Phys529B: Topics of Quantum Theory

Lecture 15: identifying Non-Fermi liquids via RGE

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Λ_{UV} $\quad \phi_0(x) \qquad H(g_{01}, g_{02}, \dots; m, \Lambda_{UV})$

Λ $\quad \phi(x) \qquad H(g_1(\Lambda), g_2(\Lambda), \dots; m(\Lambda), \Lambda; Z(\Lambda))$

$\phi(x) = Z^{-1/2}\phi_R(x)$

or $\quad = Z^{1/2}\phi_0(x)$

Λ_{IR} $\quad \phi_R(x) \qquad H(g_{1R}, g_{2R}, \dots; m_R)$

- Callan-Symanzik approach (also Coleman-Weinberg applications) : Utilizing Green's functions to obtain RGEs and understand scale Transformations

- An interesting application to cold gases can be found in Yang, Jiang and Zhou, PRL 124 (22), 225701, 2020, Tricritical physics in 2D Superfluids.

Approach 1: $\phi(x) = Z^{1/2}(\Lambda)\,\phi_R(x)$

$$\langle 0|T\phi(x_1)\phi(x_2)\cdots\phi(x_n)|0\rangle = G^{(n)}(x_1,\dots x_n)$$

$$G^{(n)}(x_1,\dots x_n) = Z^{n/2}(\Lambda)\,\underbrace{\langle 0|T\phi_R(x_1)\cdots\phi_R(x_2)|0\rangle}_{\text{independent of ``}\Lambda\text{''.}}$$

Computed with $H(g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda); m(\Lambda), \Lambda)$

$$G^{(n)} = G^{(n)}(\cdots;\ \tilde{g}_1, \tilde{g}_2, \dots \tilde{g}_n(\Lambda); \tilde{m}(\Lambda), t)$$

$$t = \ln\Lambda$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} \right\} G^{(n)} = \left\{ \frac{n}{2} \underbrace{\Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde{g})} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} - \frac{n}{2} \gamma(\tilde{g}) \right\} G^{(n)} = 0$$

Note: $Z(t=0, \tilde{g}) = 1$

Approach II : $\phi(x) = Z^{-1/2}(\Lambda)\,\phi_0(x)$

$$\langle 0|T\phi(x_1)\phi(x_2)\cdots\phi(x_n)|0\rangle = G^{(n)}(x_1,\ldots x_n)$$

$$G^{(n)}(x_1,\ldots x_n) = Z^{-n/2}(\Lambda)\underbrace{\langle 0|T\phi_0(x_1)\cdots\phi_0(x_n)|0\rangle_c}_{\text{independent of ``}\Lambda\text{''.}}$$

Computed with $H(g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda); m(\Lambda), \Lambda)$

$$G^{(n)} = G^{(n)}(\cdots;\ \tilde{g}_1, \tilde{g}_2, \ldots \tilde{g}_n(\Lambda); \tilde{m}(\Lambda), t)$$

$$t = \ln\Lambda$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} \right\} G^{(n)} = -\left\{ \frac{n}{2} \underbrace{\Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde{g})} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} + \frac{n}{2} \gamma(\tilde{g}) \right\} G^{(n)} = 0$$

Note: $Z\left(t = \ln \frac{\Lambda_{UV}}{\Lambda_{IR}}, \tilde{g}\right) = 1$

Relations between Approach I and Approach II

App I: $\phi(x) = Z_{I}^{1/2}\,\phi_R(x)$; App II: $\phi(x) = Z_{II}^{-1/2}\,\phi_0(x)$

$$Z_{I}^{1/2}\, Z_0^{-1} = Z_{II}^{-1/2}$$ or

$$\boxed{\frac{1}{Z_{I}}\frac{\partial}{\partial t} Z_{I} = -\frac{1}{Z_{II}}\frac{\partial}{\partial t} Z_{II}}$$

$$\left\{\frac{\partial}{\partial t} + \beta(\tilde{g})\frac{\partial}{\partial \tilde{g}}\right\} G^{(n)} = \left\{\frac{n}{2}\underbrace{\Lambda\frac{\delta Z}{\delta \Lambda}\frac{1}{Z}}_{\gamma(\tilde{g})}\right\} G^{(n)}$$

$$\left\{\frac{\partial}{\partial t} + \beta(\tilde{g})\frac{\partial}{\partial \tilde{g}} - \gamma(\tilde{g})\right\} G^{(n)} = 0$$

Note: $Z(t=0, \tilde{g}) = 1$

How to use them ?

