Phys529B: Topics of Quantum Theory

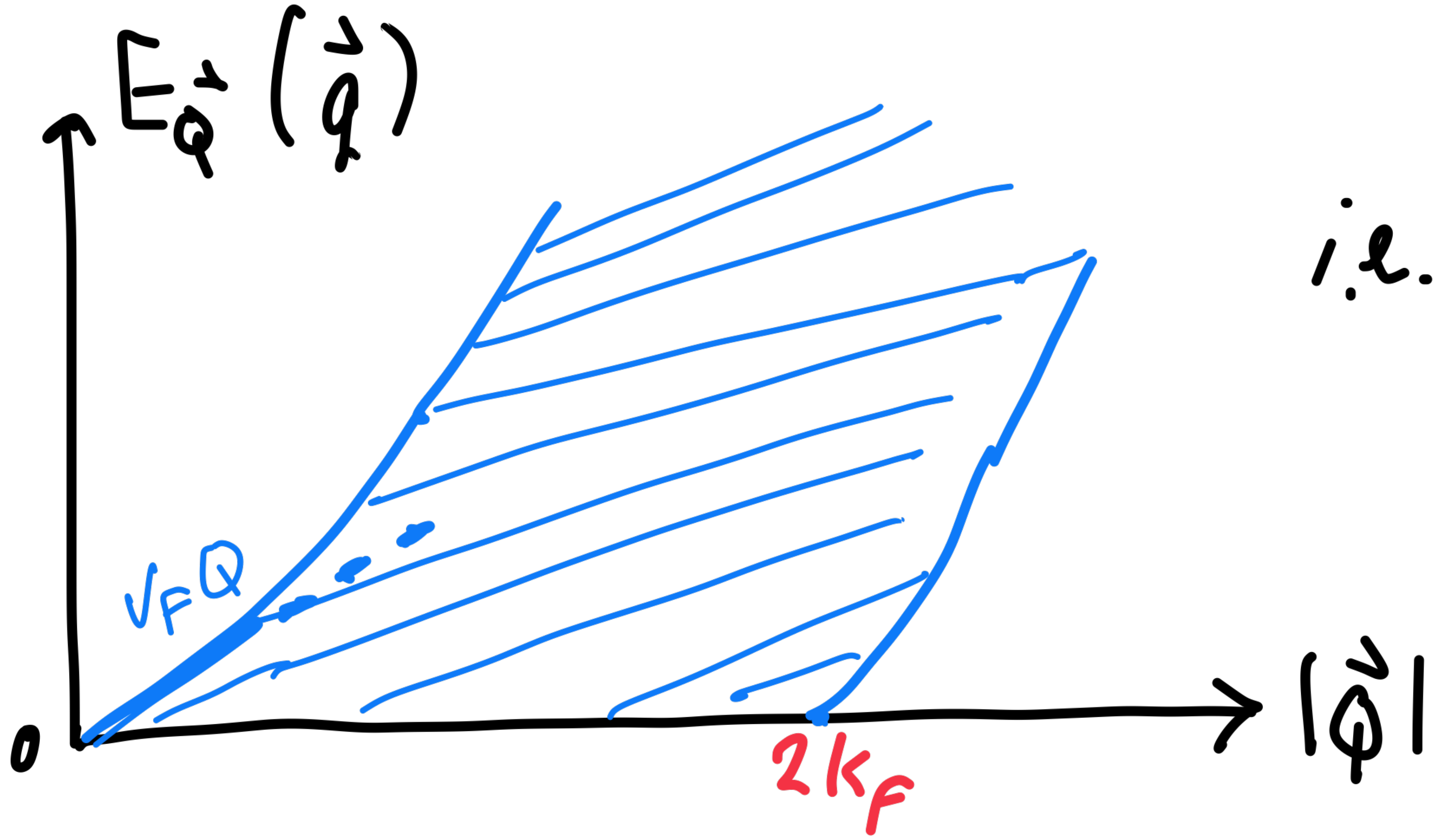
Lecture 16: introduction to 1D Fermi Gas and NFL

instructor: Fei Zhou

- Fermi liquid is a well defined phase in high dimensions; however, Generically, FL is not a phase in 1D; instead for interacting fermions, they generically form a phase of CFT liquids.

- Main differences between 1D and High D
- 1) Bosonic excitations are elementary ones with or without interactions.
- 2) All low energy excitations can be completely described by bosons leaving no Fermion-like excitations , I.e. Non-Fermi Liquid. This further leads to the idea of bosonization and/or CFT liquids.

"F. G."

