Phys529B: Topics of Quantum Theory

Lecture 18: introduction to 1D LL and bosonization

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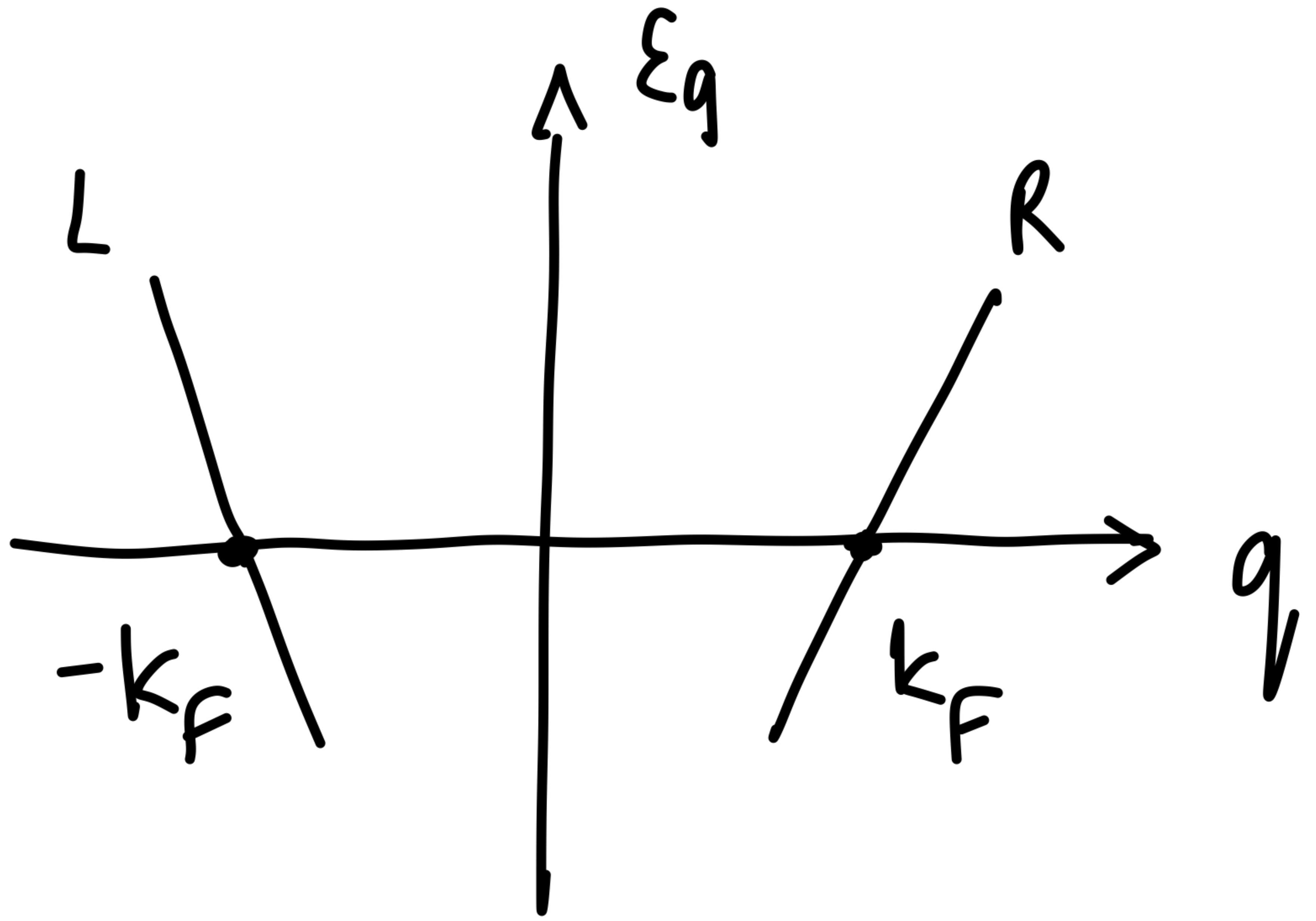
Relations to Dirac fermions Via Trucation



$$\psi(x) = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\,\psi_k$$

$$= e^{ik_f x}\,\psi_R(\vec{x}) + e^{-ik_f x}\,\psi_L(\vec{x})$$

$$\sum_k e^{i\vec{k}\cdot\vec{x}}\,\psi_k \simeq \sum_{k,R} e^{ik_f x + i\vec{k}\cdot\vec{x}}\,\psi_{kR} + \sum_{k,L} e^{-ik_f x + i\vec{k}\cdot\vec{x}}\,\psi_{kL}$$

$$H_0 = \int \left(\psi_R^{+}, \psi_L^{+}\right) \mathcal{H}_0 \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} d\vec{x}$$

$$\mathcal{H}_0 = \begin{bmatrix} -i\partial_x & 0 \\ 0 & +i\partial_x \end{bmatrix} = \sigma_z \otimes \mathbb{1} \left(-i\partial_x\right)$$

$\mathbb{1} = 1$ for Spinless case

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for Spinful case.

