Phys529B: Topics of Quantum Theory

Lecture 21: introduction to 2D an 3D CFT states: non-relativistic ones

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— Conformal invariance

$$\vec{r} \to \vec{r}\,' = \frac{\vec{r}}{1-\lambda t}, \quad t \to t' = \frac{t}{1-\lambda t},$$

$$\psi(\vec{r}, t) \to \frac{1}{(1-\lambda t)^d} e^{i \frac{\vec{r}^2}{2} \frac{\lambda}{1-\lambda t}} \psi(\vec{r}\,', t')$$

E. O. M invariant.

Rep of Conformal group $SO(2,1)$

Maki, Zhou 2019

$$\begin{cases} G_H = \partial_t, & \text{temporal translation} \\ G_D = 2t\frac{\partial}{\partial t} + \vec{r}\cdot\nabla_{\vec{r}} + \frac{d}{2} & \text{Space-time dilation} \\ G_C = t^2\frac{\partial}{\partial t} + t\left(\vec{r}\cdot\nabla_{\vec{r}} + \frac{d}{2}\right) + i\frac{\vec{r}^{\,2}}{2} & \end{cases}$$

SO(2,1) Algebra Or (SL(2,R))

$$\begin{cases} [G_D, G_H] = -2\,G_H \\ [G_D, G_C] = 2\,G_C \\ [G_H, G_C] = +\,G_D \end{cases}$$

$$[\sigma_z, \sigma_-] = -2\,\sigma_-$$

$$[\sigma_z, \sigma_+] = 2\,\sigma_+$$

$$[\sigma_-, \sigma_+] = -\,\sigma_z$$

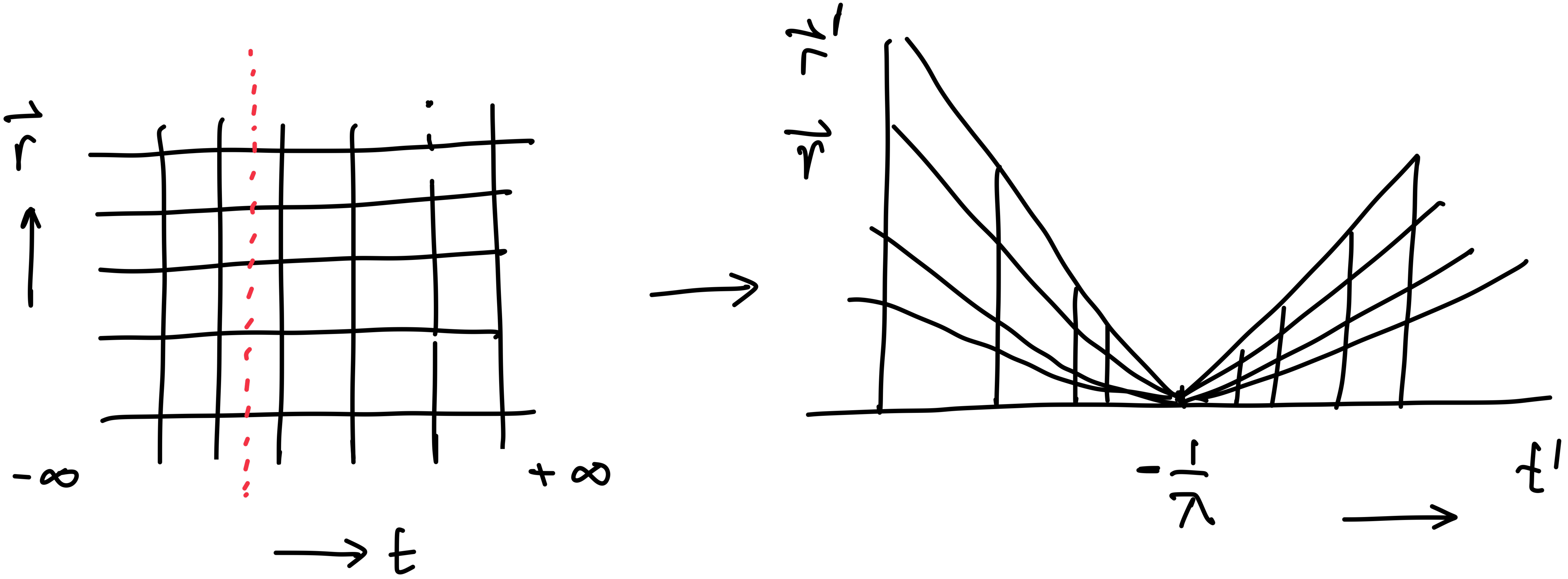
Ref.