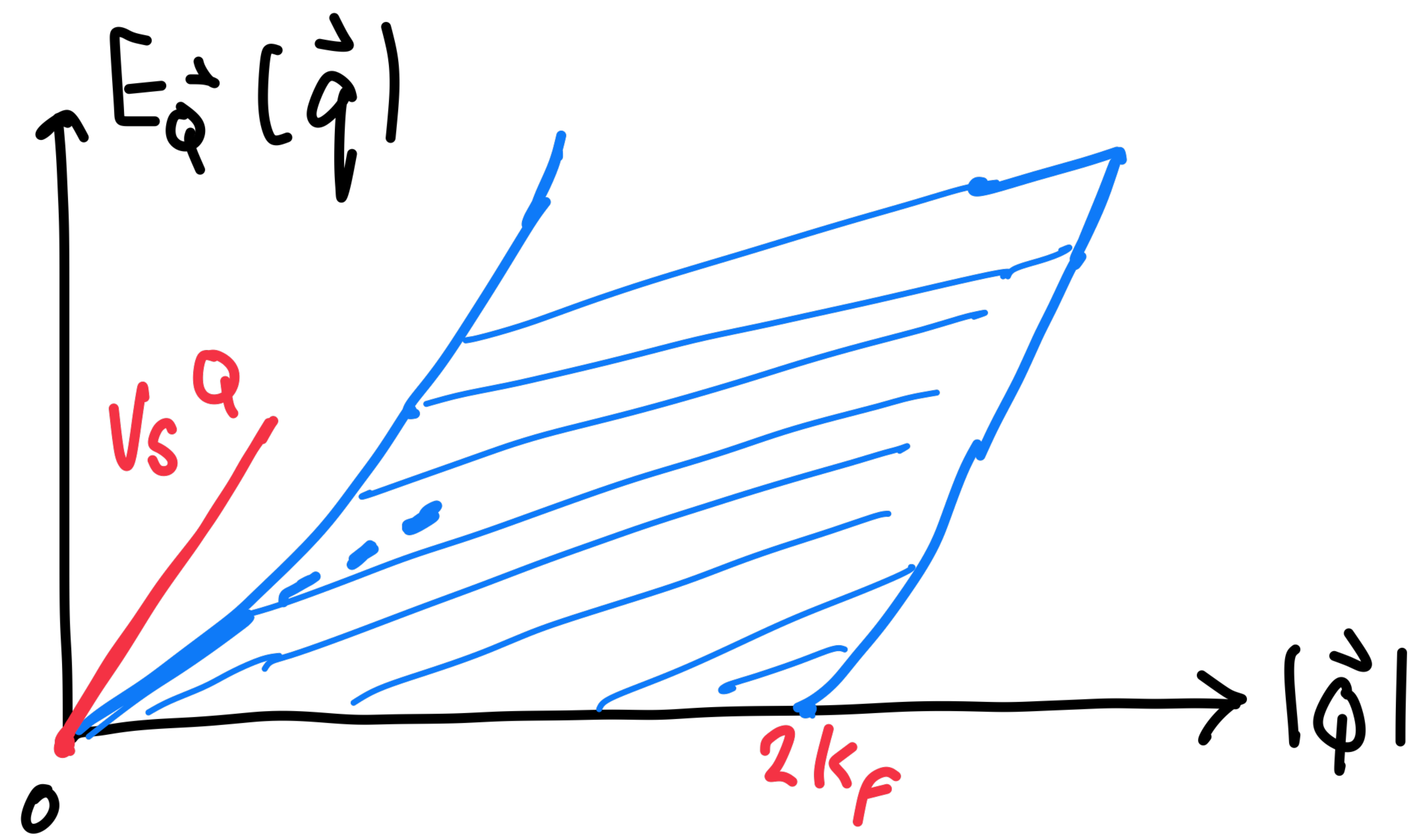


Continuum for bosonic states

i.e. No bosonic "particle"s in F. G.

In F. G., $G_B(\vec{Q}, \Omega)$ doesn't have isolated poles.

"F. L"



In F. L., $G_B(\vec{Q}, \Omega)$ has simple isolated poles, i.e. emergent bosonic fields.

("zero sound")

$v_S = v_S(v_F, \tilde{g}) > v_F$

Composite bosonic states as a continuum

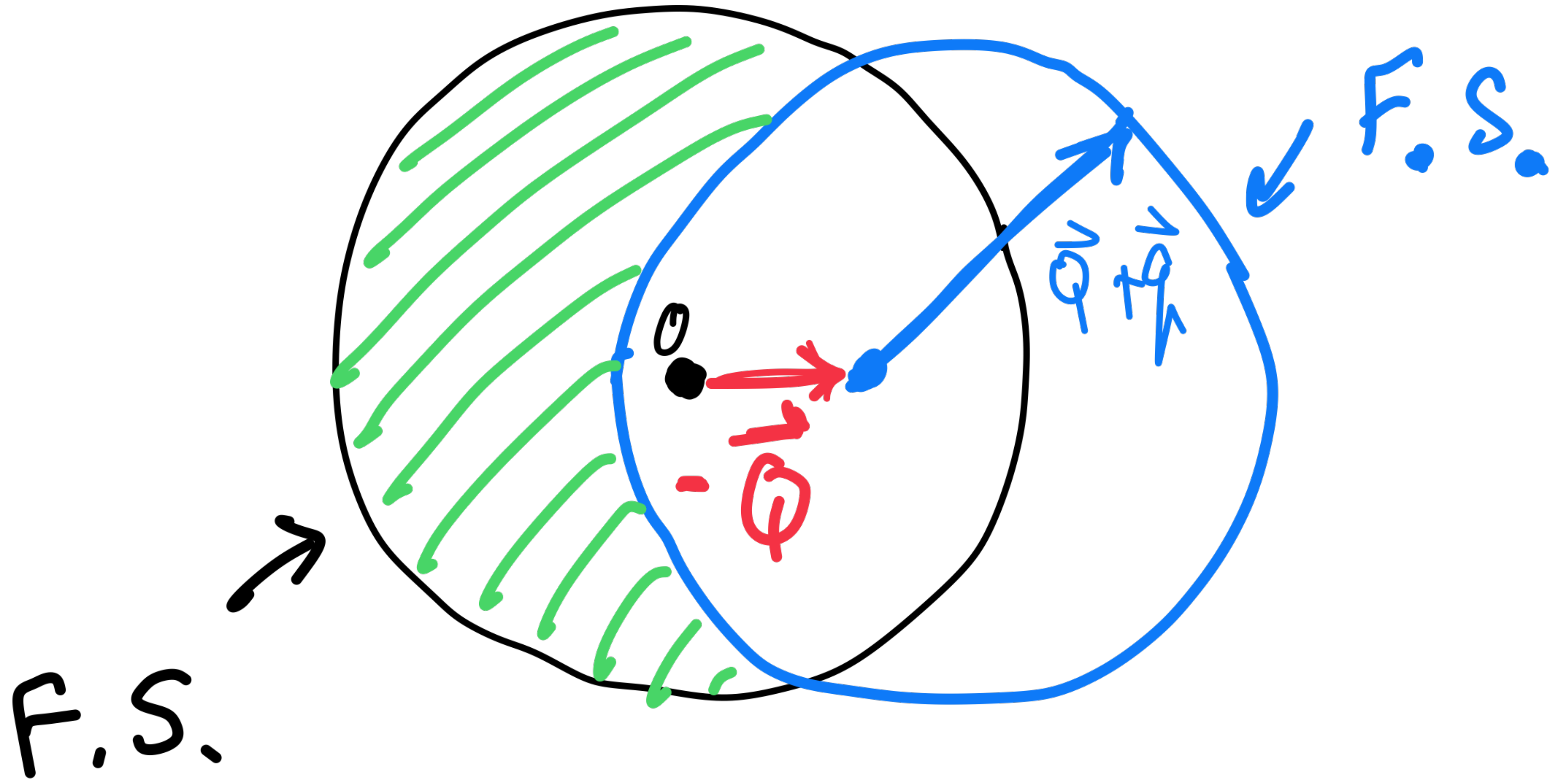
$$\phi^{\dagger}_{Q} = \sum_{\vec{q}} A_{Q,\vec{q}} \, \psi^{\dagger}_{\vec{Q}+\vec{q}} \, \psi_{\vec{q}} \qquad (\text{“}Q=0 \text{ Sector”})$$

