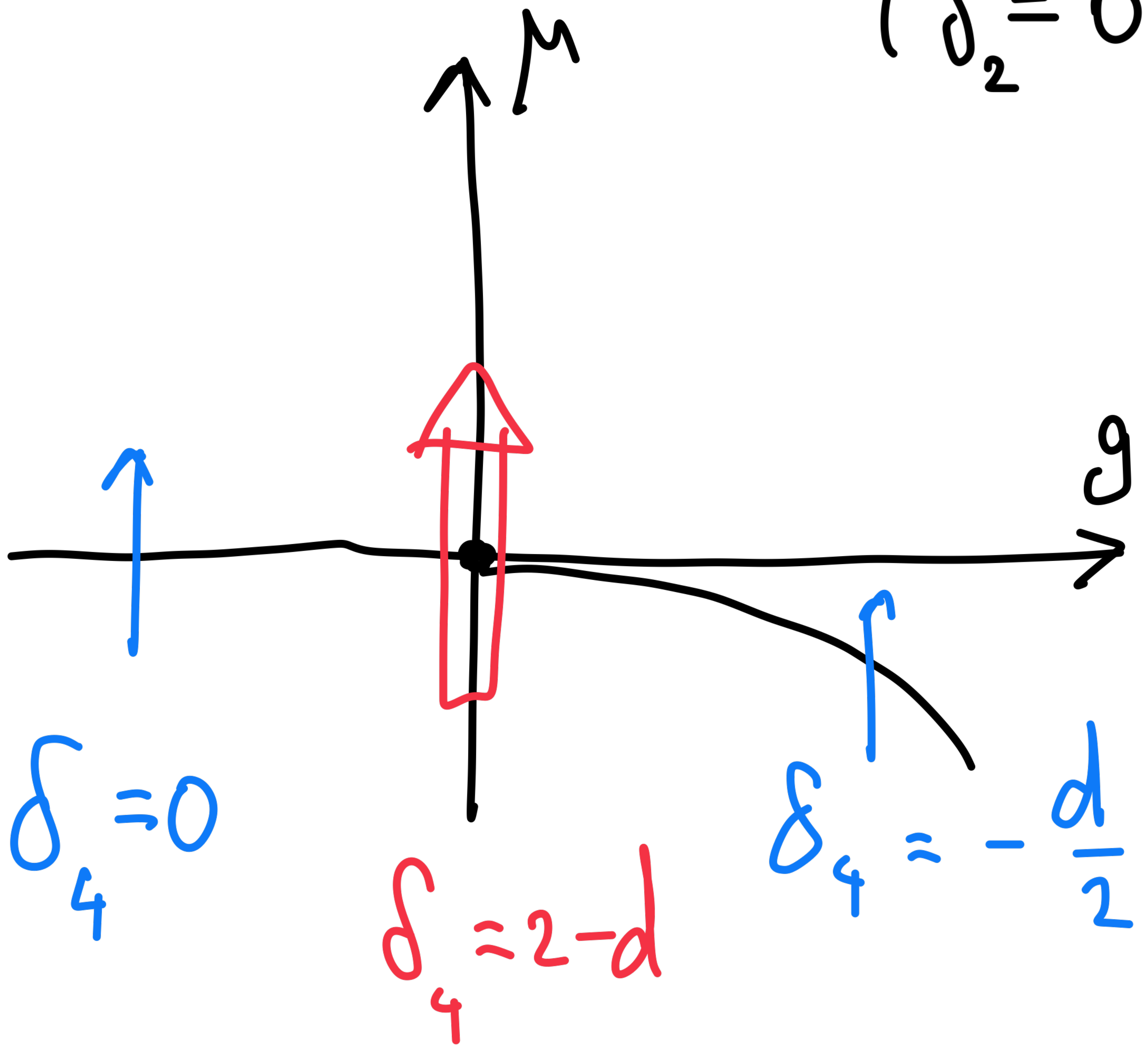


$$G_F(\vec{r}, t) = \langle -i T \psi(\vec{r}, t) \psi^\dagger(0) \rangle_{g.s.}$$

$$G_F(\vec{r}, t) \sim \frac{1}{t^{\frac{d}{2}+\delta_2}} e^{i\frac{r^2}{2t}}$$

Characterizing SO(2,1) CFT via Green's functions

$(\delta_2 = 0, \ \delta_4 = -1)$



$$G_{4f}(\vec{r}, t) \sim \frac{1}{t^{d+\delta_4}} e^{i\frac{r^2}{t}}$$

$$d+\delta_4 = \begin{cases} d, & \text{F.F.} \\ 4, & SO(2,1) \\ d/2, & \text{F.B} \end{cases}$$

$$G_{4f}(\vec{r}, t) = \langle -i\, T\, \psi_\uparrow(\vec{r}, t)\, \psi_\downarrow(\vec{r}, t)\, \psi_\downarrow(0,0)\, \psi_\uparrow(0,0) \rangle$$

$$\frac{dZ}{dt} = 0 \quad \longrightarrow \quad \delta_2 = 0$$

$$\frac{d\tilde{g}}{dt} = (d-2)\tilde{g} + \tilde{g}^2$$

A) $\rightarrow \dfrac{d\tilde{g}}{dt} = (d-2)\tilde{g}$ near $g=0$, $\mathrm{Dim}[\psi^4] = (d+2) + (d-2)$
$= 2d$

B) $\rightarrow \dfrac{d\delta\tilde{g}}{dt} \simeq (2-d)\delta\tilde{g}$ near $\tilde{g}^*$

$$\mathrm{Dim}[\psi^4] = \underbrace{(2+d) + (2-d)}_{4}, \quad \text{near } \tilde{g}^* \quad (4 > d > 2)$$

Relativistic Conformal Group (~ SO(4,2))

Poincare Group:
translation Group and Lorentz Group (~SO(3,1) with SL(2, C) as its covering)
Dilation transformation
Special Conformal Group

$$P_\mu = -i\partial_\mu \qquad \text{translation}$$

$$L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu) \qquad \text{Lorentz Generator}$$

$(SO(3,1)$ or its Covering $SL(2,\mathbb{C}))$

$$D = -i x^\mu \partial_\mu$$

$$K_\mu = -i(2x_\mu x^\nu \partial_\nu - x^2 \partial_\mu) \qquad \text{Special Conformal Group}$$

Conformal Group

Special Conformal group (Space time)

$$\frac{x'^{\mu}}{x'^{2}} = \frac{x^{\mu}}{x^{2}} - b^{\mu}$$

or

$$x'^{\mu} = \frac{x^{\mu} - b^{\mu} x^{2}}{1 + b^{2} x^{2} - 2\, b \cdot x}$$

In a linear Representation,

Conformal Group $\sim SO(\overset{1,4}{d}, 2)$

Generators =

1 dilation + d translations + $\frac{d(d-1)}{2}$ Rotations

+ d Conformal transformal Transformations

$$= \frac{(d+1)(d+2)}{2} \text{ Generators} \quad \begin{cases} d=4, & N=15 \\ d=2, & N=6 \end{cases}$$