Interactions

$$H_I = g_1 \int \left[\psi_R^\dagger \psi_R \psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L \psi_L^\dagger \psi_L \right] \text{ - forward Scattering}$$

$$+ g_2 \int \psi_R^\dagger \psi_R \psi_L^\dagger \psi_L \qquad \text{- back Scattering}$$

$$g_1 = V(q \approx 0), \qquad g_2 = V(0) - V(2k_F)$$

if $$H_I = \int \psi^\dagger(\vec{x})\,\psi(\vec{x})\,V(\vec{x}-\vec{x}')\,\psi^\dagger(\vec{x}')\,\psi(\vec{x}')\,d^d\vec{x}\,d^d\vec{x}'$$

Note that

$$\rho(x) = \psi_R^\dagger(x)\psi_R(x) + \psi_L^\dagger(x)\psi_L(x)$$

$$+ \psi_R^\dagger(x)\psi_L(x)\,e^{-i2k_F x} + \psi_L^\dagger(x)\psi_R(x)\,e^{i2k_F x}$$

Useful algebraic Relations for current algebras

$$[AB, C] = A[B, C] + [A, C]B$$

$$[A, BC] = \{A, B\}C - B\{A, C\}$$

Current operator and Current Algebras

$$J_L = \psi_L^\dagger \psi_L \qquad J_R = \psi_R^\dagger \psi_R$$

$$\left[\psi_R^\dagger(x+\epsilon)\psi_R(x+\epsilon),\ \psi_R^\dagger(x)\psi_R(x)\right]$$

$$= \psi_R^\dagger(x+\epsilon)\left[\psi_R(x+\epsilon),\ \psi_R^\dagger(x)\psi_R(x)\right] \rightarrow \overbrace{\left\{\psi_R(x+\epsilon),\ \psi_R^\dagger(x)\right\}}^{\delta(\epsilon)}\psi_R(x)$$

$$+ \left[\psi_R^\dagger(x+\epsilon),\ \psi_R^\dagger(x)\psi_R(x)\right]\psi_R(x+\epsilon) \rightarrow \underbrace{-\left\{\psi_R^\dagger(x+\epsilon),\ \psi_R(x)\right\}}_{-\delta(\epsilon)}\psi_R^\dagger(x)$$

$$= \left\{\psi_R^\dagger(x+\epsilon)\psi_R(x) - \psi_R^\dagger(x)\psi_R(x+\epsilon)\right\}\delta(\epsilon)$$

$$\psi_R^+(x+\epsilon)\,\psi_R(x) \sim \langle \psi_R^+(x+\epsilon)\,\psi_R(x) \rangle = \frac{i}{2\pi\epsilon}$$

$$[J_R(x'), J_R(x)] = \frac{i}{\pi}\frac{\partial}{\partial x'}\delta(x'-x)$$

by the same token,

$$\psi_L^+(x+\epsilon)\,\psi_L(x) \sim \langle \psi_L^+(x+\epsilon)\,\psi_L(x) \rangle = \frac{-i}{2\pi\epsilon}$$

$$[J_L(x'), J_L(x)] = -\frac{i}{\pi}\frac{\partial}{\partial x'}\delta(x'-x)$$

$$J_0 = J_R + J_L \,, \quad J = J_R - J_L$$

$$[J_0(x), J_0(x')] = 0 \,, \quad [J(x), J(x')] = 0$$

$$[J_0(x), J(x')] = + \frac{i}{\pi} \frac{\partial}{\partial x} \delta(x - x')$$

(Following more careful analysis)

Suggests the following bosonization

$$J(x) = \frac{1}{\sqrt{\pi}} \partial_x \phi, \qquad J_0(x) = \frac{1}{\sqrt{\pi}} \Pi,$$

$$[\Pi(x), \phi(x')] = -i\,\delta(x-x')$$

So that $$[J_0(x), J(x)] = +\frac{i}{\pi} \frac{\partial}{\partial x} \delta(x-x')$$

$$H_0 = \frac{V_F}{2} \int \left[\Pi(x)^2 + (\partial_x \phi(x))^2 \right]$$

$$J_R J_R + J_L J_L = \frac{1}{2}\left[J_0^2 + J^2\right]$$ forward scattering

$$2 J_R J_L = \frac{1}{2}\left[J_0^2 - J^2\right]$$ backward scattering

$$H_{I_1} = \int \frac{g_1}{2}\left[J_0^2 + J^2\right] \rightarrow \int \frac{g_1}{2\pi}\left[\Pi^2 + (\partial_x \phi)^2\right]$$

$$H_{I_2} = \int \frac{g_2}{2\pi}\left[\Pi^2 - (\partial_x \phi)^2\right]$$

$$H = \frac{v}{2}\int\left\{\frac{1}{K}\Pi^2 + K\left(\partial_x\phi\right)^2\right\}$$

$$K = \sqrt{\frac{v_F + \frac{g_1}{\pi} + \frac{g_2}{\pi}}{v_F + \frac{g_1}{\pi} - \frac{g_2}{\pi}}} \qquad v = \sqrt{\left(v_F + \frac{g_1}{\pi}\right)^2 - \left(\frac{g_2}{\pi}\right)^2}$$

"$K = 1$ when $g_2 = 0$, effectively free"

illustration of Mandelstam Construction