$SU(2) \sim SO(3)$

$SO(2,1)$

Geometry

$$\vec{r}\,' = \frac{\vec{r}}{1-\lambda t} \qquad t' = \frac{t}{1-\lambda t} \qquad \begin{cases} \vec{r} = \dfrac{\vec{r}\,'}{1+\lambda t'} \\ t = \dfrac{t'}{1+\lambda t'} \end{cases}$$



$H = H'$

Conformal Symmetry

$$i\partial_t \psi(\vec{r},t) = [\psi, H] \longrightarrow i\partial_{t'} \psi'(\vec{r}',t') = [\psi', H']$$

$$U(t) \longrightarrow U'(t')$$

$$U(t\to\infty) \longrightarrow U'(t' = -\tfrac{1}{\lambda} + \epsilon) \qquad \epsilon \to 0$$

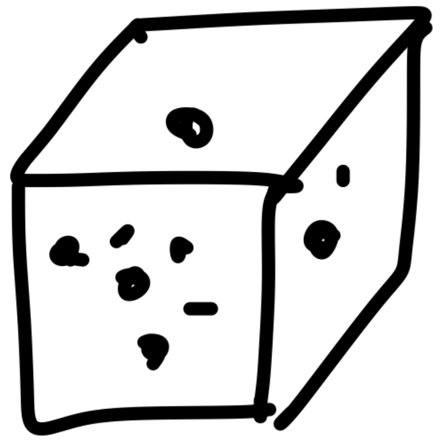
$$\simeq U'(-\tfrac{1}{\lambda}) + O(\epsilon)$$

All dynamics become frozen

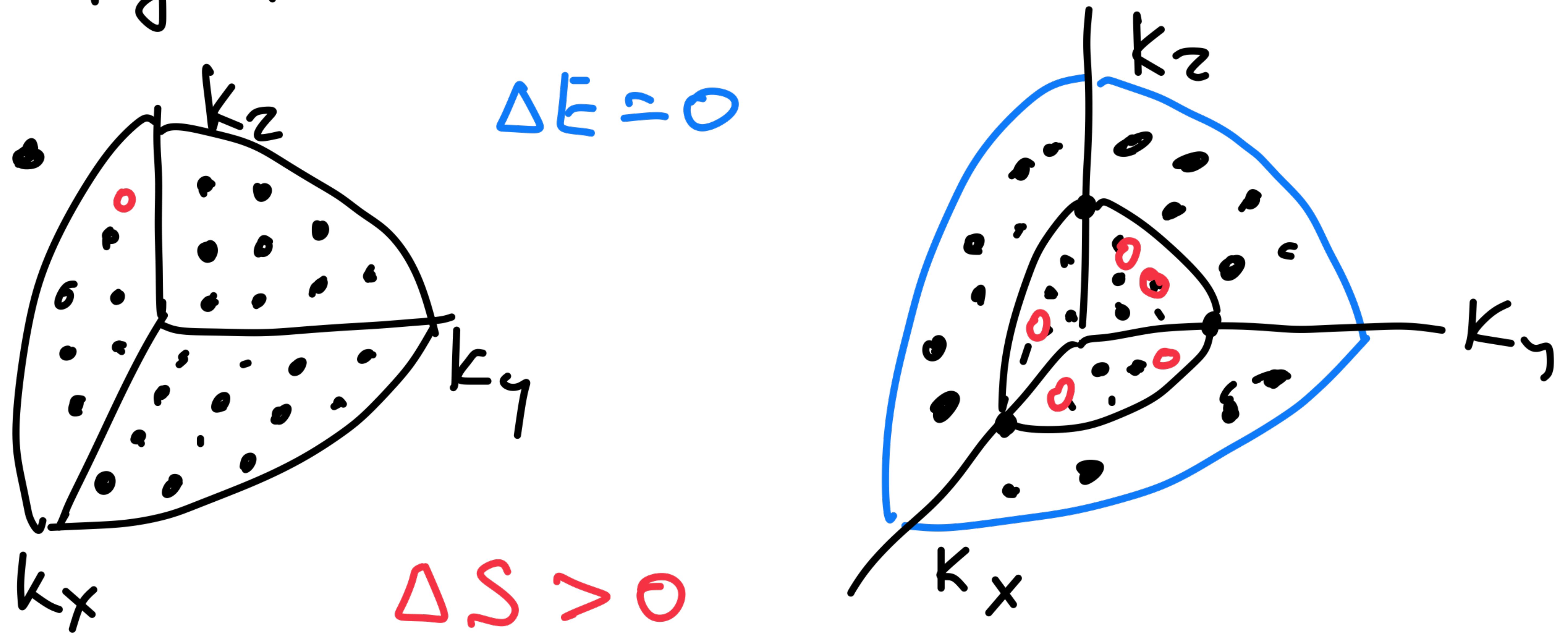
$$\Delta t \simeq \Delta t' \lambda^2 t^2$$

