Phys529B: Topics of Quantum Theory

Lecture 11: identifying Non-Fermi liquids

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- Reading materials on Non-Fermi liquids
- 1) Stripe physics and Nematic Liquid Model
- E. Fradkin and S. A. Kivelson, Phys. Rev. B 59, 8065 (1999); V. Oganesyan. S. Kivelson, and E. Fradkin, Phys. Rev. B 64, 195109 (2001).
- Or Metal near QPC or critical metals (See Sachdev et al.).
- Fitzpatrick, Karchru, Kaplan, Raghu, PRB 89, 165114(2014).
- 2) un-particles
- Philip Philips, Langley, Hutasoit, https://arxiv.org/pdf/1305.0006.pdf
- Karch et al., https://arxiv.org/pdf/1511.02868.pdf;
- Hartnoll,and Karch, https://arxiv.org/abs/1501.03165.

A Model for Fermi Surface + Wilson-Fisher bosons

$$H = H_B + H_F + H_{BF}$$

$$H_B = \int [\nabla \varphi(\vec{x})]^2 + \Pi^2(\vec{x}) + m^2 \varphi^2 + \lambda \varphi^4 ,$$

$$H_F = \int \psi^\dagger(\vec{x}) \left(-\frac{\nabla^2}{2} - \mu\right) \psi(\vec{x}) ,$$

$$H_{BF} = g \int \varphi(\vec{x})\, \psi^\dagger(\vec{x})\, \psi(\vec{x}) + h.c$$

$$[\varphi(\vec{x}), \Pi(\vec{x}')] = i\delta(\vec{x}-\vec{x}')$$

$$\{\psi(\vec{x}), \psi^\dagger(\vec{x}')\} = \delta(\vec{x}-\vec{x}')$$

Supplementary stuff on RGE

$$H = H(m, \lambda, \dots ; \Lambda) = H(\tilde{m}, \tilde{\lambda}, \dots, Z(\Lambda) ; \Lambda)$$

for any given Λ, there shall be a $\tilde{m}(\Lambda)$, $\tilde{\lambda}(\Lambda)$ ...

so that H leads to the same physics.

$$\frac{d\tilde{m}}{dt} = \beta_m(\tilde{m}, \tilde{\lambda}, \dots) \qquad \frac{dZ(\Lambda)}{dt} = \gamma(\tilde{m}, \tilde{\lambda}, Z)$$

$$\frac{d\tilde{\lambda}}{dt} = \beta_{\tilde{\lambda}}(\tilde{m}, \tilde{\lambda}, \dots) \qquad t = \ln\frac{\Lambda}{\Lambda_{uv}}$$