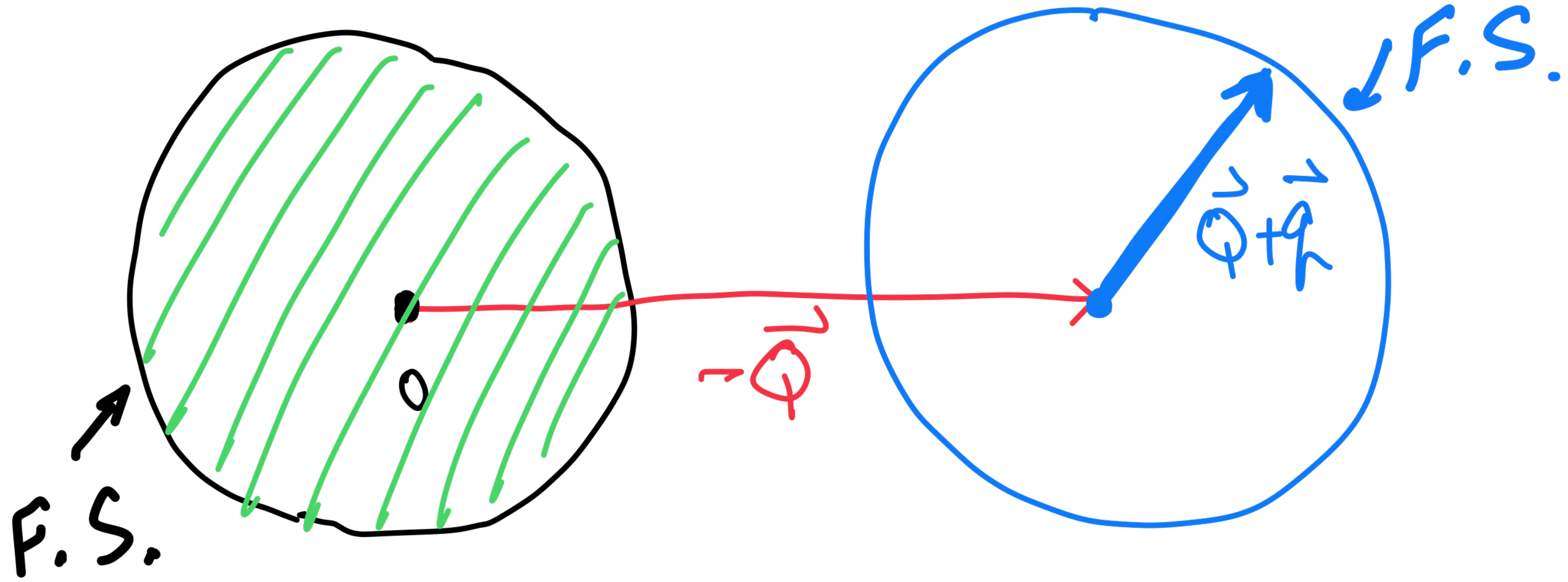
charge neutral sector

$$|\vec{q}| < k_F, \quad |\vec{Q}+\vec{q}| > k_F$$

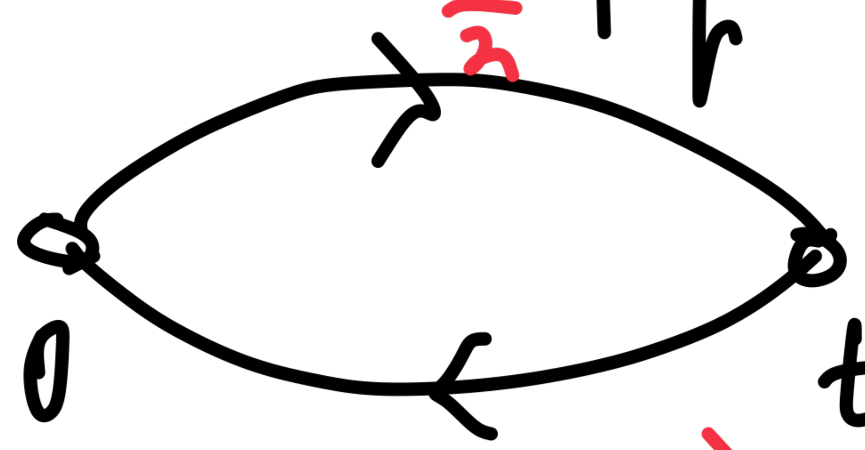
$|\vec{Q}| < 2k_F$

$|\vec{Q}| > 2k_F$

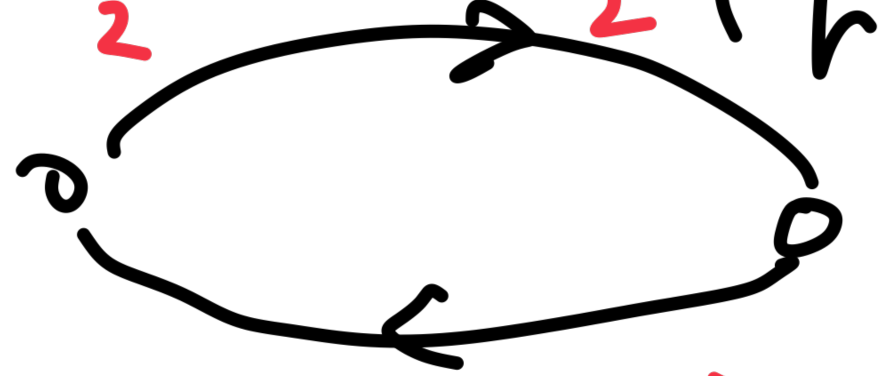


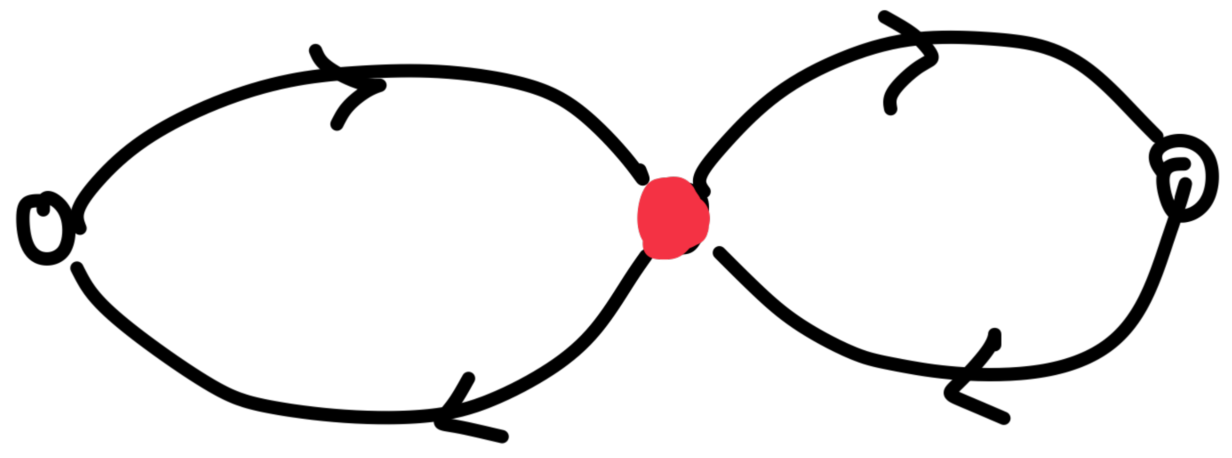
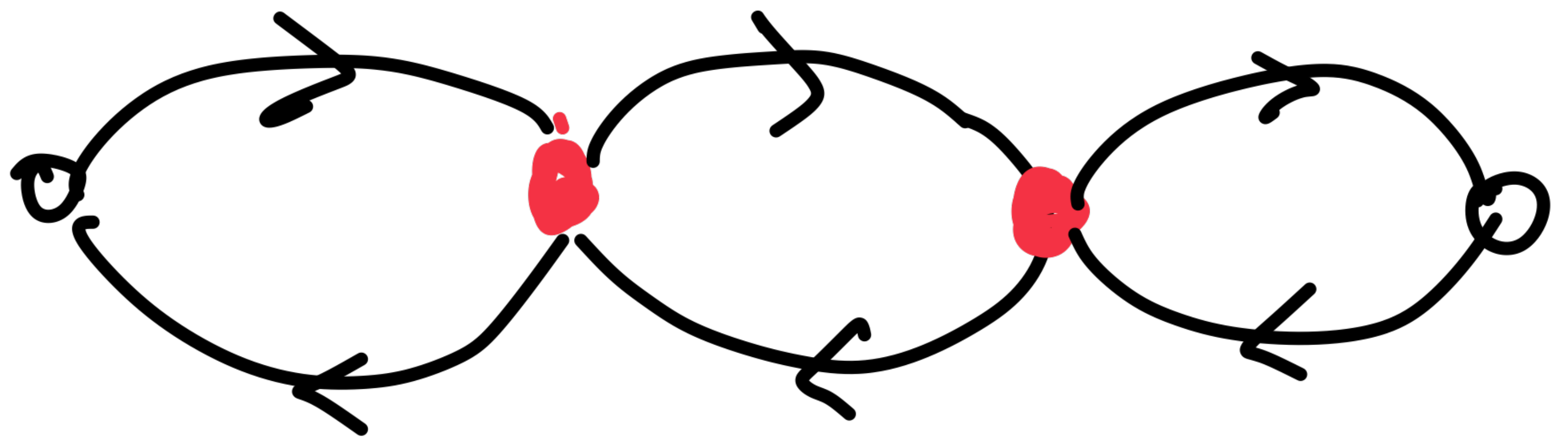


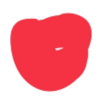
