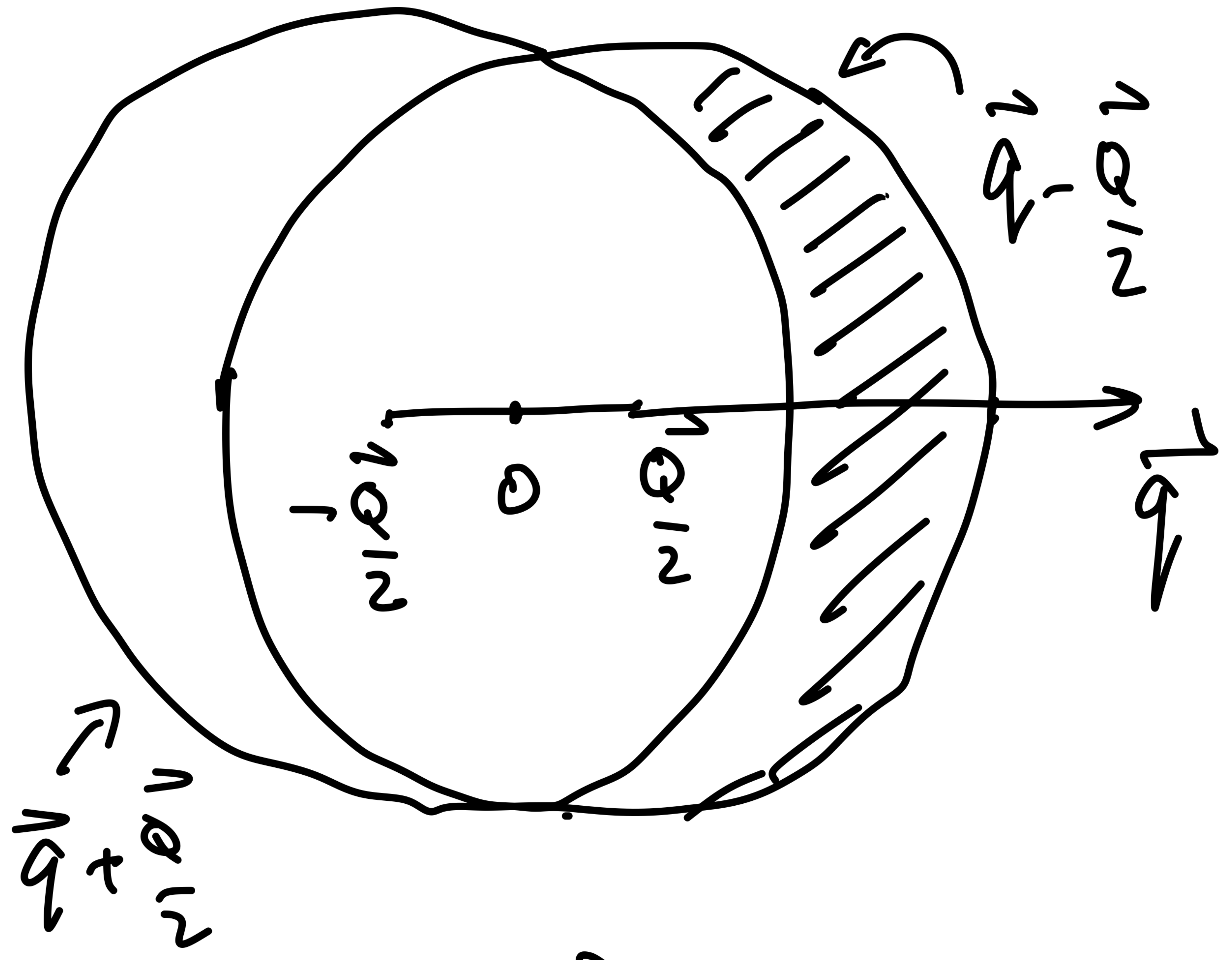
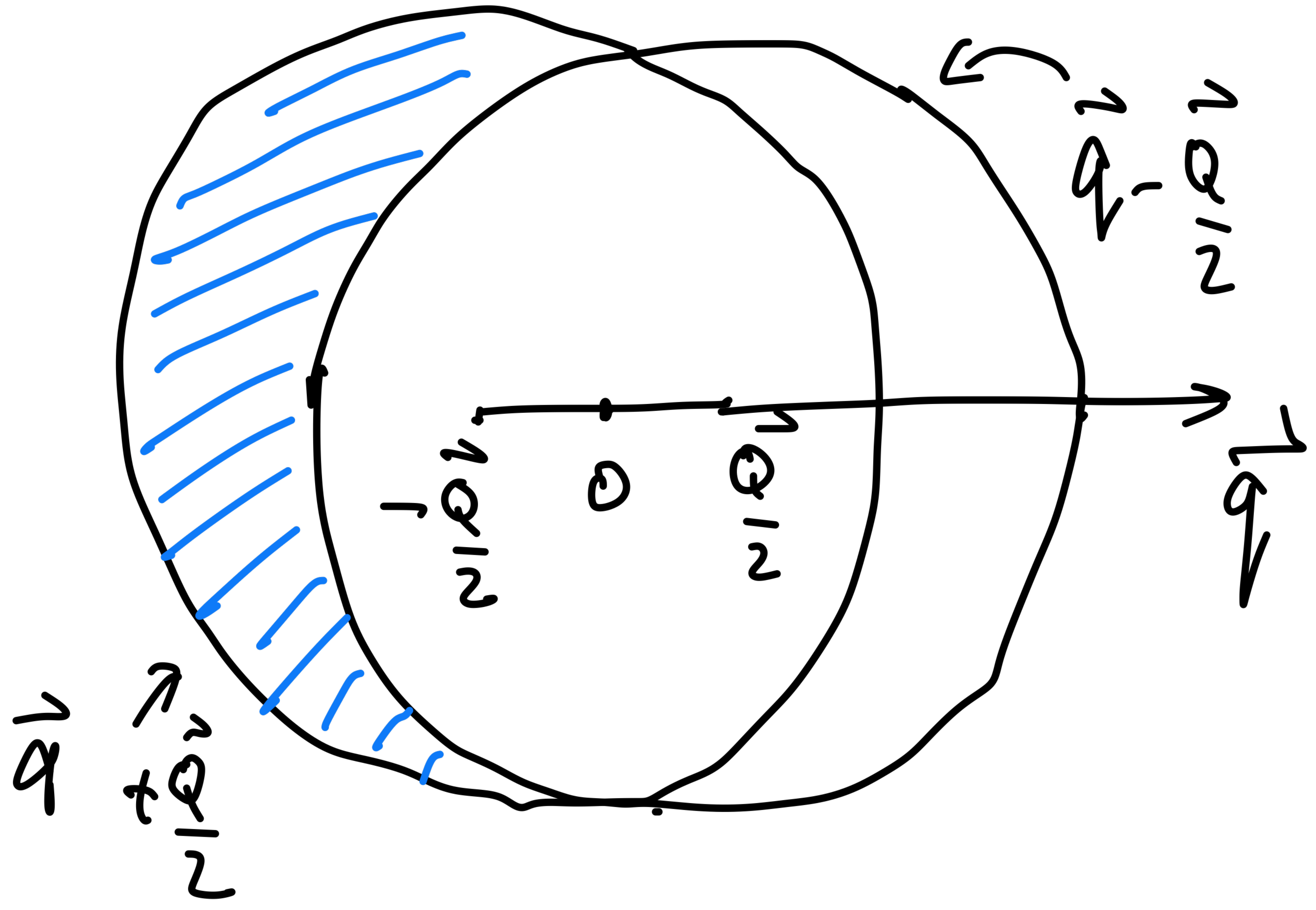
Phys529B: Topics of Quantum Theory

