Phys529B: Topics of Quantum Theory

Lecture 20: introduction to 2D an 3D CFT states: non-relativistic ones

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- Final presentation:

- Holographic matter, Hartnoll, Lucas and Sachdev, Chapter 1(New ways of thinking of RGE, UV/IR mixing), Chapter 3 and 5 (transport physics near QCPs and metal without quasi-particles).

Conformal field Theory states I: Non-Relativistic CFT

Hagen, 72; Niederer, 72; Nishida, Son, 07; on CFT

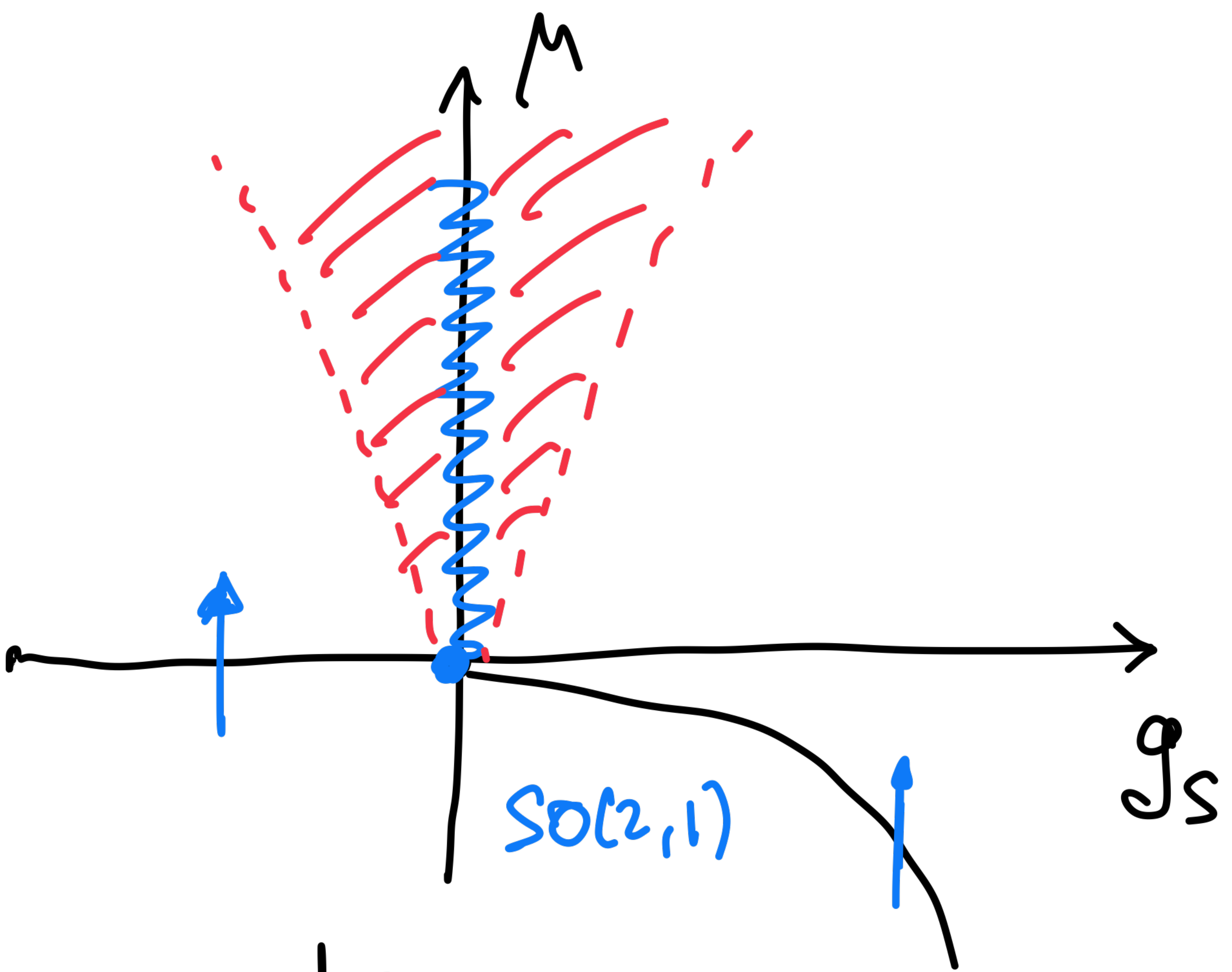
Maki and Zhou, PRA 98, 13602 (2018)

PRA 100, 23601 (2019)

