Phys529B: Topics of Quantum Theory

Lecture 14: identifying Non-Fermi liquids via RGE

instructor: Fei Zhou

Supplementary stuff on RGE

$$H = H(m, \lambda, \dots ; \Lambda) = H(\tilde{m}, \tilde{\lambda}, \dots, Z(\Lambda) ; \Lambda)$$

for any given Λ, there shall be a $\tilde{m}(\Lambda)$, $\tilde{\lambda}(\Lambda)$ ...

so that H leads to the same physics.

$$\frac{d\tilde{m}}{dt} = \beta_m(\tilde{m}, \tilde{\lambda}, \dots) \qquad \frac{dZ(\Lambda)}{dt} = \gamma(\tilde{m}, \tilde{\lambda}, Z)$$

$$\frac{d\tilde{\lambda}}{dt} = \beta_{\tilde{\lambda}}(\tilde{m}, \tilde{\lambda}, \dots) \qquad t = \ln\frac{\Lambda}{\Lambda_{uv}}$$

Renormalization Group Equations

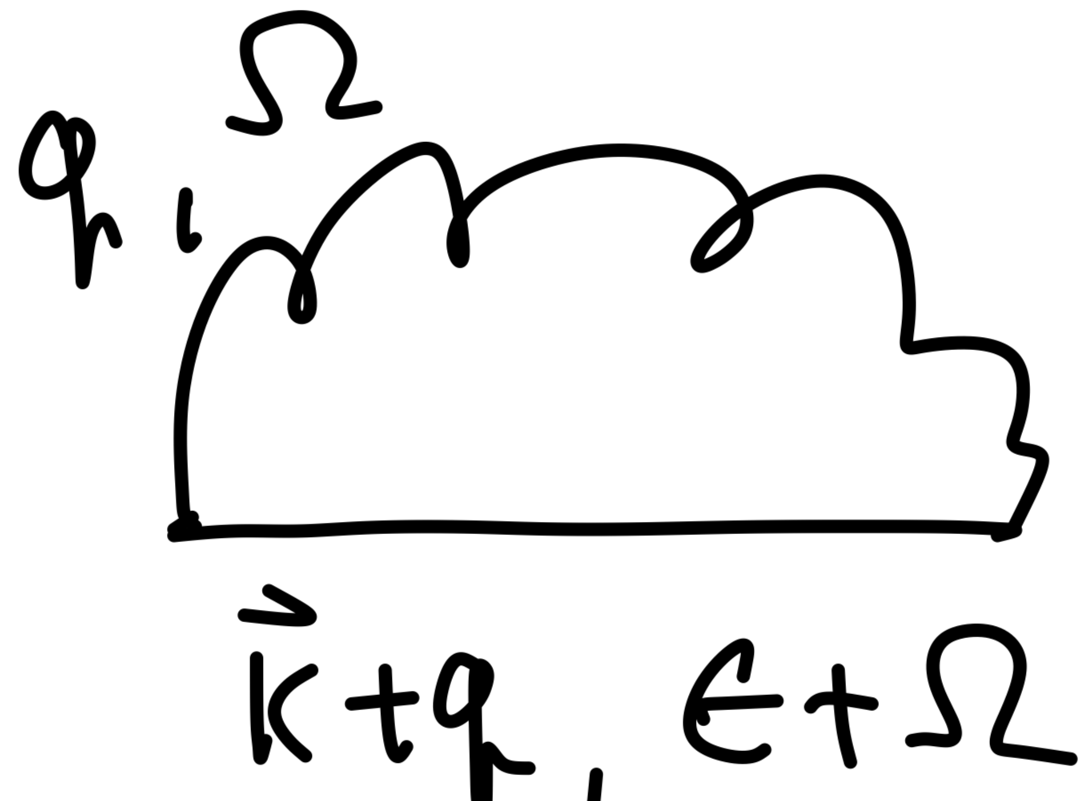
Running scale

Generally $G = G_0 + G_0 \Sigma G$ or $G^{-1} = G_0^{-1} - \Sigma$

pertrubation theory :

$$G_k = \langle 0 | -iT\, \psi_k(0)\, \psi_k^{\dagger}(0) | 0 \rangle$$

$\delta G = G_0 \Sigma^{(2)} G_0$, $\quad \Sigma^{(2)}(\vec{k}, \epsilon) = i \int D_0(\vec{q}, \Omega)\, G_0(\vec{k}+\vec{q}, \epsilon+\Omega) \times g^2$



$$D_0(\vec{q}, \Omega) = \frac{1}{\Omega^2 - w_{\vec{q}}^2 + i\delta}$$

$$G_0(\vec{k}, \epsilon) = \frac{1}{\epsilon - (\epsilon_{\vec{k}} - \mu) + i\delta_{\vec{k}}}$$

- The basic idea of renormalizability: 1) physical measueables shall be independent of the UV scale at which you formulate your theory; I.e. “scale independent”. 2) there can be infinite numbers of field theories of one single physical reality; they are all equivalent. 3) different theories is connected by Renormalization group equations.

