Phys529B: Topics of Quantum Theory

Lecture 19: introduction to 2D an 3D CFT states

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• Holographic matter, Hartnoll, Lucas and Sachdev, Chapter 3 and 5 (transport physics near QCPs and metal without quasi-particles).

Constructing fermions (Mandelstam 1975)
$$N(x) = e^{i\sqrt{\pi} \Theta(x) + i\sqrt{\pi} \Phi(x)}$$

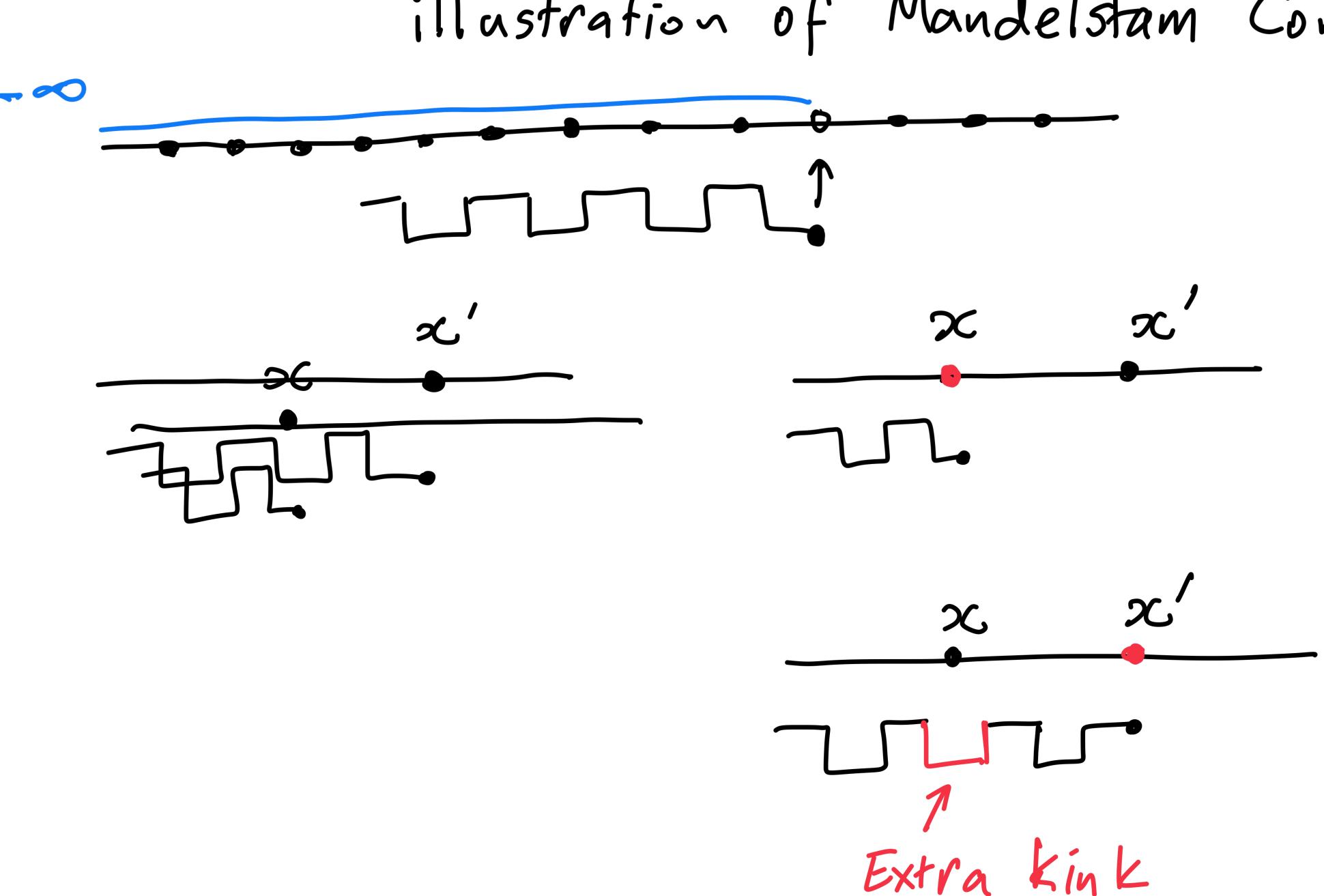
$$J_{\sigma}(x) = \frac{1}{\sqrt{\pi}} \Pi(x), \quad \Theta(x) = \int_{-\infty}^{x} J_{\sigma}(x') dx'$$

$$J(x) = \frac{1}{\sqrt{\pi}} \partial_{x} \Phi(x), \quad \Phi(x) = \int_{-\infty}^{x} J_{\sigma}(x') dx'$$

$$[\Phi(x), \Pi(x')] = i\delta(xx') \rightarrow [\Phi(x), \theta(x')] = i\Phi(xx')$$

$$\{\psi(x), \psi(x)\} = 0$$
 if $x \neq x'$
Via Baker-Bansdorff Relation
 $e^A e^B = e^B e^A e^{-[A,B]}$
 $\psi(x)\psi(x') = \psi(x')\psi(x) e^{-i}\phi(x,x')$
 $\psi(x)\psi(x') = -[\pi \Theta(x) + \pi \phi(x), \pi \Theta(x') + \pi \phi(x')]$
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 $\psi(x)\psi(x) = -[\pi \Theta(x) + \pi \phi(x), \pi \Theta(x') + \pi \phi(x')]$

illustration of Mandelstam Construction



Conformal field Theory States I: Non-Relativistic CFT Hagen, 72; Niederer, 72; Nishida, Soy, 07; on CFT Maki and Zhou, PRA.98, 13602 (2018) PRA 100, 23601 (2019) PRA 102,063319 (2025) PRL 128,04040] (2022)

$$H - \mu \hat{N} = \int \psi^{\dagger} (-\frac{\nabla^{2}}{2} - \mu) \psi - g \int \psi^{\dagger} 6y \psi^{\dagger} \psi 6y \psi$$