Lecture 17: introduction to 1D LL and bosonization

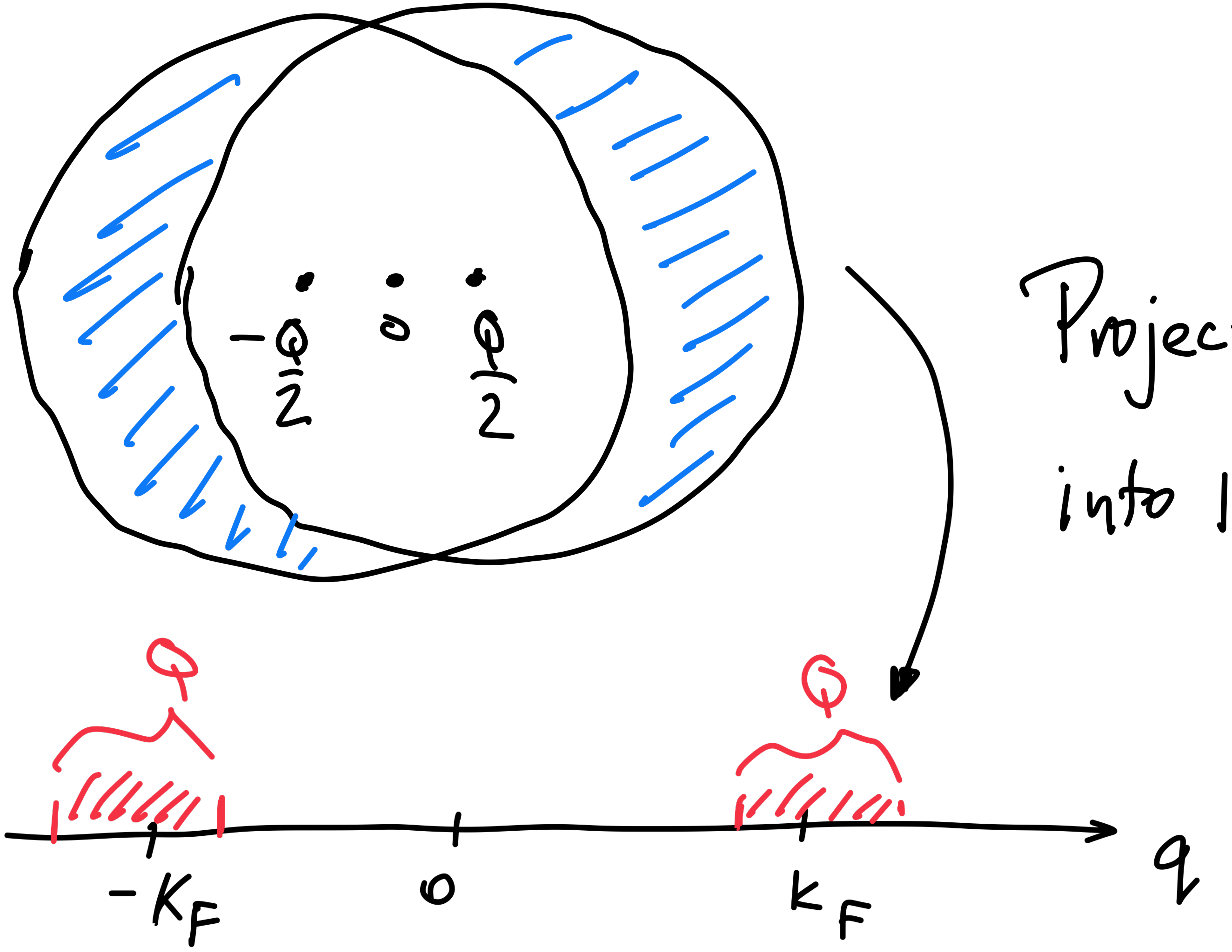
instructor: Fei Zhou



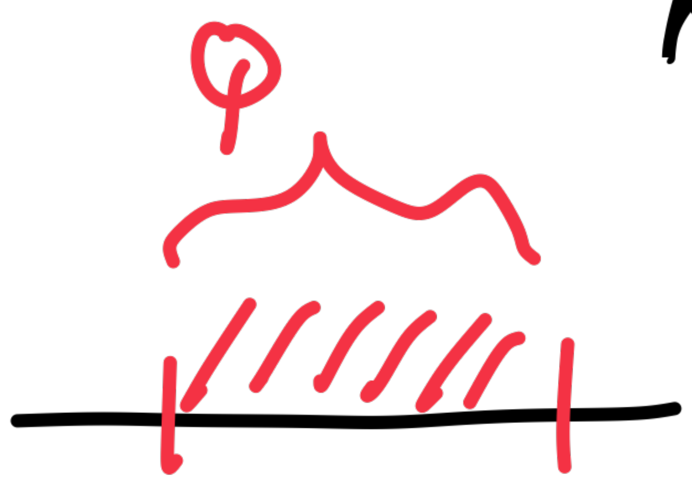


$$\frac{Q}{\Omega - v_F Q + i\delta} \qquad - \frac{Q}{\Omega + v_F Q - i\delta}$$

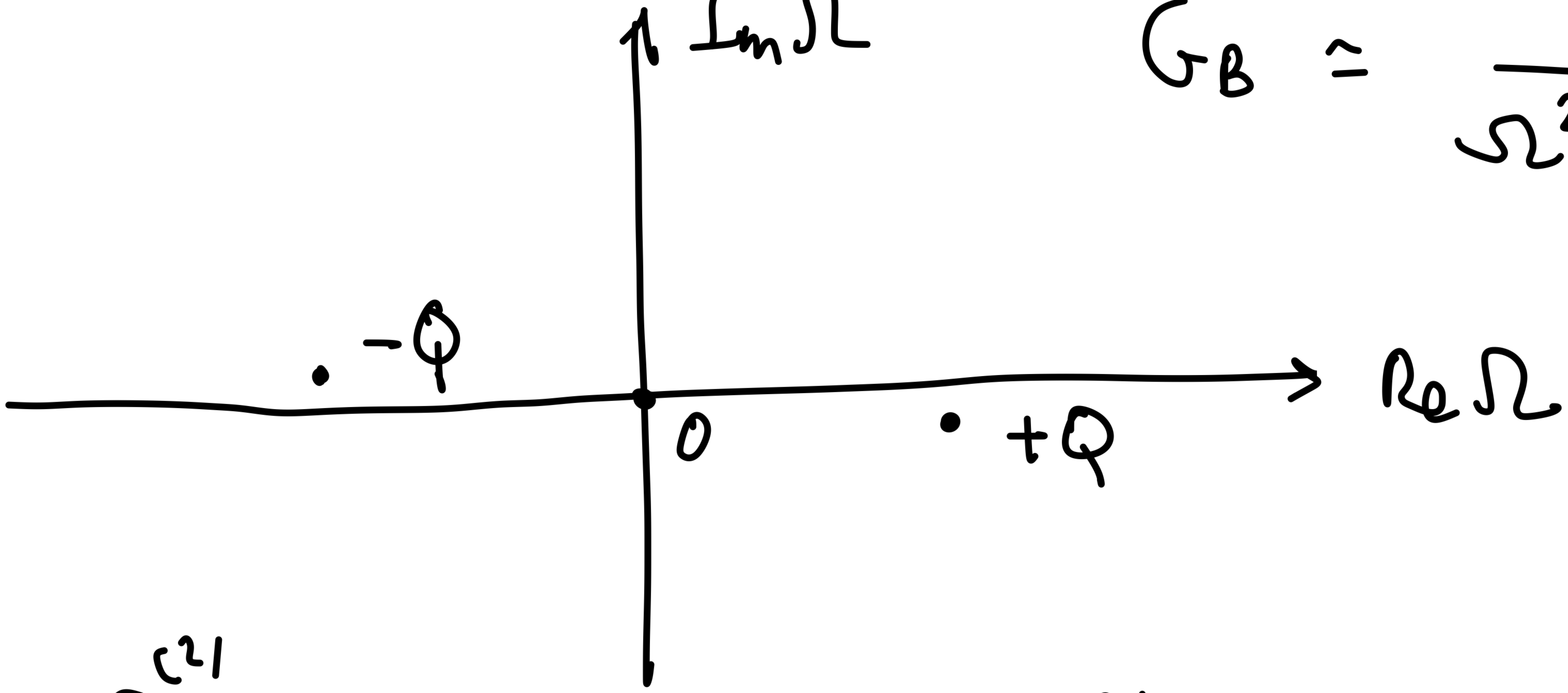
$$\text{Total} = \frac{Q^2}{\Omega^2 - Q^2}$$