$$G_B(\vec{Q}, t) = \langle 0| -i\, T\, \phi_{\vec{Q}}(t)\, \phi^{\dagger}_{\vec{Q}}(0) |0\rangle, \quad \phi^{\dagger}_{\vec{Q}} = \sum_{\vec{q}} \psi^{\dagger}_{\frac{\vec{Q}}{2}+\vec{q}}\, \psi_{\vec{q}-\frac{\vec{Q}}{2}}$$

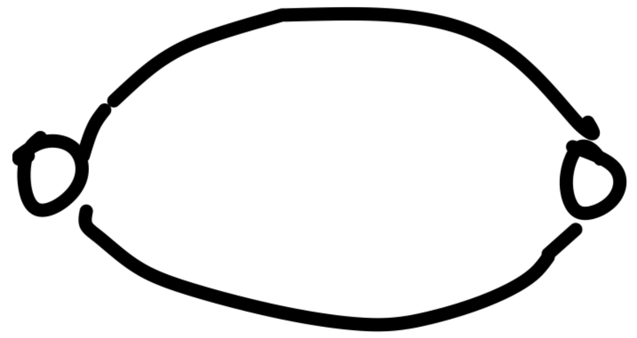


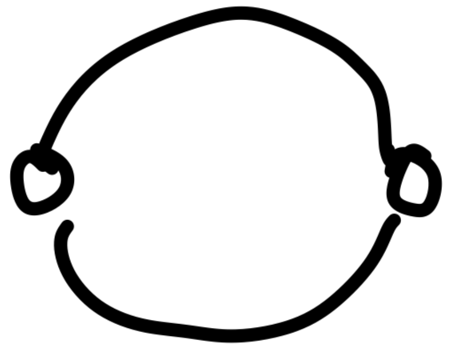
$\longrightarrow G_B(\vec{Q}, \Omega)$

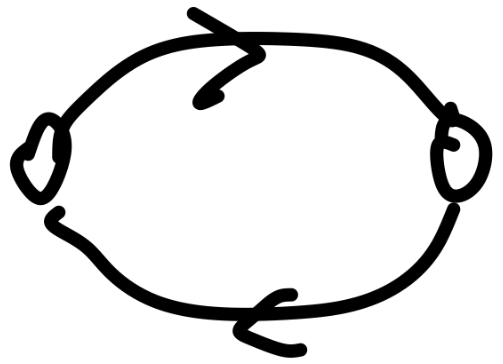
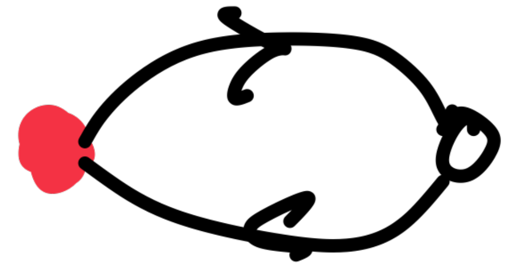