Absence of Entropy Production

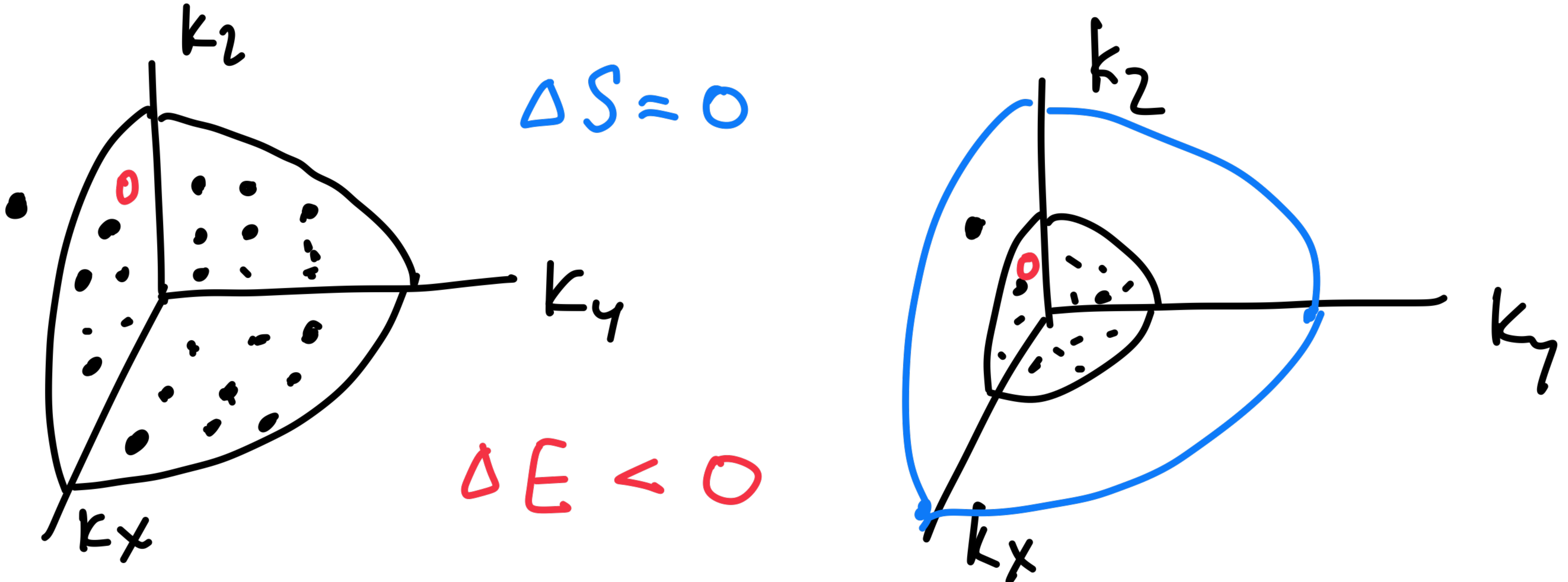
When $t \to \infty$ $\quad \Delta S = 0$ $\quad \Delta E = 0$