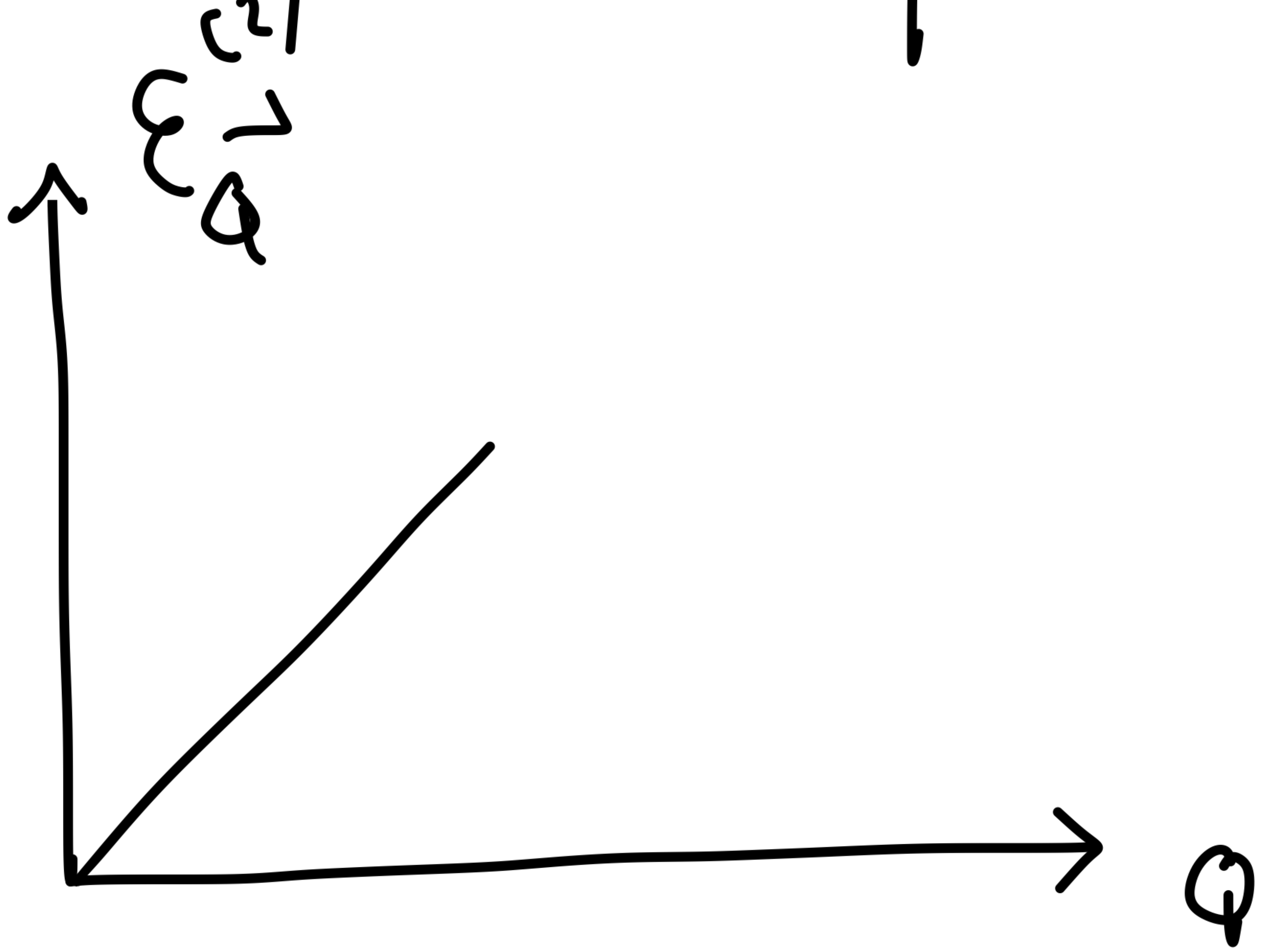
Projection into 1D



The momenta contributing to $G_B(\Omega, \vec{Q})$



$$G_B \simeq \frac{Q^2}{\Omega^2 - Q^2}$$



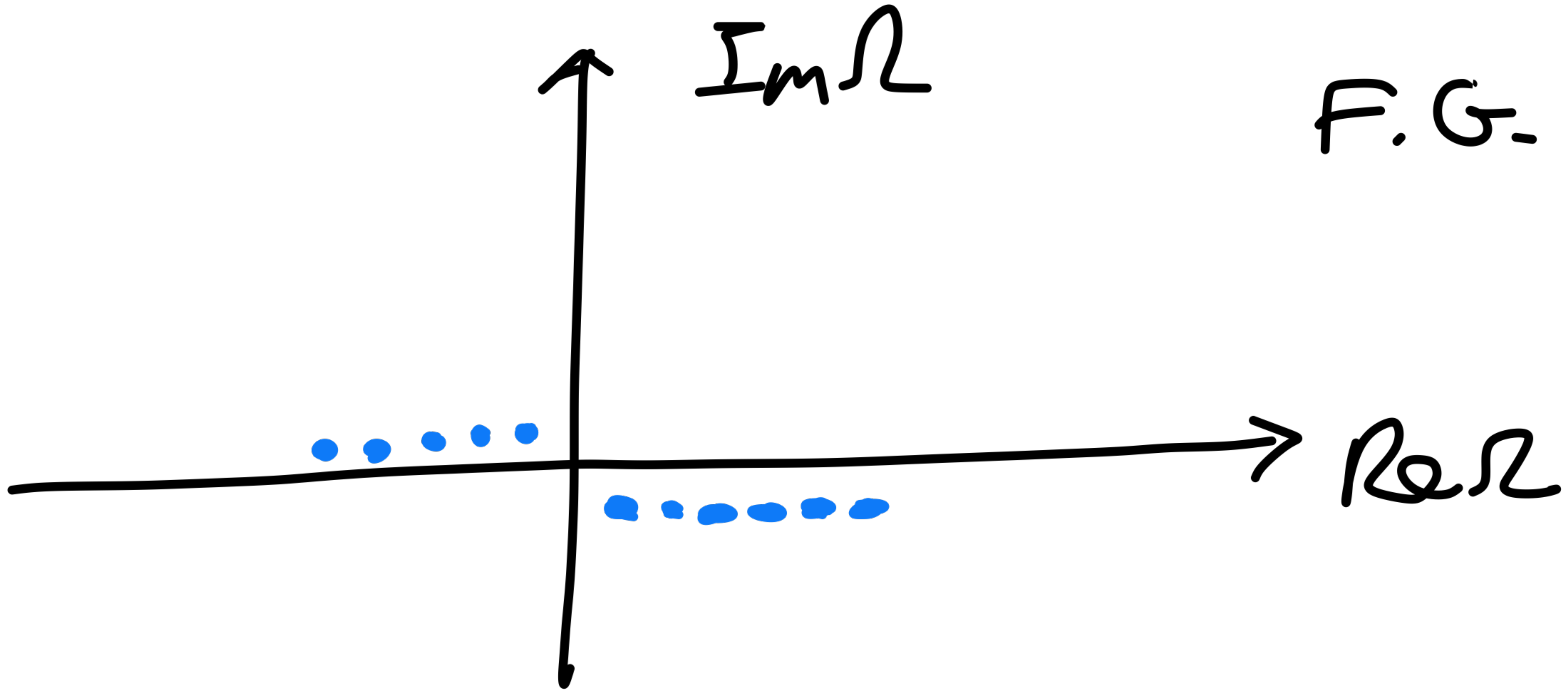
$$\xi^{(2)}_{\vec{Q}} = \xi_{\frac{\vec{Q}}{2}+\vec{q}} - \mu + \mu - \xi_{-\frac{\vec{Q}}{2}+\vec{q}}$$