Renormalization Group Equations

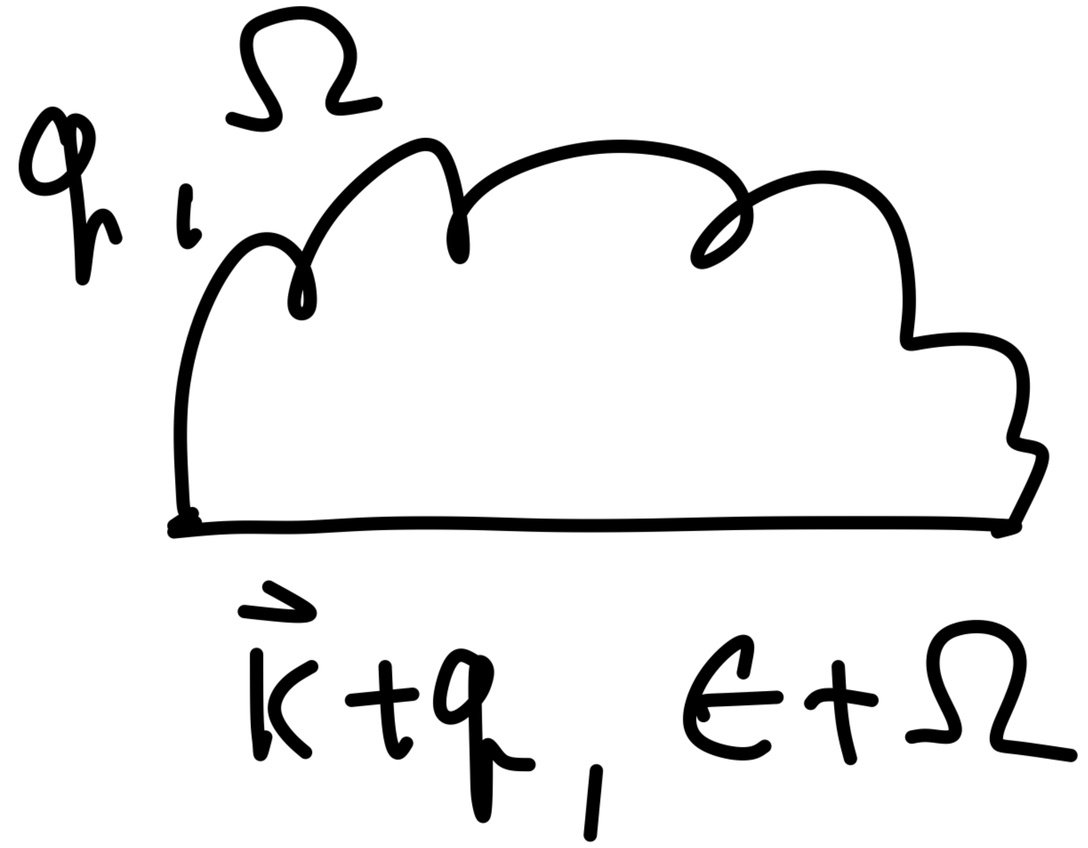
Running scale

Generally $\quad G = G_0 + G_0 \Sigma G \quad$ or $\quad G^{-1} = G_0^{-1} - \Sigma$

perturbation theory :

$$G_k = \langle 0 | -i T \psi_k(0) \psi_k^{\dagger}(0) | 0 \rangle$$

$$\delta G = G_0 \Sigma^{(2)} G_0 \,, \quad \Sigma^{(2)}(\vec{k}, \epsilon) = i \int D_0(\vec{q}, \Omega)\, G_0(\vec{k}+\vec{q}, \epsilon+\Omega) \times g^2$$



$$D_0(\vec{q}, \Omega) = \frac{1}{\Omega^2 - w_{\vec{q}}^2 + i\delta}$$

$$G_0(\vec{k}, \epsilon) = \frac{1}{\epsilon - (\epsilon_{\vec{k}} - \mu) + i\delta_{\vec{k}}}$$