Free Expansion

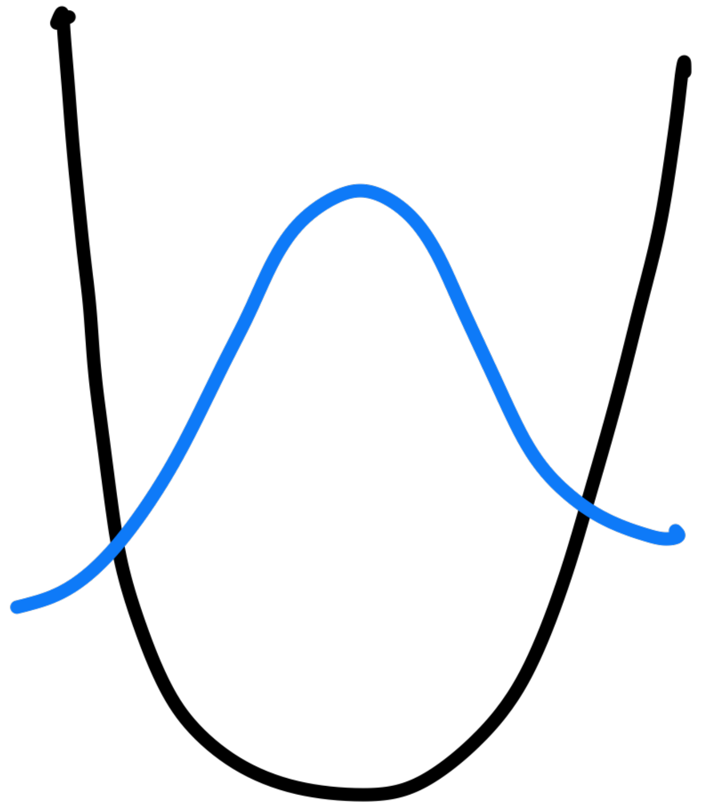


Adiabatic Expansion

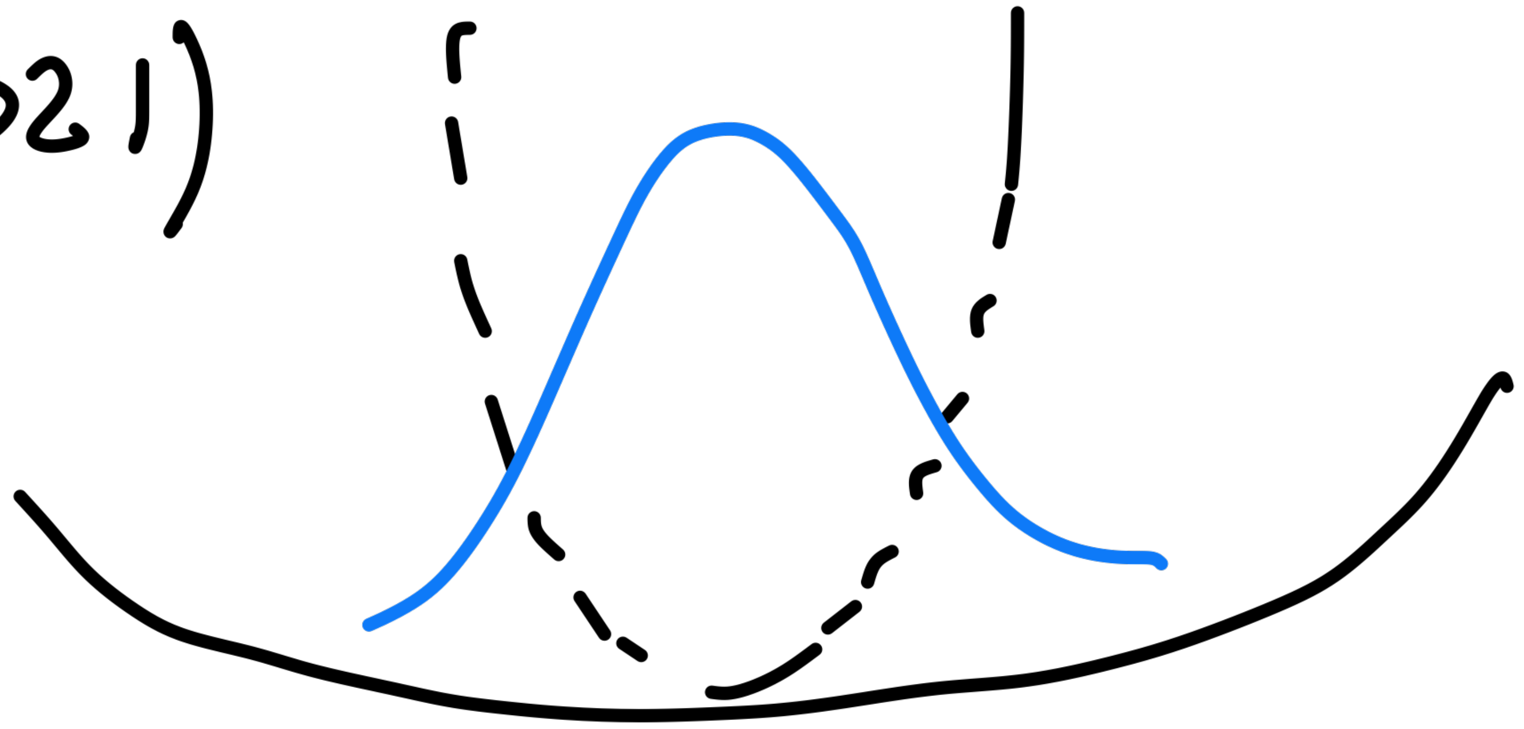


Conformal Dynamics
(Maki, Zhou, 2021)