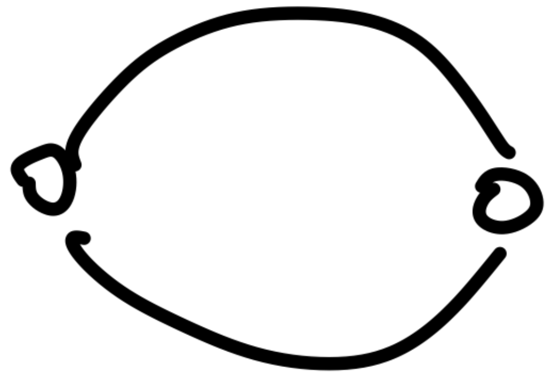
$+$  $+$  $+ \cdots$

 : interaction "g"

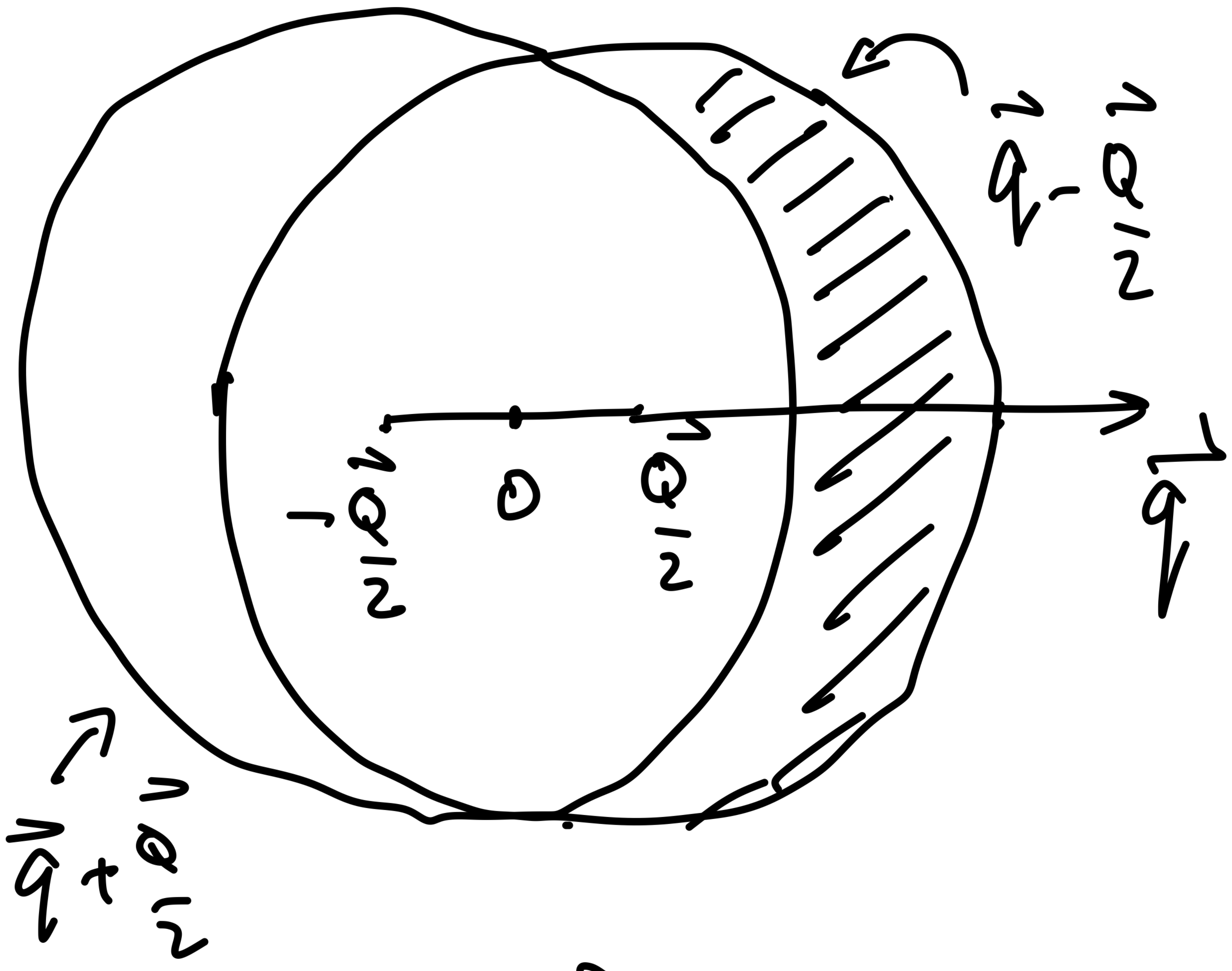
$G_B(\Omega, \vec{Q}) =$ 

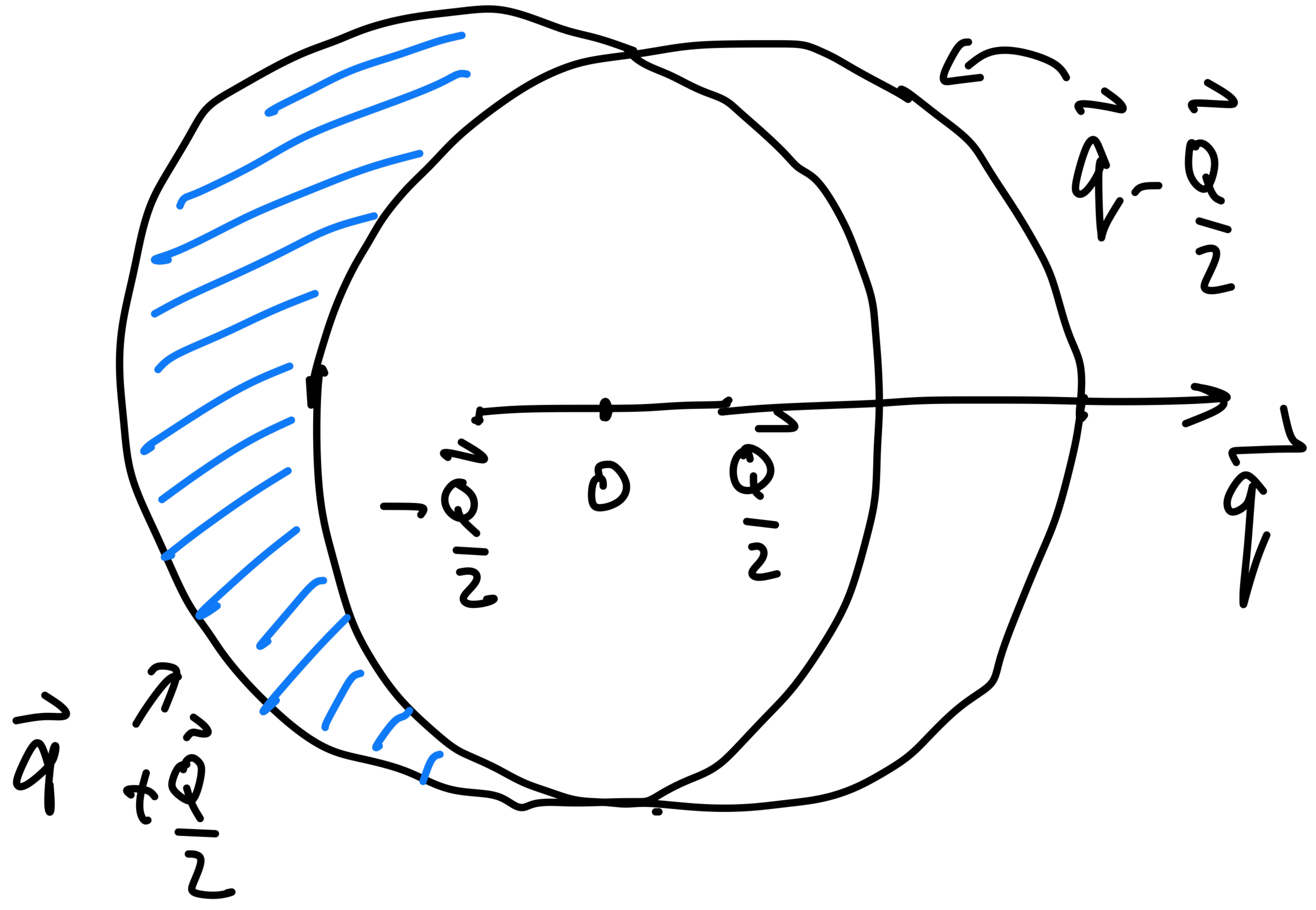
$1 - g$  ,

g  $=$ 

 $-i\int \frac{d\epsilon}{2\pi}\frac{d\vec{q}}{(2\pi)^3}\, G\left(\frac{\Omega}{2}+\epsilon, \frac{\vec{Q}}{2}+\vec{q}\right) G\left(-\frac{\Omega}{2}+\epsilon, -\frac{\vec{Q}}{2}+\vec{q}\right)$