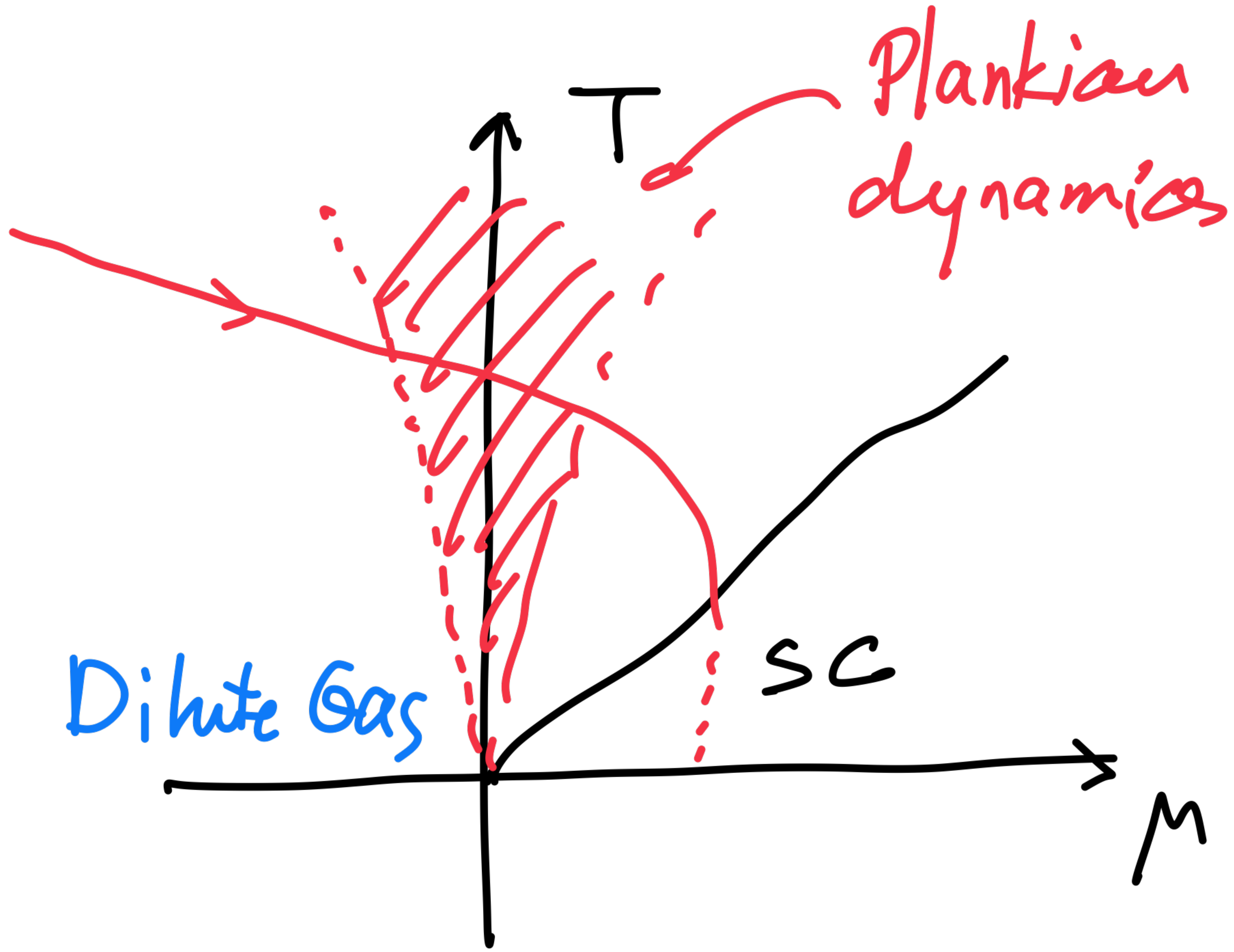
PRA 102, 063319 (2020)

PRL 128, 040401 (2022)