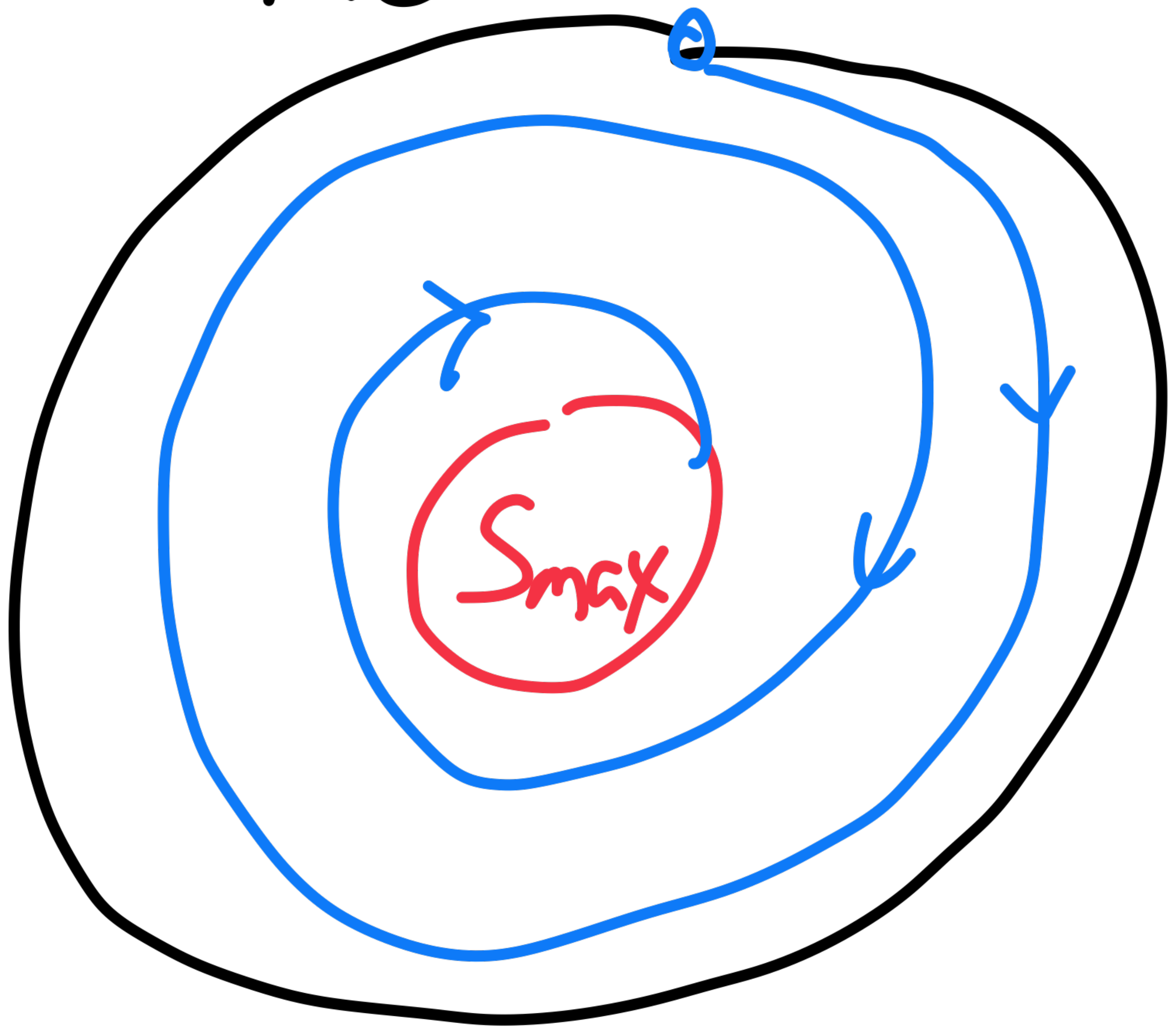
$t = 0^-$



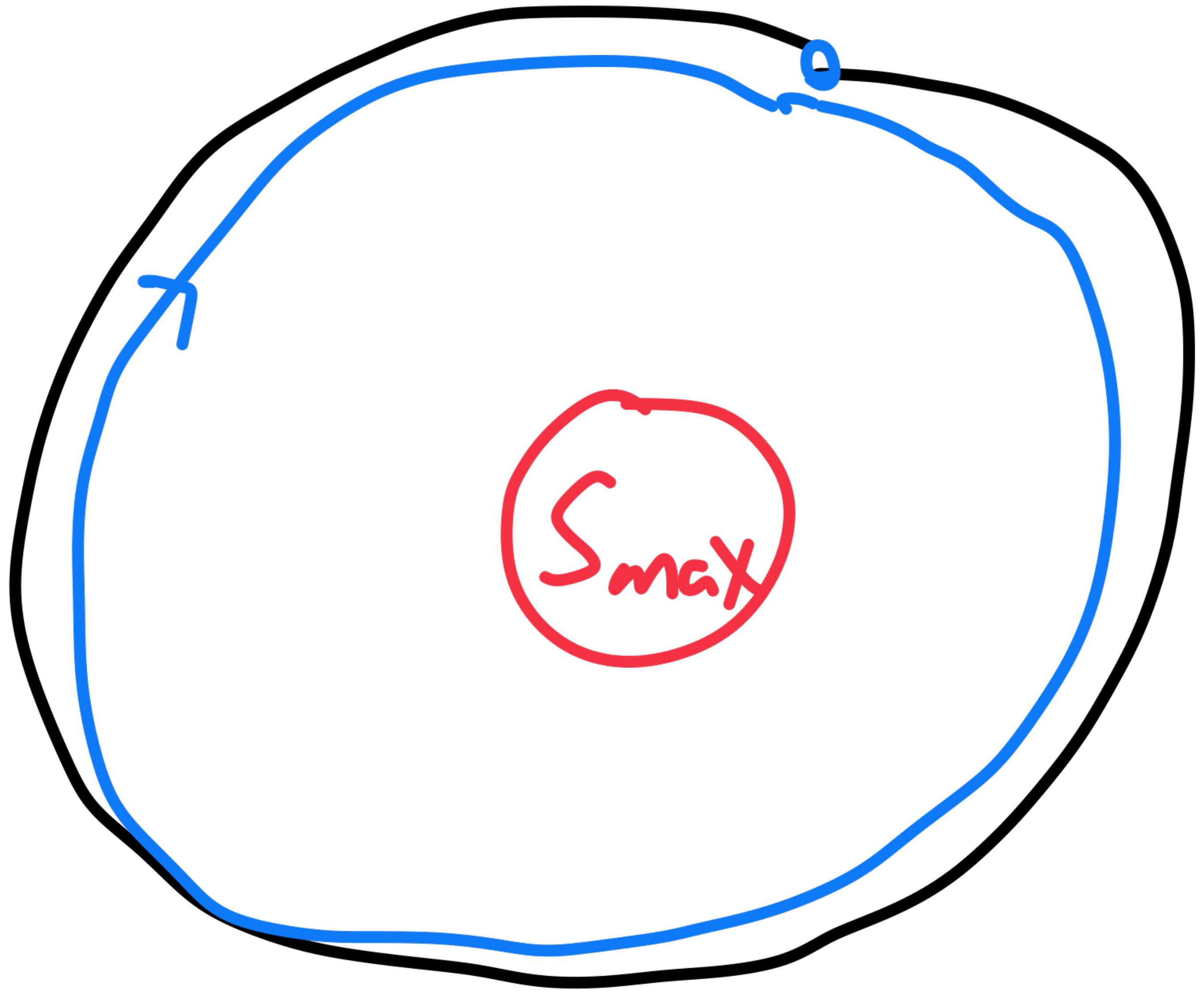
$t = 0^+$



Thermalization



Conformal Dynamics



$$\frac{\partial S}{\partial t} = \frac{\partial E}{\partial t} = 0$$