What to do beyond perturbation theory : RGE

idea

Only Λ, Not Λ_{uv}

$$P = P(g;\Lambda) = \Lambda^{p}\tilde{P}(\tilde{g}), \qquad H = H(g;\Lambda)$$

$$0 = \frac{\partial P}{\partial t} = p P + \Lambda^{p}\frac{\partial \tilde{P}(\tilde{g})}{\partial \tilde{g}}\beta(\tilde{g}), \qquad \beta(\tilde{g}) = \frac{\partial \tilde{g}}{\partial t}$$

$$\beta(\tilde{g}) = -p\,\frac{\tilde{P}}{\frac{\partial \tilde{P}}{\partial \tilde{g}}}$$

↙ a function of $\tilde{g}$.

Supplementary stuff on RGE

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for any given Λ, there shall be a $\tilde{m}(\Lambda), \tilde{\lambda}(\Lambda) \dots$

so that H leads to the same physics.

$$\frac{d\tilde{m}}{dt} = \beta_m(\tilde{m}, \tilde{\lambda}, \dots) \qquad \frac{dZ(\Lambda)}{dt} = \gamma(\tilde{m}, \tilde{\lambda}, Z)$$

$$\frac{d\tilde{\lambda}}{dt} = \beta_{\tilde{\lambda}}(\tilde{m}, \tilde{\lambda}, \dots) \qquad t = \ln\frac{\Lambda}{\Lambda_{uv}}$$

Renormalization Group Equations

Running scale

Λ_{UV} $\phi_0(x)$ $H(g_{01}, g_{02}, \dots; m, \Lambda_{UV})$

Λ $\phi(x)$ $H(g_1(\Lambda), g_2(\Lambda), \dots; m(\Lambda), \Lambda; Z(\Lambda))$

$\phi(x) = Z^{-\frac{1}{2}}\phi_R(x)$

or $= Z^{\frac{1}{2}}\phi_0(x)$

Λ_{IR} $\phi_R(x)$ $H(g_{1R}, g_{2R}, \dots; m_R)$

- Callan-Symanzik approach (also Coleman-Weinberg applications) : Utilizing Green’s functions to obtain RGEs and understand scale Transformations

- An interesting application to cold gases can be found in Yang, Jiang and Zhou, PRL 124 (22), 225701, 2020, Tricritical physics in 2D Superfluids.

Approach 1: $\phi(x) = Z^{1/2}(\Lambda)\,\phi_R(x)$

$$\langle 0|T\phi(x_1)\phi(x_2)\cdots\phi(x_n)|0\rangle = G^{(n)}(x_1,\dots x_n)$$

$$G^{(n)}(x_1,\dots x_n) = Z^{n/2}(\Lambda)\,\underbrace{\langle 0|T\phi_R(x_1)\cdots\phi_R(x_1)|0\rangle_c}_{\text{independent of ``}\Lambda\text{''.}}$$

Computed with $H(g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda); m(\Lambda), \Lambda)$

$$G^{(n)} = G^{(n)}(\cdots; \tilde{g}_1, \tilde{g}_2, \dots \tilde{g}_n(\Lambda); \tilde{m}(\Lambda), t)$$

$$t = \ln\Lambda$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} \right\} G^{(n)} = \left\{ \frac{n}{2} \underbrace{\Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde{g})} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde{g}) \frac{\partial}{\partial \tilde{g}} - \gamma(\tilde{g}) \right\} G^{(n)} = 0$$

Note : $Z(t=0, \tilde{g}) = 1$

Approach II : $\phi(x) = Z^{-1/2}(\Lambda)\,\phi_0(x)$

$$\langle 0 | T\,\phi(x_1)\,\phi(x_2) \cdots \phi(x_n) | 0 \rangle = G^{(n)}(x_1, \ldots\, x_n)$$

$$G^{(n)}(x_1, \ldots x_n) = Z^{-n/2}(\Lambda)\, \underbrace{\langle 0 | T\,\phi_0(x_1) \cdots \phi_0\ x_2) | 0 \rangle}_{\text{independent of ``}\Lambda\text{''.}}{}_c$$

Computed with $H(g_1(\Lambda), g_2(\Lambda), \cdots g_n(\Lambda); m(\Lambda), \Lambda)$

$$G^{(n)} = G^{(n)}(\cdots ; \tilde{g}_1, \tilde{g}_2, \ldots \tilde{g}_n(\Lambda); \tilde{m}(a), t)$$

$$t = \ln \Lambda$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde g) \frac{\partial}{\partial \tilde g} \right\} G^{(n)} = -\left\{ \frac{n}{2} \underbrace{\Lambda \frac{\delta Z}{\delta \Lambda} \frac{1}{Z}}_{\gamma(\tilde g)} \right\} G^{(n)}$$

$$\left\{ \frac{\partial}{\partial t} + \beta(\tilde g) \frac{\partial}{\partial \tilde g} + \gamma(\tilde g) \right\} G^{(n)} = 0$$

Note: $Z\left(t = \ln \frac{\Lambda_{uv}}{\Lambda_{IR}}, \tilde g\right) = 1$