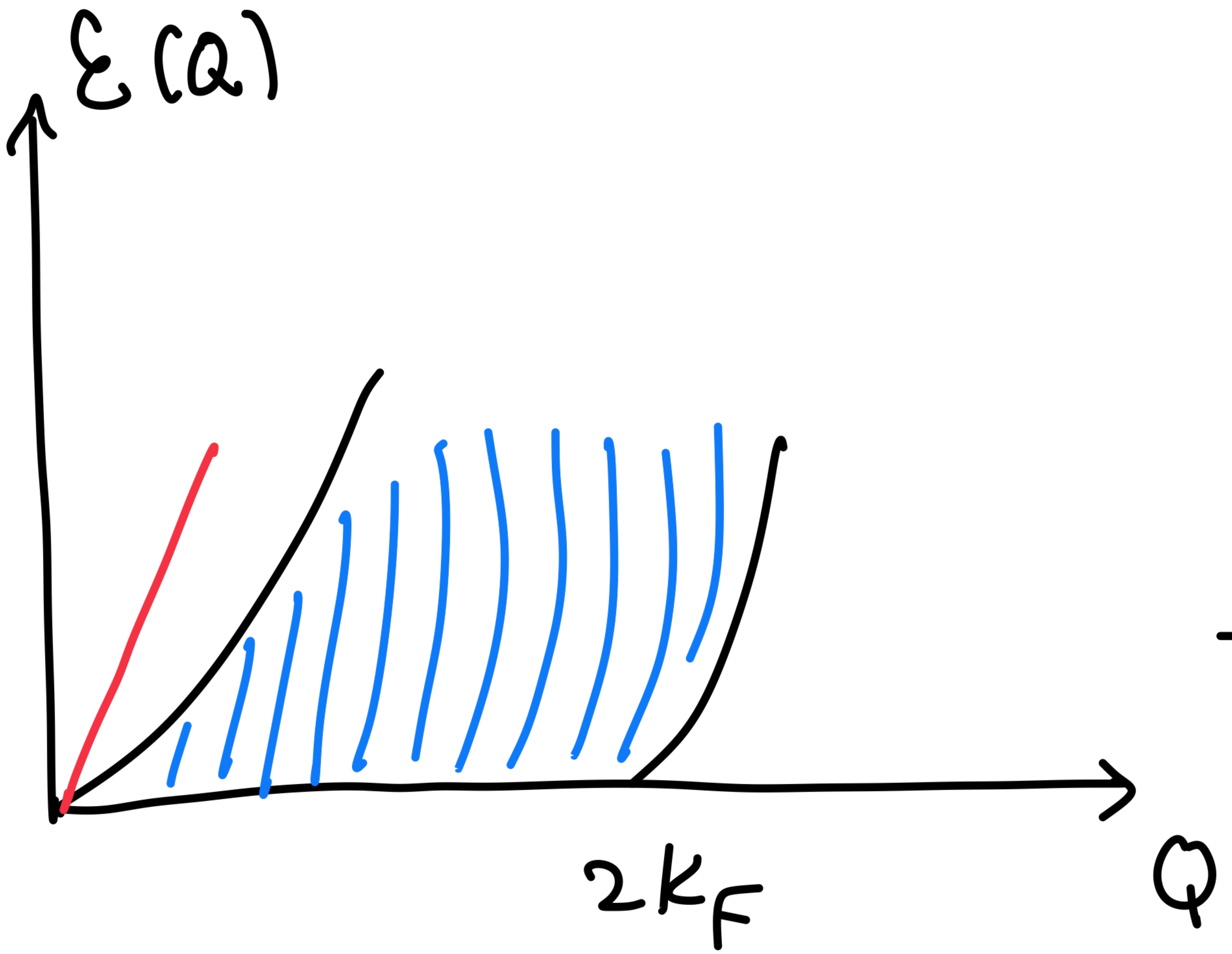
$$\simeq v_F Q$$

Comparison 3D

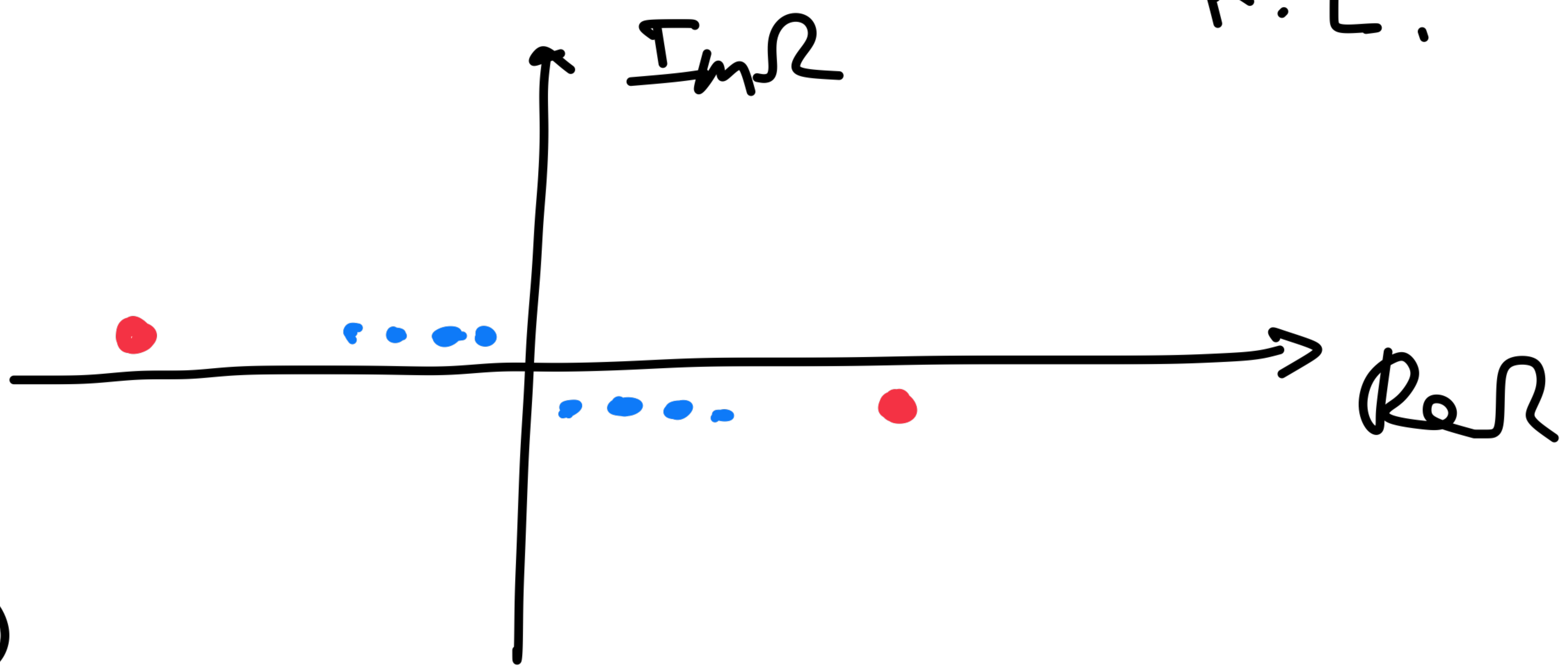
G_B

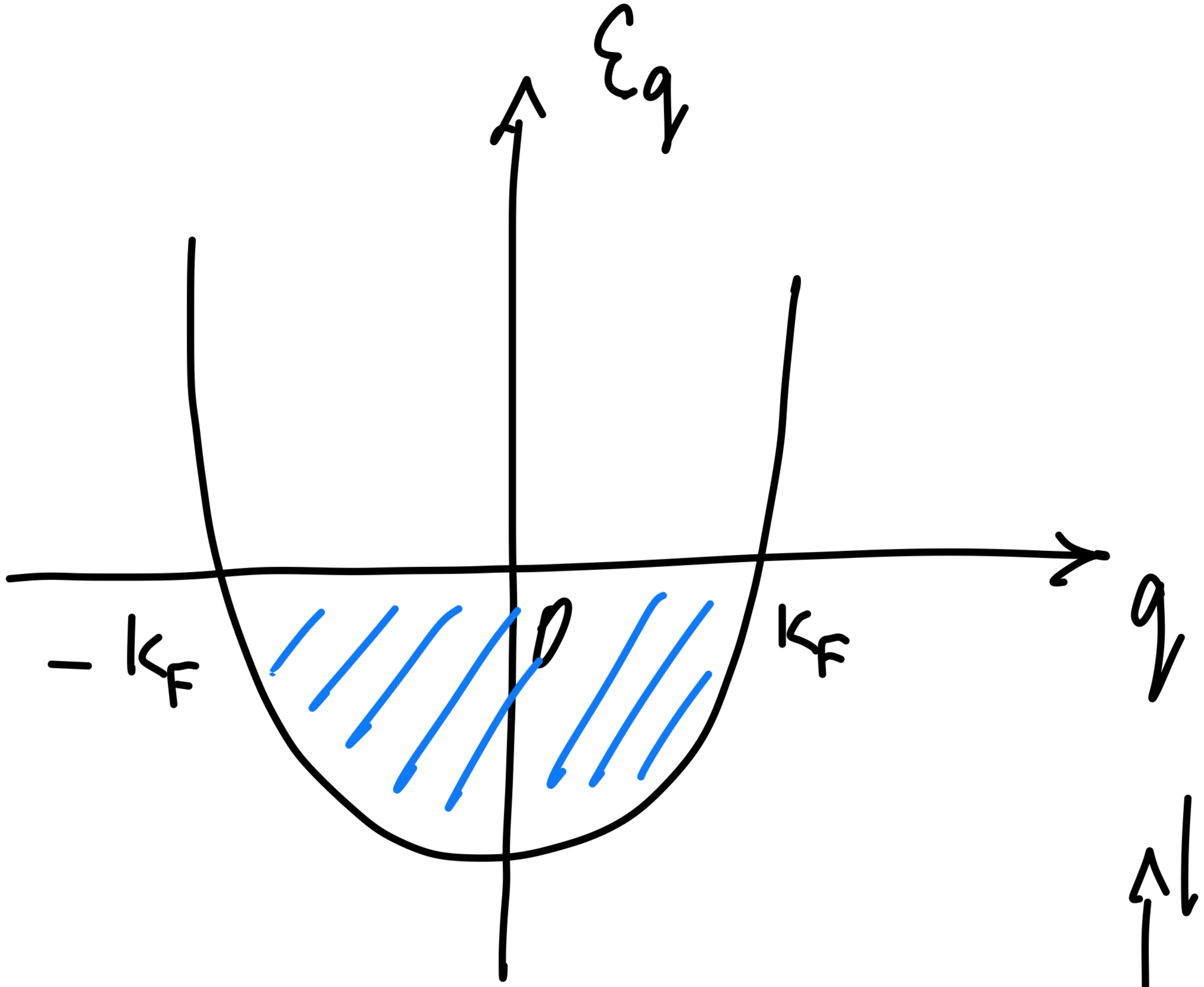
F.G.