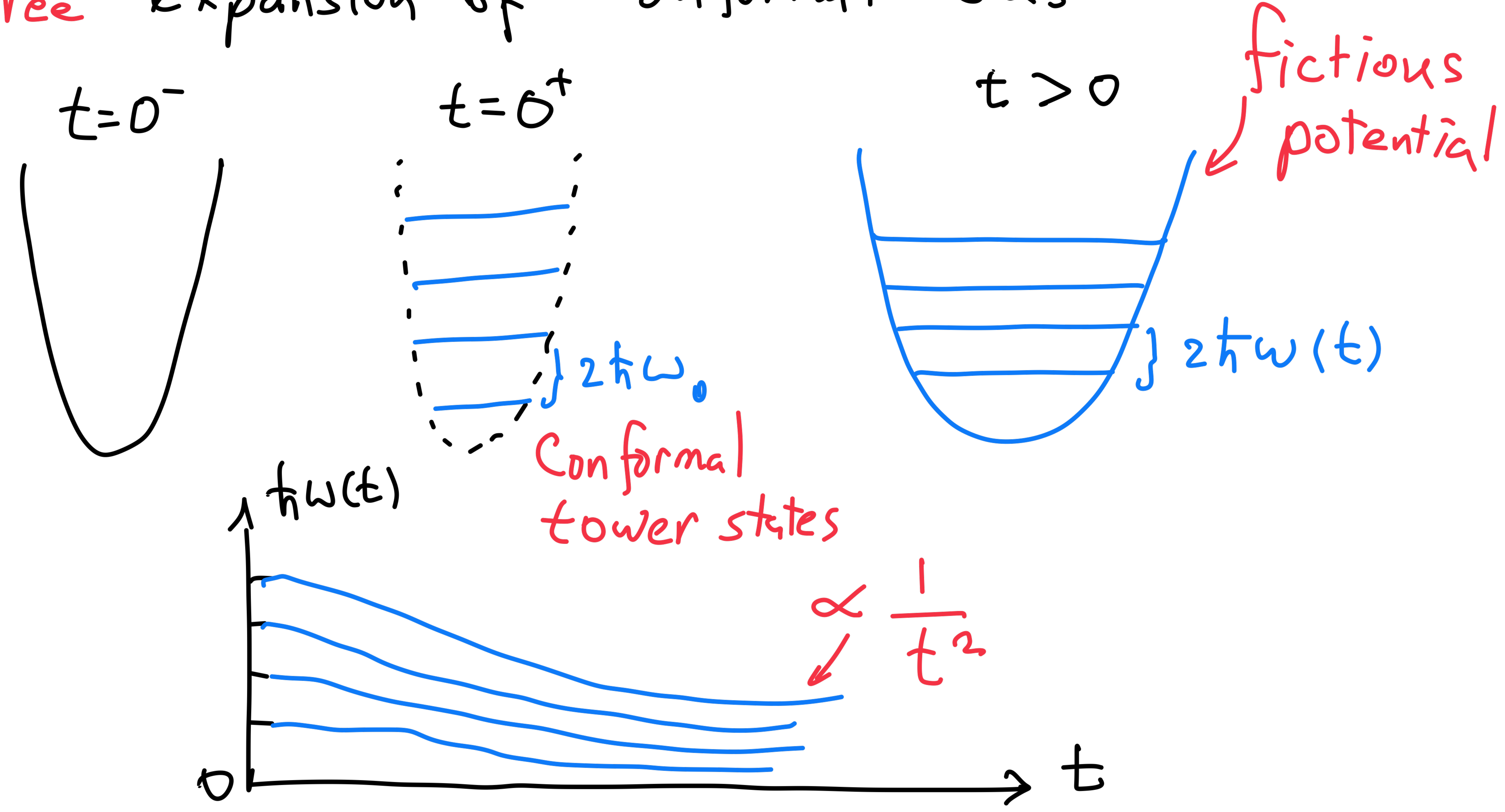
Adiabatic Process: $\Delta E = W, \quad \Delta S = 0$

Isothermal Process: $\Delta E = 0, \quad \Delta S = \frac{\Delta Q}{T} = +\frac{W}{T} \neq 0$