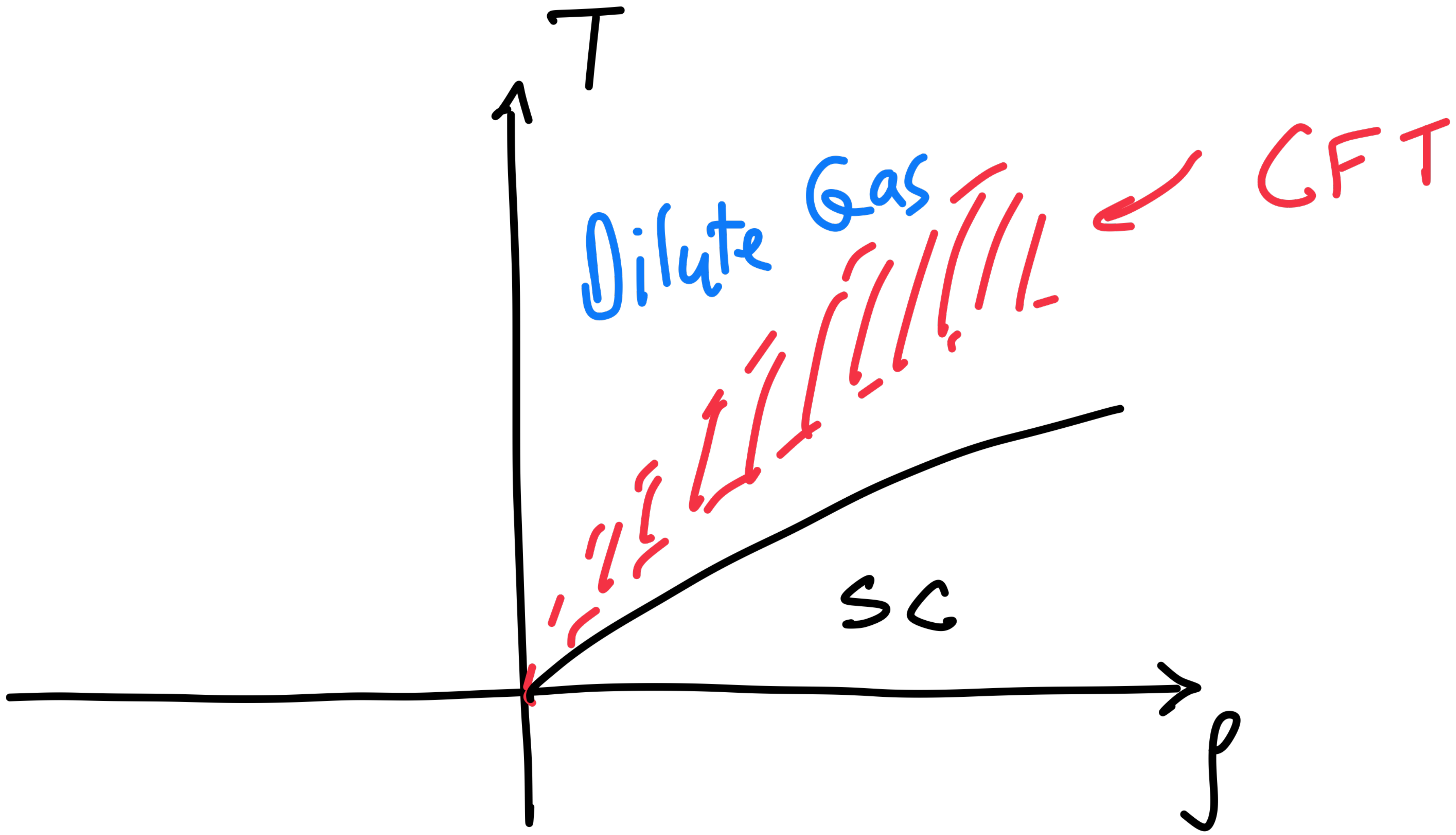
$$H-\mu\hat{N} = \int \psi^\dagger\left(-\frac{\nabla^2}{2}-\mu\right)\psi - g_s\int \psi^\dagger\sigma_y\psi^\dagger\,\psi\sigma_y\psi$$



$d > 2$



$g_s = g^*$



$$g = g^*$$

Invariance / Symmetry of Equation of Motion

$$i\partial_t \psi(\vec{r},t) = [\psi(\vec{r},t), H] \leftarrow \text{E.O.M.}$$

- Scale Symmetry $\quad H \to \lambda^2 H \quad$ or $\quad \tilde{H} \to \tilde{H}$

$$\vec{r} \to \vec{r}\,' = \frac{\vec{r}}{\lambda}, \quad t \to t' = \frac{t}{\lambda^2}, \quad \psi(\vec{r},t) \to \frac{1}{\lambda^d}\psi(\vec{r}\,',t')$$

E.O.M invariant.

— Conformal invariance

$$\vec{r} \to \vec{r}\,' = \frac{\vec{r}}{1-\lambda t}\,, \quad t \to t' = \frac{t}{1-\lambda t}\,,$$

$$\psi(\vec{r},t) \to \frac{1}{(1-\lambda t)^d}\, e^{i \frac{\vec{r}^{\,2}}{2} \frac{\lambda}{1-\lambda t}}\, \psi(\vec{r}\,', t')$$

E.O.M invariant.

Rep of Conformal group $SO(2,1)$

Maki, Zhou 2019

$$\begin{cases} G_H = \partial_t , & \text{temporal translation} \\ G_D = 2t\frac{\partial}{\partial t} + \vec{r}\cdot\nabla_{\vec{r}} + \frac{d}{2} & \text{Space-time dilation} \\ G_c = t^2\frac{\partial}{\partial t} + t\left(\vec{r}\cdot\nabla_{\vec{r}} + \frac{d}{2}\right) + i\frac{\vec{r}^{\,2}}{2} & \end{cases}$$

$SO(2,1)$ Algebra (Or $SL(2,R)$)

$$\begin{cases} [G_D, G_H] = -2G_H \\ [G_D, G_C] = 2G_C \\ [G_H, G_C] = +G_D \end{cases}$$

$$[\sigma_z, \sigma_-] = -2\sigma_-$$

$$[\sigma_z, \sigma_+] = 2\sigma_+$$

$$[\sigma_-, \sigma_+] = -\sigma_z$$

Ref.

$SU(2) \sim SO(3)$

$SO(2,1)$

Geometry

$$\vec{r}\,' = \frac{\vec{r}}{1-\lambda t} \qquad t' = \frac{t}{1-\lambda t} \qquad \begin{cases} \vec{r} = \dfrac{\vec{r}\,'}{1+\lambda t'} \\[2ex] t = \dfrac{t'}{1+\lambda t'} \end{cases}$$