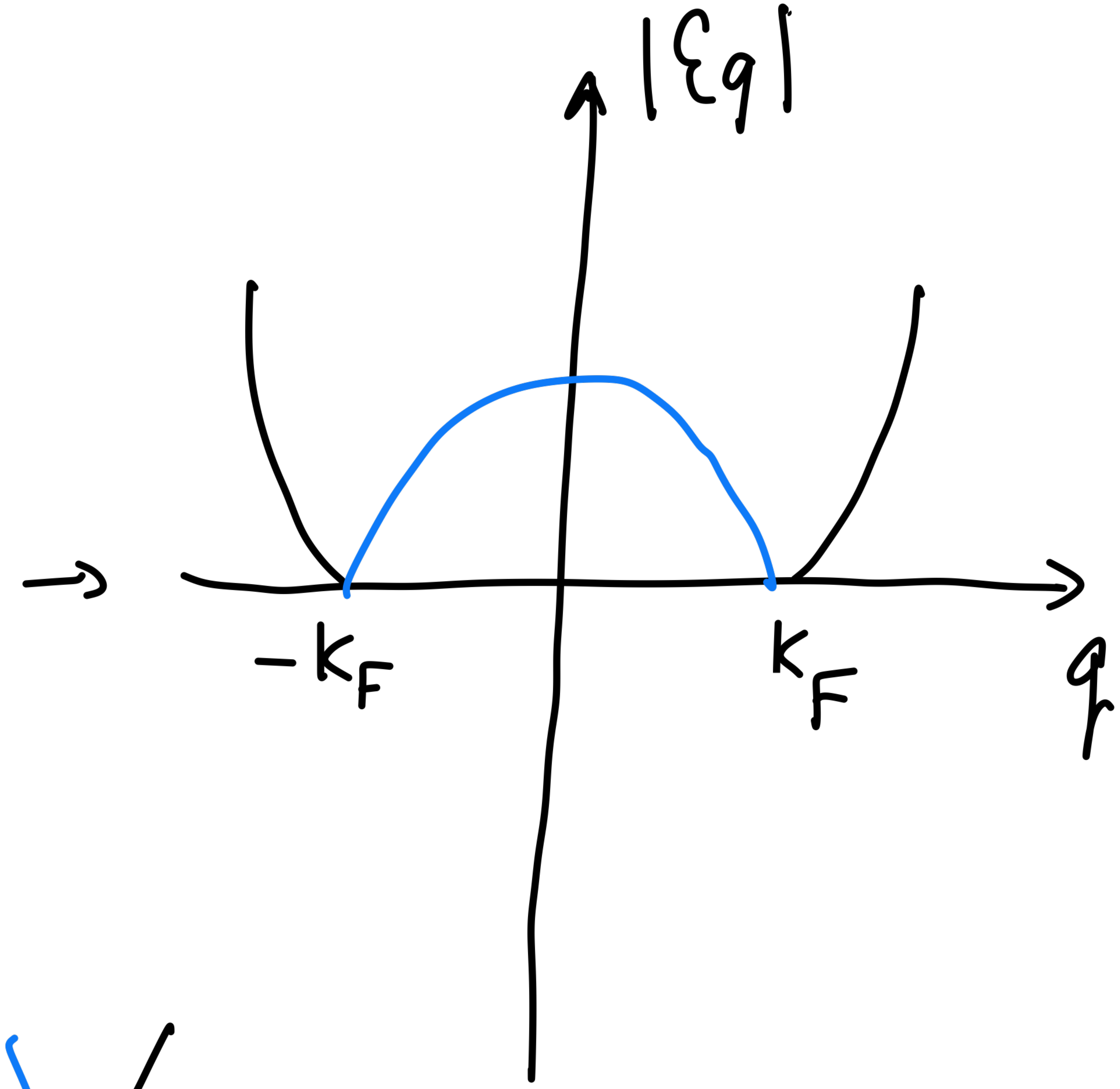


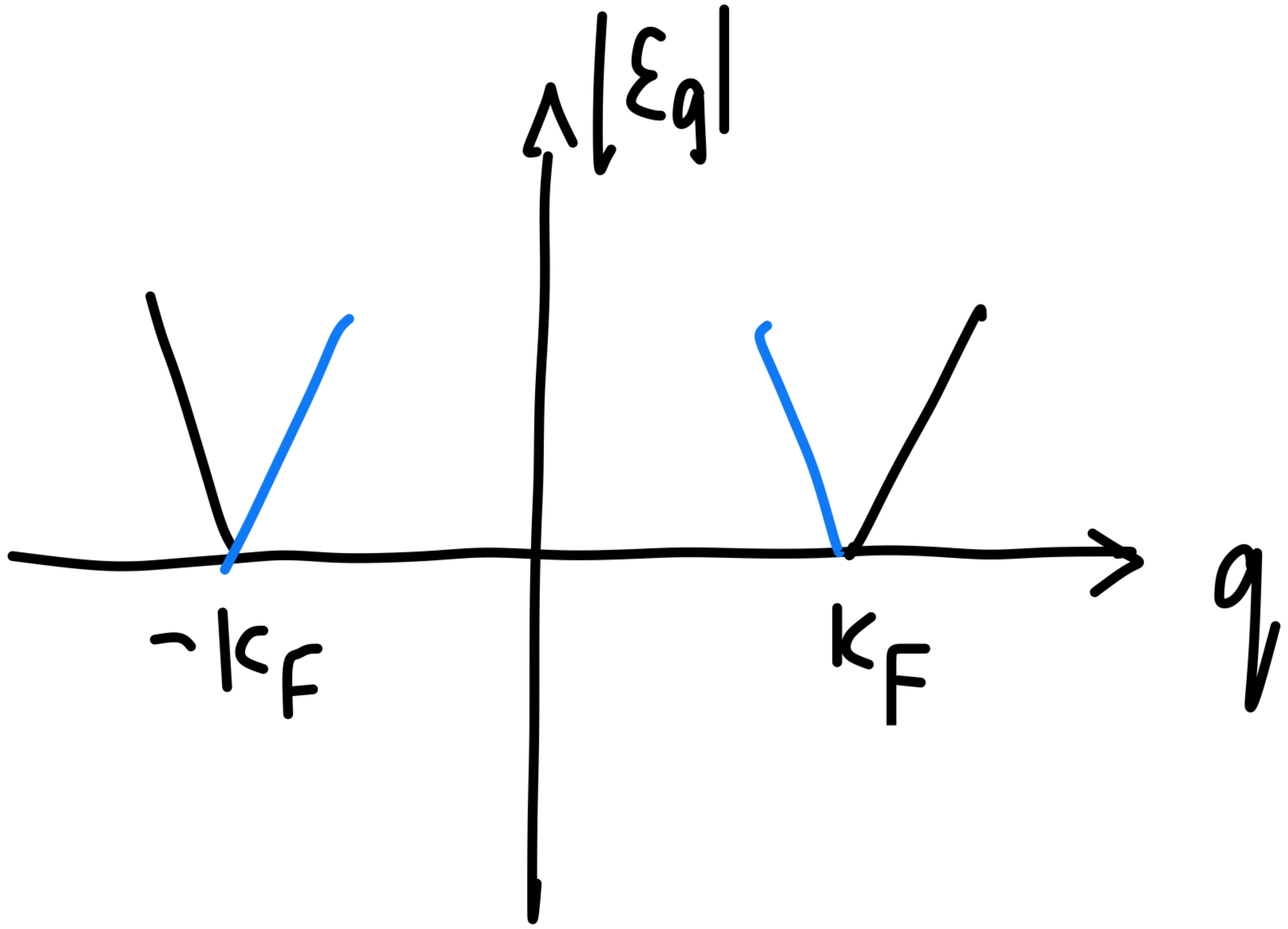


F.L.



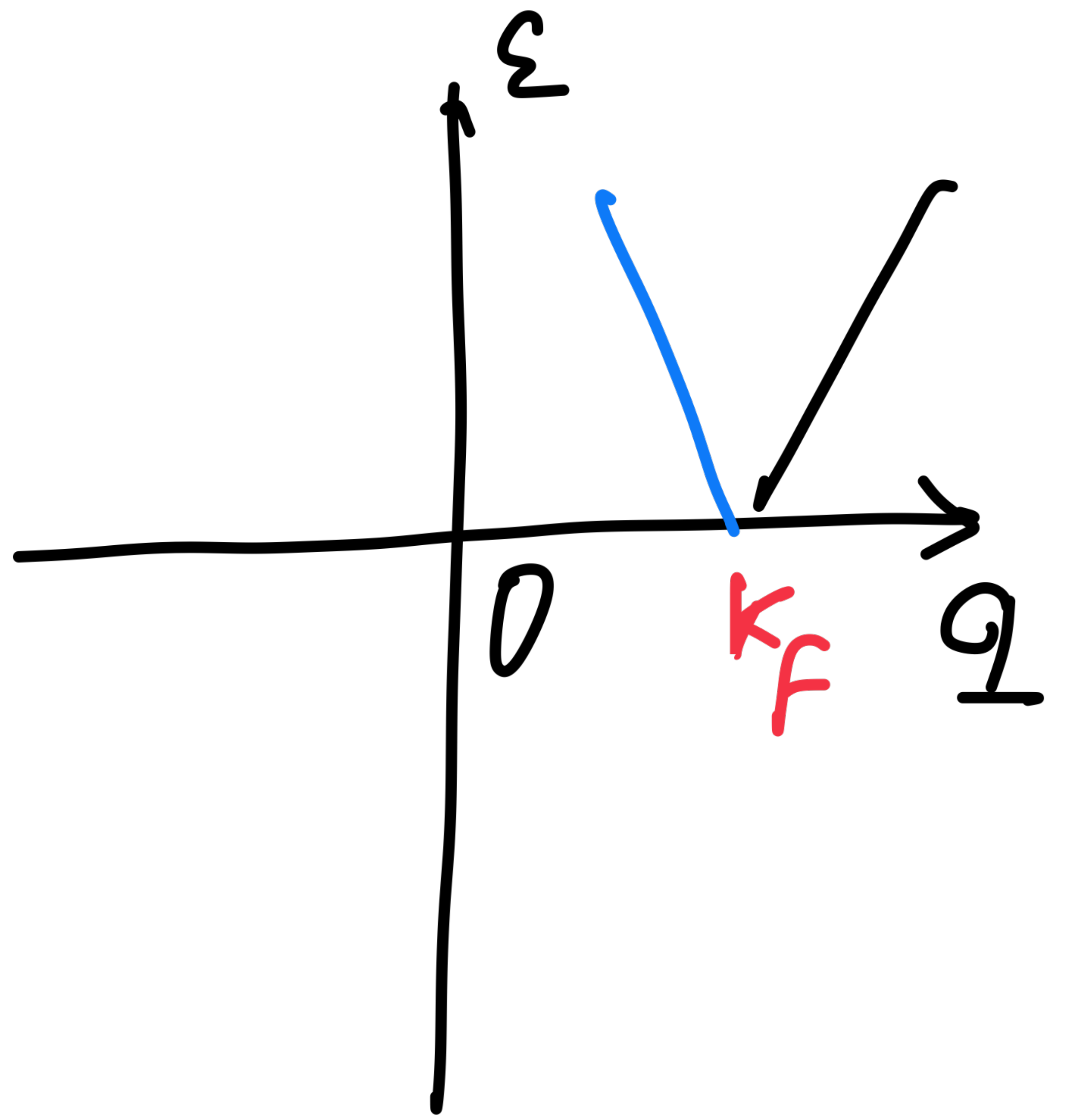

ε_q
q
$-k_F$
0
k_F


$|\varepsilon_q|$
$-k_F$