Two-point $\quad n=2$

I: $\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} - \gamma(\tilde{g}) \right\} G(p) = 0 \,, \quad G^{(2)}(p) \sim \frac{Z}{p^2}$

$$Z(\Lambda = \Lambda_{2R}, \tilde{g}) = \mathbb{1} \longrightarrow \frac{\partial}{\partial \tilde{g}} G^{(2)}(p) = 0$$

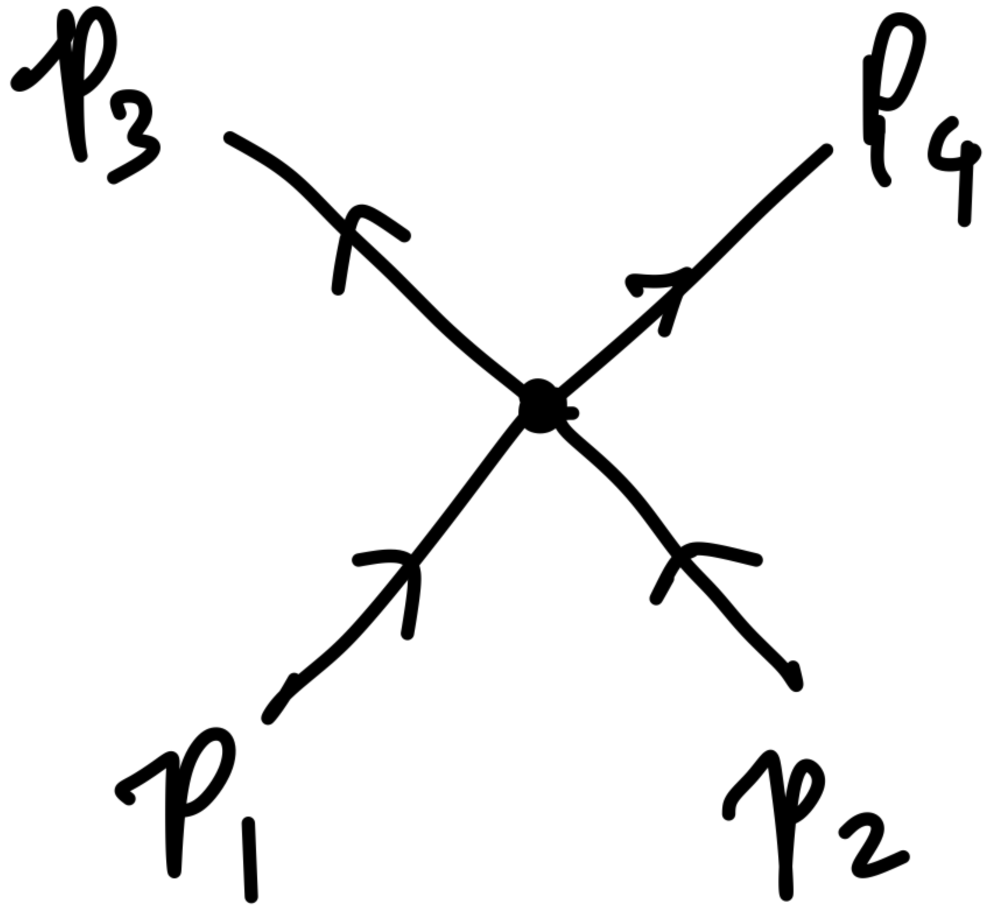
$$\gamma(\tilde{g}) = G(p)^{-1} \frac{\partial}{\partial t} G(p)$$