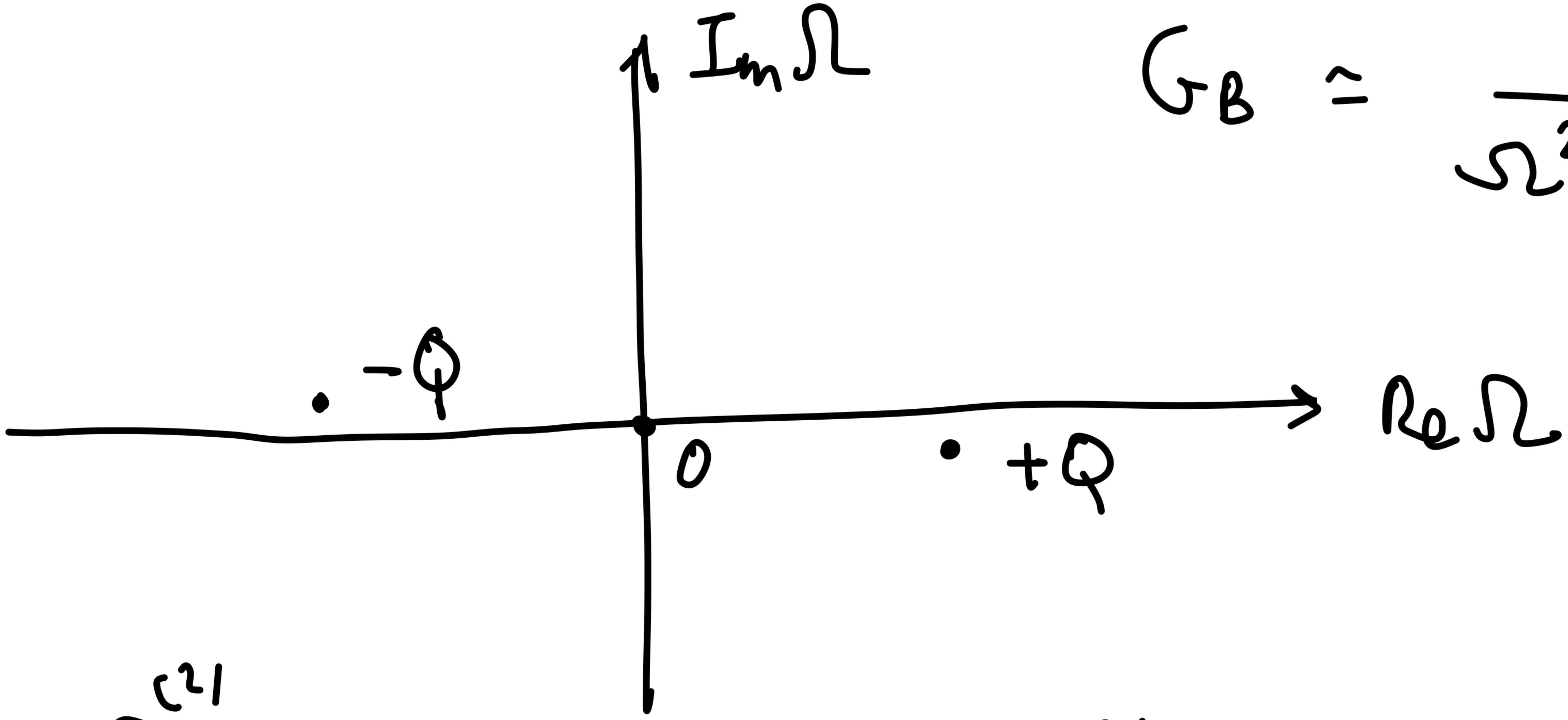
$$= \int \frac{d\vec{q}}{2\pi}\left[\frac{1}{\Omega - \vec{q}\cdot\vec{Q} + i\delta}\,\Theta\left(\xi_{\frac{\vec{Q}}{2}+\vec{q}}\right)\Theta\left(-\xi_{-\frac{\vec{Q}}{2}+\vec{q}}\right) + \frac{1}{-\Omega + \vec{q}\cdot\vec{Q} + i\delta}\,\Theta\left(\xi_{-\frac{\vec{Q}}{2}+\vec{q}}\right)\Theta\left(-\xi_{\frac{\vec{Q}}{2}+\vec{q}}\right)\right]$$