k_F
q


$|\varepsilon_q|$
$-k_F$
k_F
q

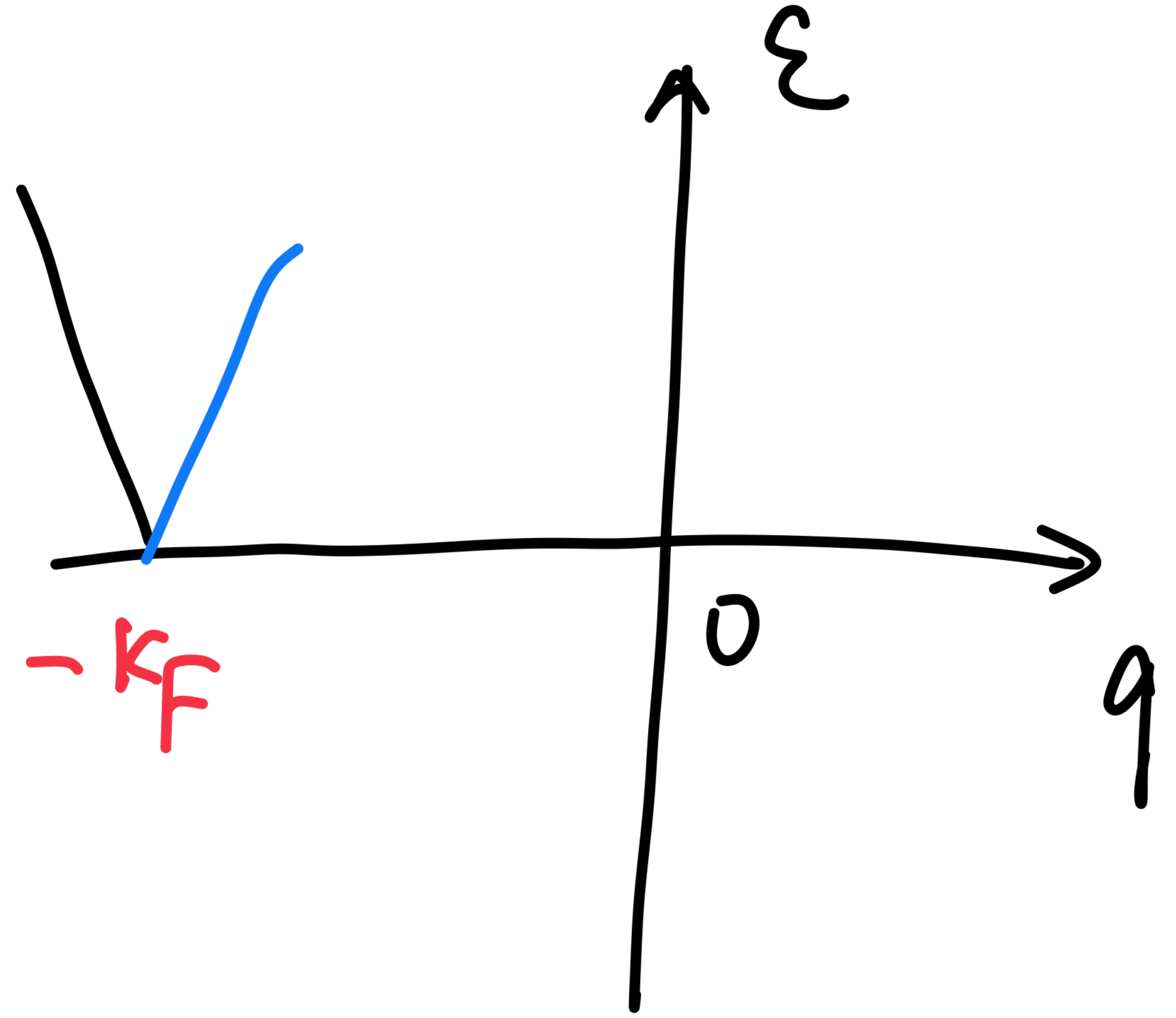
$$\varepsilon_R^{(2)} = v\cdot\left(\frac{Q}{2}+q\right) - v\cdot\left(-\frac{Q}{2}+q\right)$$