Conformal Dynamics: $\Delta E = 0, \quad \Delta S = 0$

Following Conformal Tower State Dynamics

Free expansion of Conformal Gas



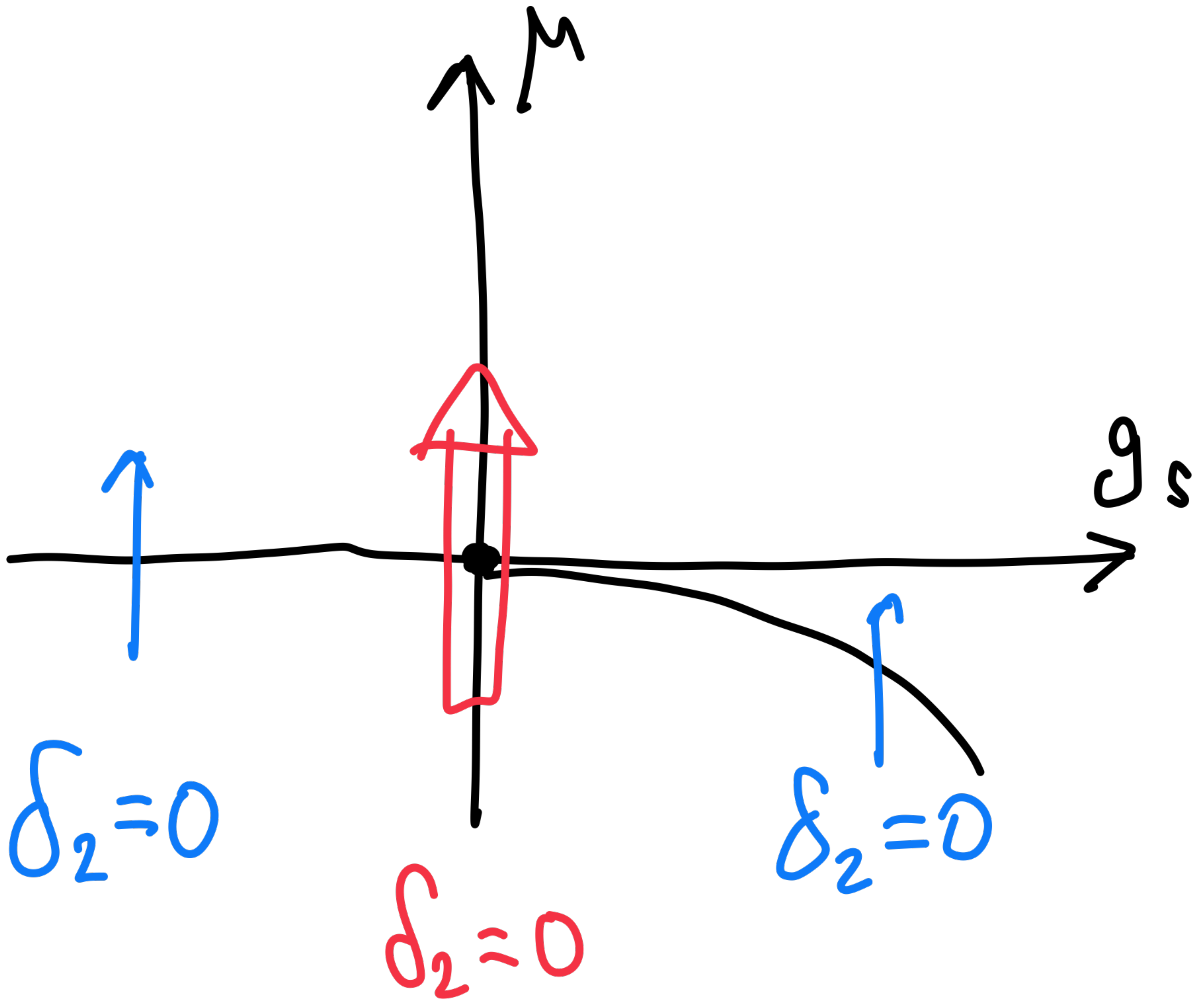
At $t>0$, $|t>$ can be mapped into an eigen state of an FICTIOUS Confined Scale-conformal invariant Many-body state.

$$H_{INT}(t)\,|t> = E_0(t)\,|t>$$

$$H_{INT}(t) = H_{free\ space} + \int \frac{M}{2} \underbrace{\omega^2(t)\, \psi^{+} \vec{r}^{\,2} \psi\, d^3\vec{r}}_{\text{fictious}}$$