RGE

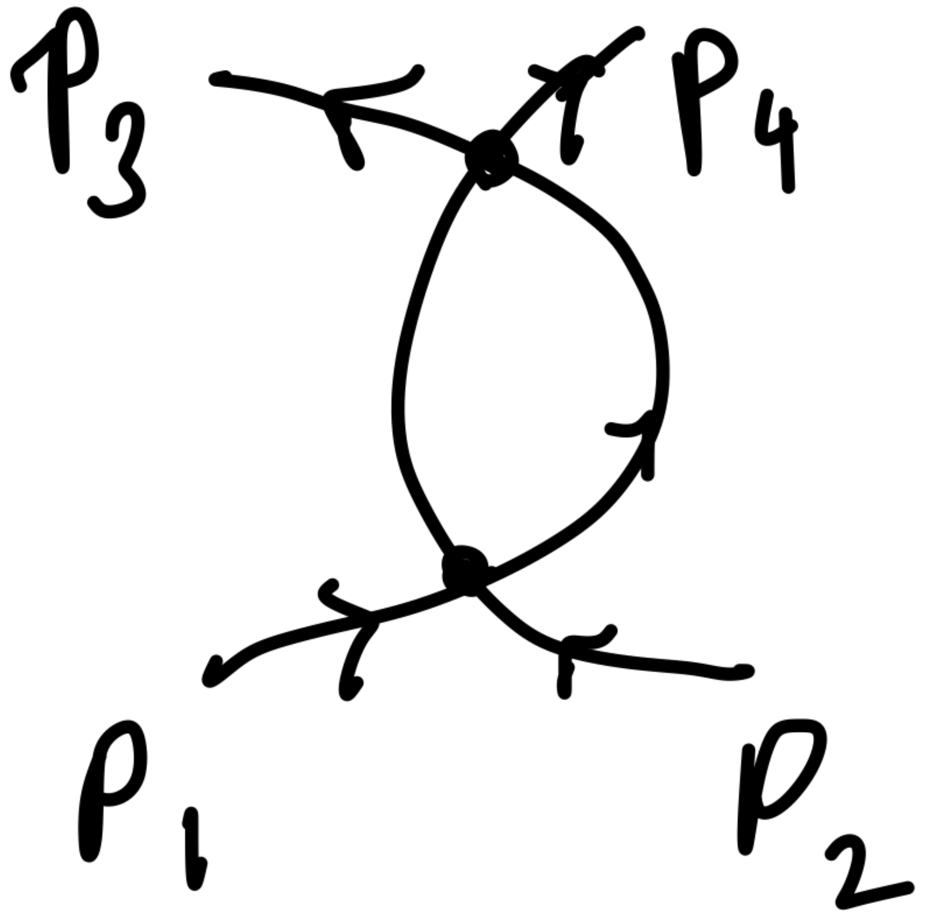
Green-function

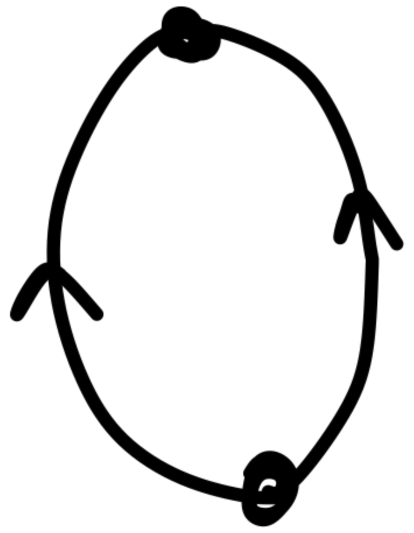
$$\mathrm{II}: \left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} - 2\gamma(\tilde{g}) \right\} G(p_1, p_2, p_3, p_4) \doteq 0$$

4-point 1PI



$$\frac{d\tilde{g}}{dt} = \beta(\tilde{g}) = 2\gamma(\tilde{g})\,\tilde{g} + \tilde{g}(d-1) + \frac{\partial}{\partial t}\left[X \right]$$



X =  + ...

$\uparrow$

1PI