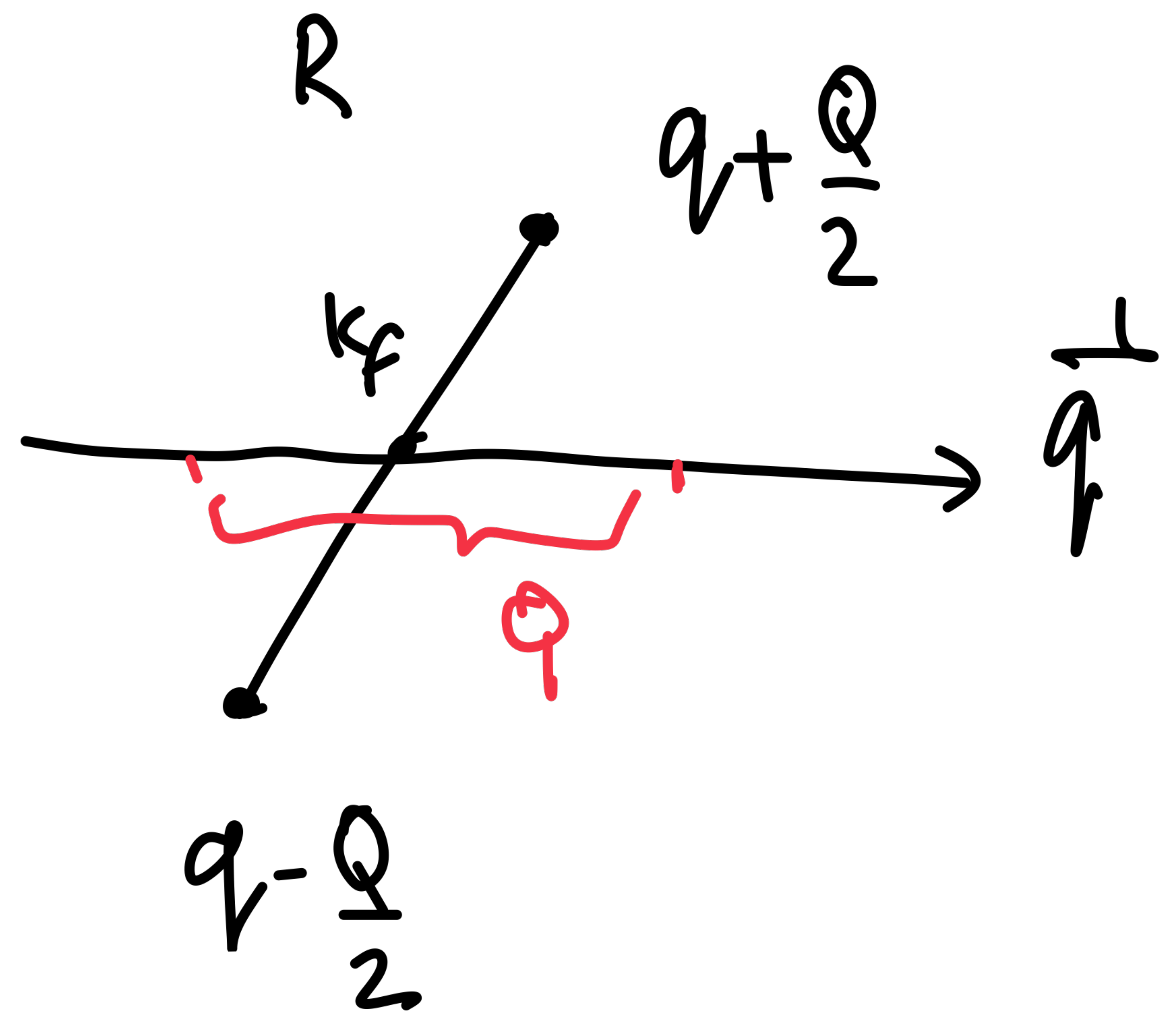
$$= v\cdot Q > 0$$



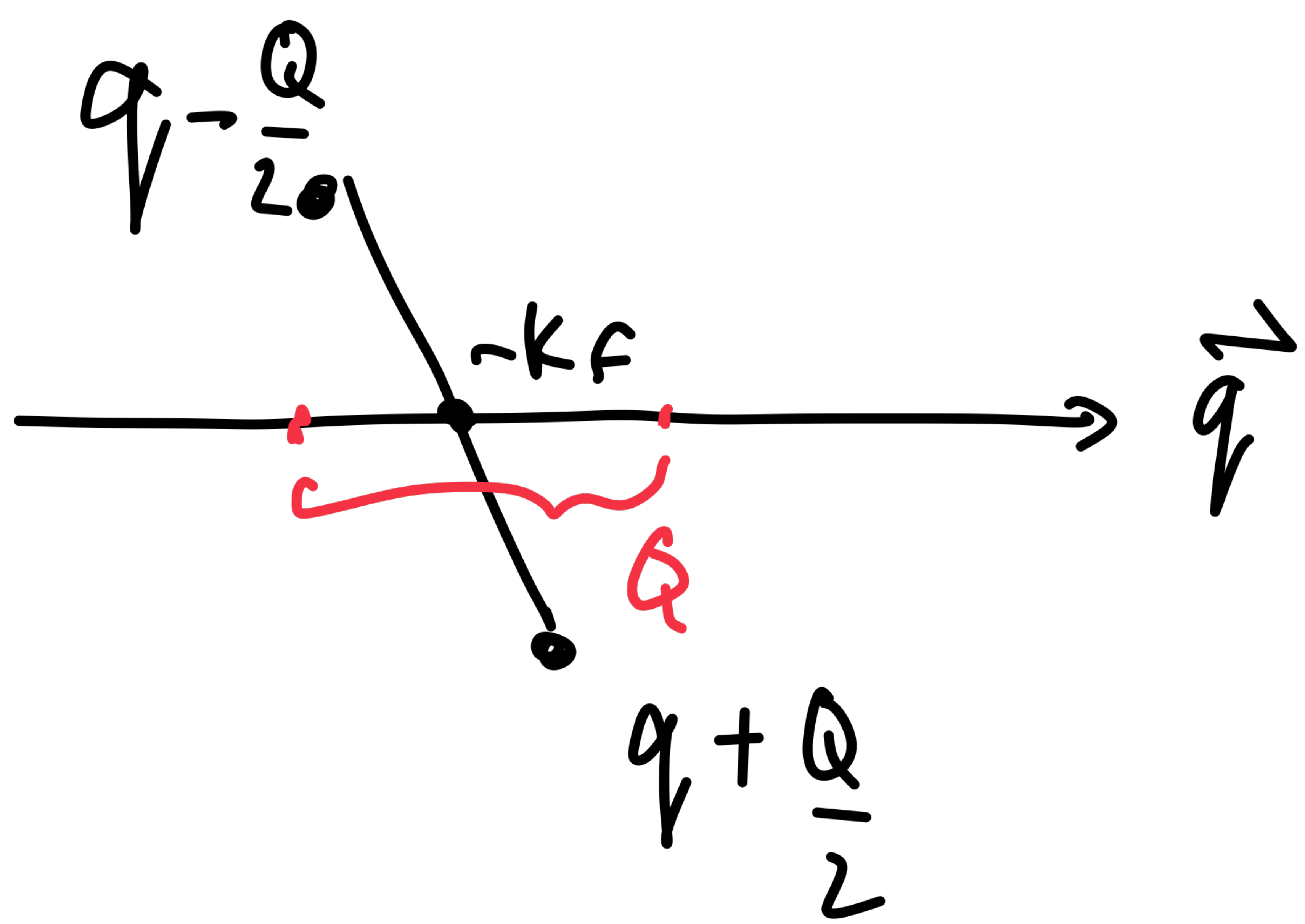
$$\varepsilon_L^{(2)} = -v\cdot\left(-\frac{Q}{2}+q\right) + v\cdot\left(\frac{Q}{2}+q\right)$$