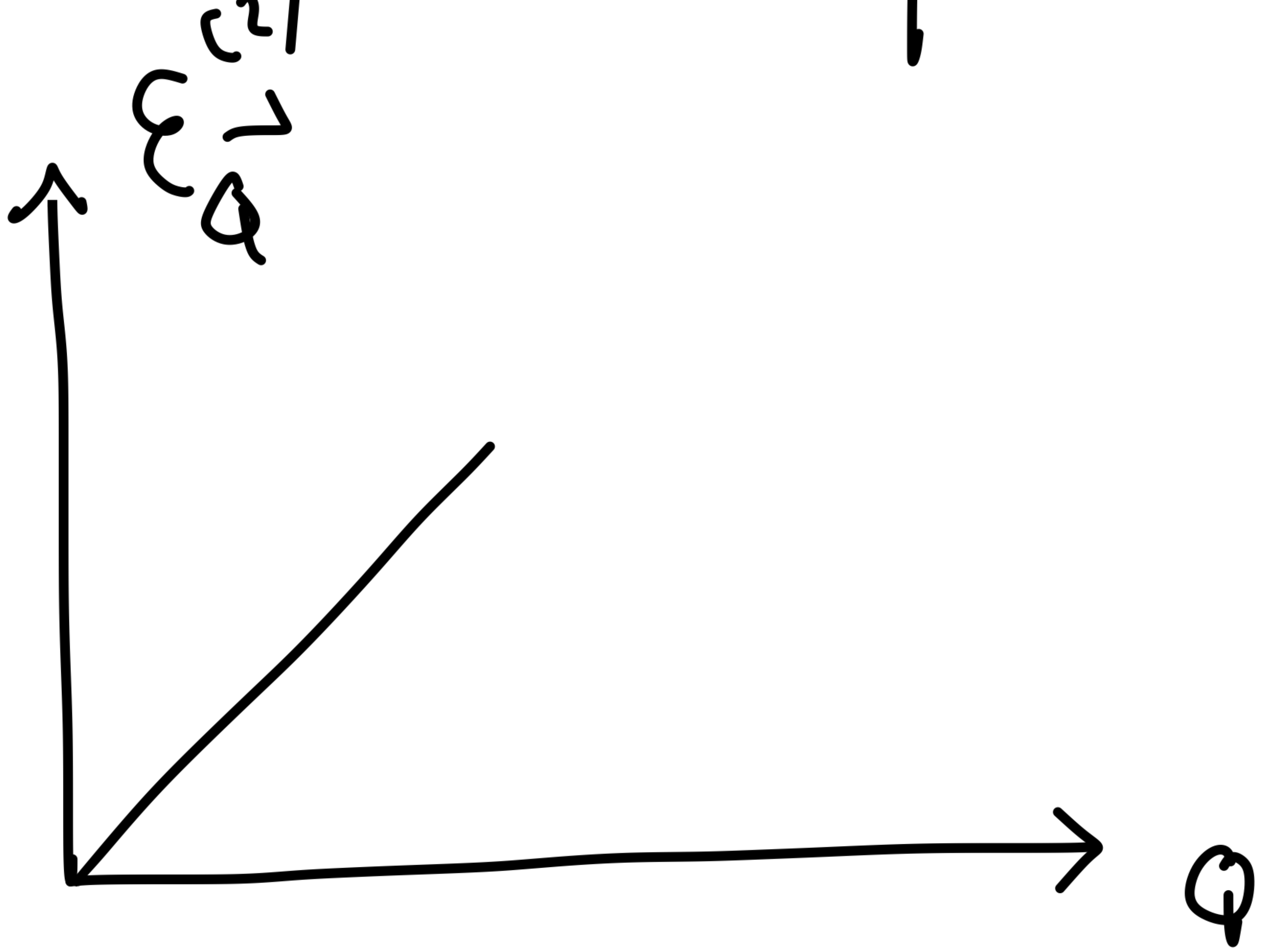


$$\frac{Q}{\Omega - v_F Q + i\delta} \qquad - \frac{Q}{\Omega + v_F Q - i\delta}$$