Characterizing SO(2,1) CFT via Green's functions

$(\delta_2 = 0,\ \delta_4 = -1)$

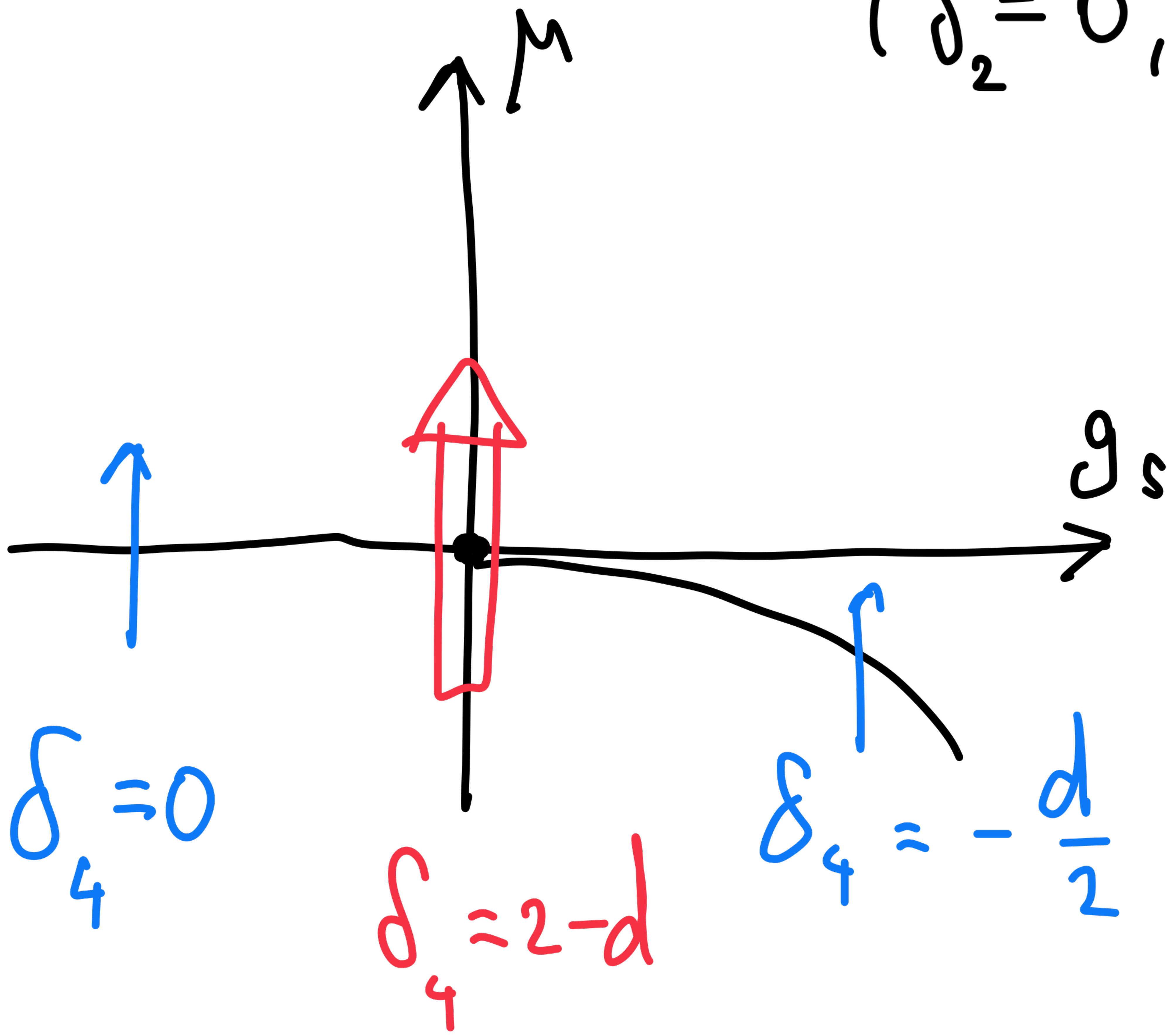


$$G_F(\vec{r}, t) = \langle -i T \psi(\vec{r}, t) \psi^\dagger(0) \rangle_{g.s.}$$

$$G_F(\vec{r}, t) \sim \frac{1}{t^{\frac{d}{2} + \delta_2}} e^{i \frac{r^2}{2t}}$$

Characterizing SO(2,1) CFT via Green's functions

($\delta_2 = 0$, $\delta_4 = -1$)



$$G_{4f}(\vec{r},t) \sim \frac{1}{t^{d+\delta_4}} e^{i\frac{r^2}{t}}$$

$$d+\delta_4 = \begin{cases} d, & \text{F. F.} \\ 4, & SO(2,1) \\ d/2, & \text{F. B} \end{cases}$$

$$G_{4f}(\vec{r},t) = \langle -i\, T\, \psi_\uparrow(\vec{r},t)\, \psi_\downarrow(\vec{r},t)\, \psi_\downarrow(0.0)\, \psi_\uparrow(0.0) \rangle$$

$$\frac{dZ}{dt} = 0 \longrightarrow \delta_2 = 0$$

$$\frac{d\tilde{g}}{dt} = (d-2)\tilde{g} + \tilde{g}^2$$

A) $\rightarrow \dfrac{d\tilde{g}}{dt} = (d-2)\tilde{g}$ near $g=0$, $\mathrm{Dim}[\psi^4] = (d+2) + (d-2)$
$= 2d$

B) $\rightarrow \dfrac{d\delta\tilde{g}}{dt} \simeq (2-d)\delta\tilde{g}$ near $\tilde{g}^*$

$$\mathrm{Dim}[\psi^4] = \underbrace{(2+d) + (2-d)}_{4},$$ near $\tilde{g}^*$