$$= v\cdot Q > 0$$



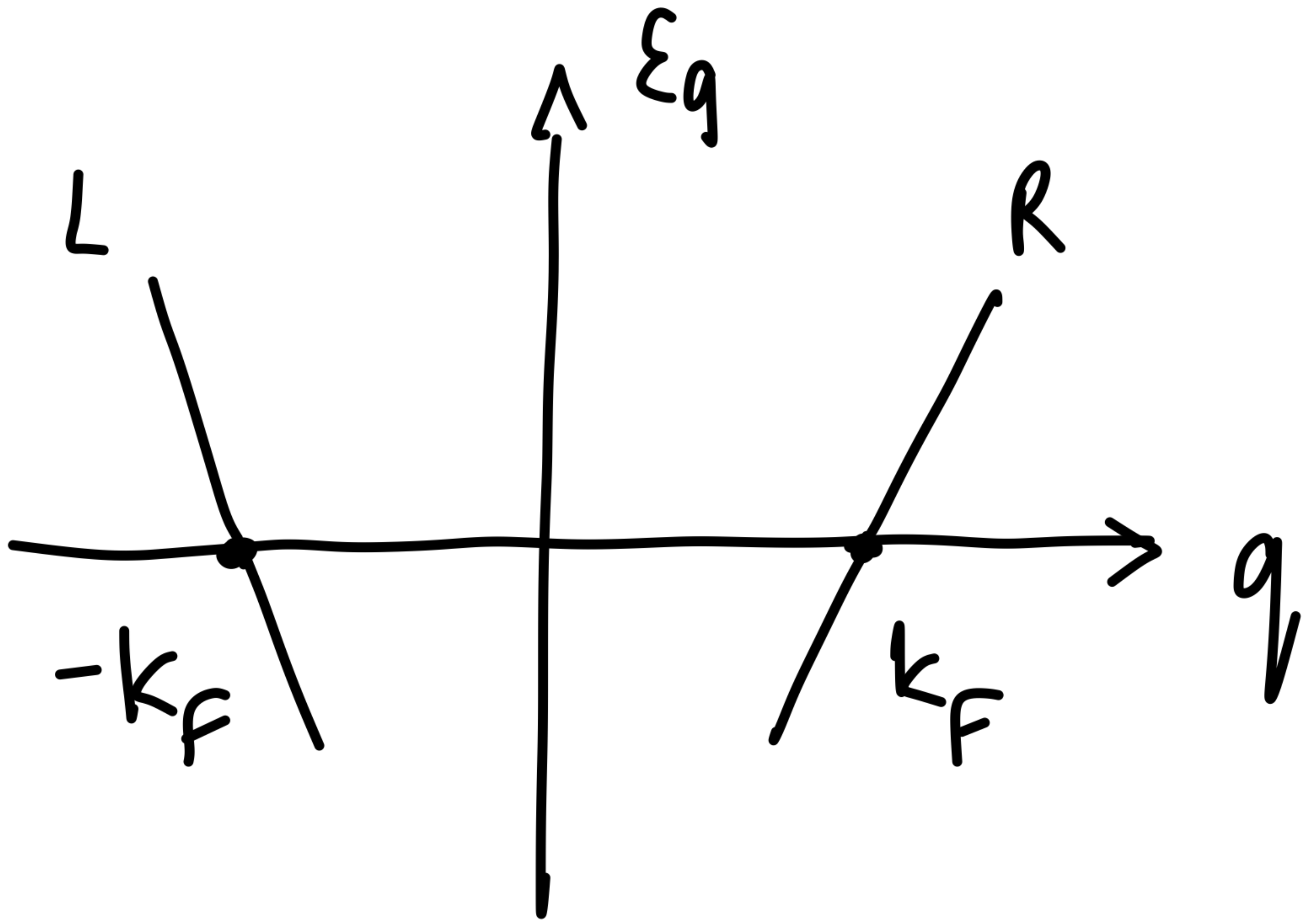


$$\mathcal{E}_R^{(2)}(Q) = u\,Q$$



$$\mathcal{E}_L^{(2)}(-Q) = (-Q)(-u) = uQ$$

Relations to Dirac fermions Via Trucation



$$\psi(x) = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\,\psi_k$$

$$= e^{ik_f x}\,\psi_R(\vec{x}) + e^{-ik_f x}\,\psi_L(\vec{x})$$

$$\sum_k e^{i\vec{k}\cdot\vec{x}}\,\psi_k \simeq \sum_{k,R} e^{ik_f x + i\vec{k}\cdot\vec{x}}\,\psi_{kR} + \sum_{k,L} e^{-ik_f x + i\vec{k}\cdot\vec{x}}\,\psi_{kL}$$

$$H_0 = \int \left(\psi_R^\dagger , \psi_L^\dagger \right) \mathcal{H}_0 \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} d\vec{x}$$

$$\mathcal{H}_0 = \begin{bmatrix} -i\partial_x & 0 \\ 0 & +i\partial_x \end{bmatrix} = \sigma_z \otimes \mathbb{1} \left(-i\partial_x \right)$$

$\mathbb{1} = 1$ for Spinless case

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ for Spinful case.

Interactions

$$H_I = g_1 \int \left[\psi_R^\dagger \psi_R \psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L \psi_L^\dagger \psi_L \right] \text{ - forward Scattering}$$

$$+ g_2 \int \psi_R^\dagger \psi_R \psi_L^\dagger \psi_L \qquad \text{- back Scattering}$$

$$g_1 = V(q \approx 0), \qquad g_2 = V(0) - V(2k_F)$$

if $H_I = \int \psi^\dagger(\vec{x}) \psi(\vec{x}) V(\vec{x} - \vec{x}') \psi^\dagger(\vec{x}') \psi(\vec{x}') \, d^d\vec{x} \, d^d\vec{x}'$

Note that

$$\rho(x) = \psi_R^\dagger(x) \psi_R(x) + \psi_L^\dagger(x) \psi_L(x)$$

$$+ \psi_R^\dagger(x) \psi_L(x) e^{-i2k_F x} + \psi_L^\dagger(x) \psi_R(x) e^{i2k_F x}$$

Useful algebraic Relations for current algebras

$$[AB, C] = A[B, C] + [A, C]B$$

$$[A, BC] = \{A, B\}C - B\{A, C\}$$

$$J_0 = J_R + J_L \,, \quad J = J_R - J_L$$

$$[J_0(x), J_0(x')] = 0 \,, \quad [J(x), J(x')] = 0$$

$$[J_0(x), J(x')] = + \frac{i}{\pi} \frac{\partial}{\partial x} \delta(x - x')$$

(Following more careful analysis)

Suggests the following bosonization

$$J(x) = \frac{1}{\sqrt{\pi}} \partial_x \phi, \qquad J_0(x) = \frac{1}{\sqrt{\pi}} \Pi,$$

$$[\Pi(x), \phi(x')] = -i\,\delta(x-x')$$

So that $$[J_0(x), J(x)] = +\frac{i}{\pi} \frac{\partial}{\partial x} \delta(x-x')$$

$$H_0 = \frac{v_F}{2} \int \left[\Pi(x)^2 + (\partial_x \phi(x))^2 \right]$$