$$\text{Total} = \frac{Q^2}{\Omega^2 - Q^2}$$

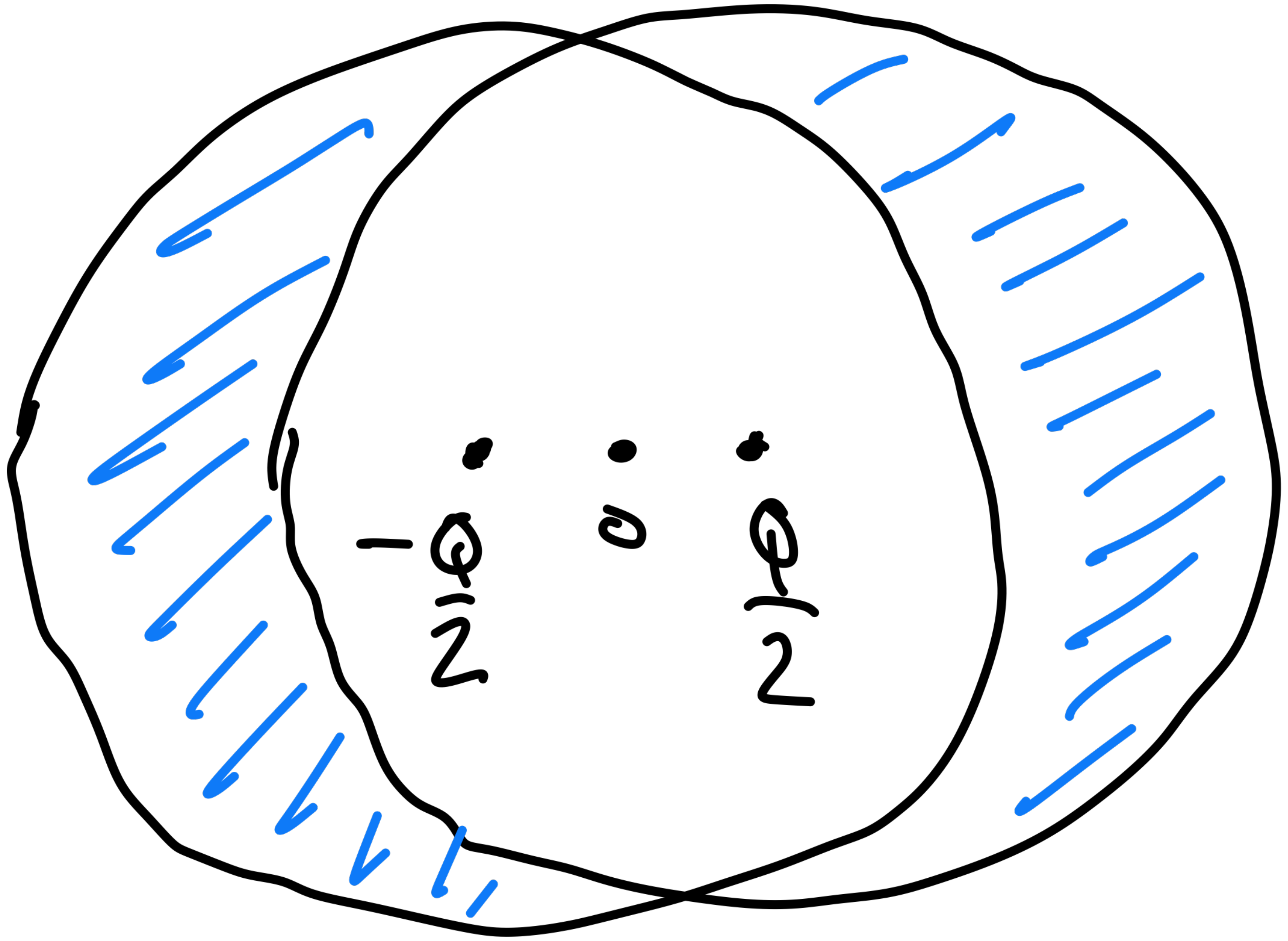


$$G_B \simeq \frac{Q^2}{\Omega^2 - Q^2}$$

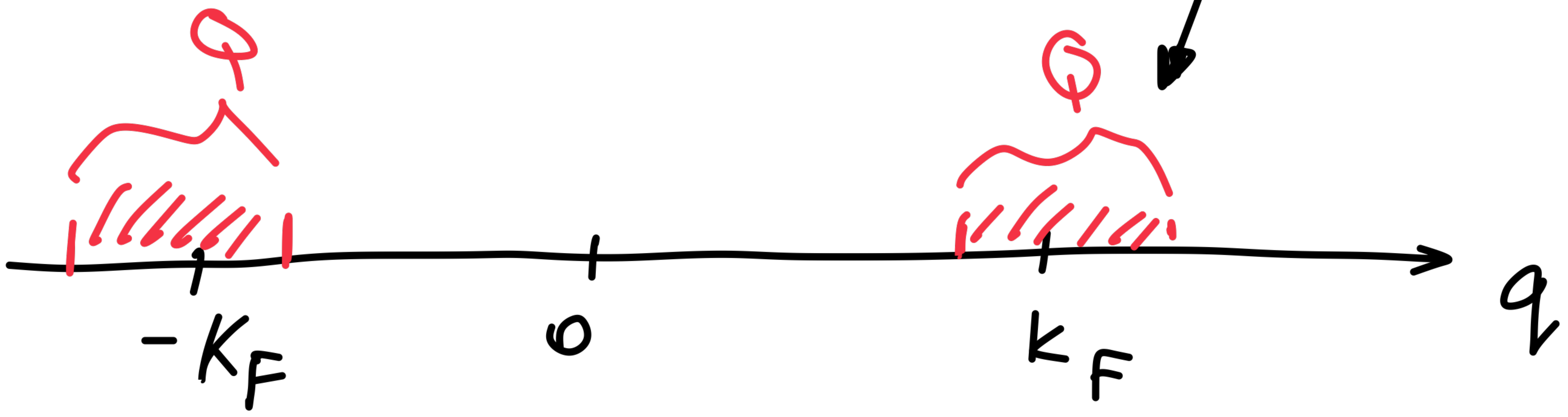


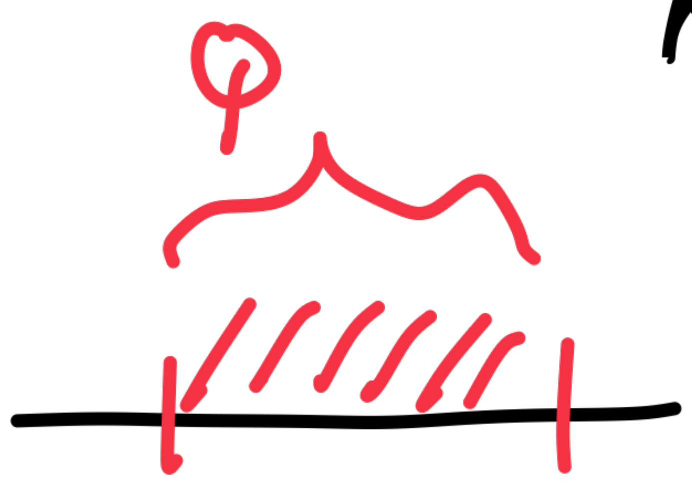
$$\xi^{(2)}_{\vec{Q}} = \xi_{\frac{\vec{Q}}{2}+\vec{q}} - \mu + \mu - \xi_{-\frac{\vec{Q}}{2}+\vec{q}}$$

$$\simeq v_F Q$$



Projection into 1D





The momenta contributing to $G_B(\Omega, \vec{Q})$