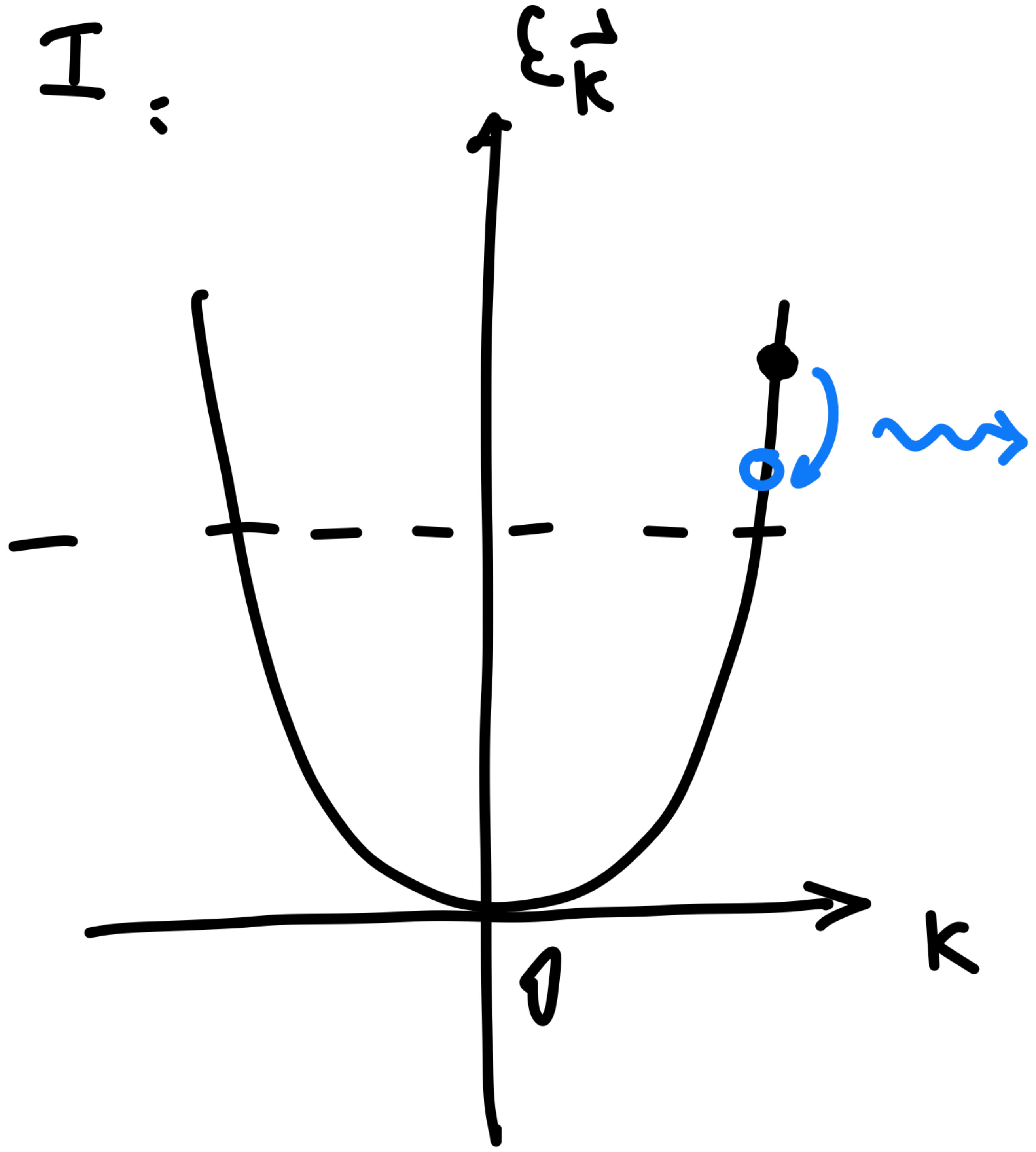
....

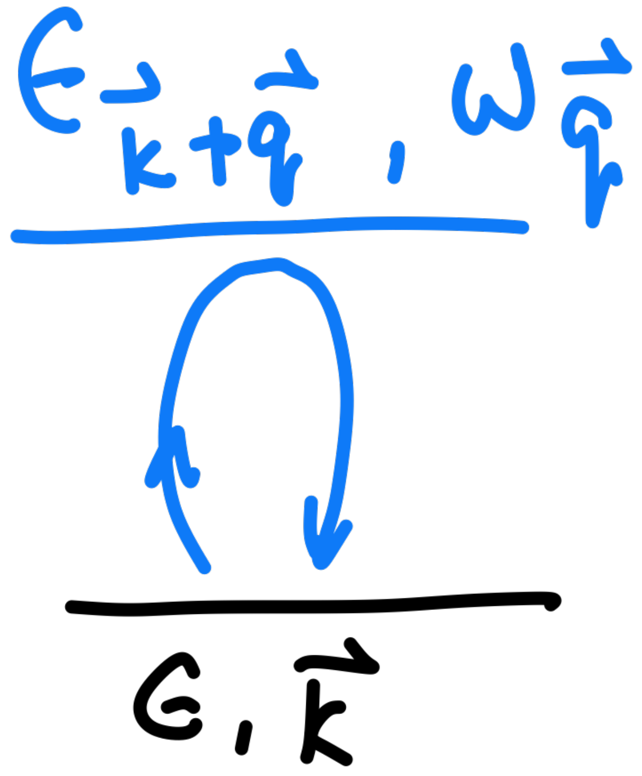
$$\Sigma(\vec{k},\epsilon) = \int_{\vec{q}} -\left[\frac{\Theta(\vec{k}+\vec{q})}{\omega_{\vec{q}}+\tilde{\epsilon}_{\vec{k}+\vec{q}}-(\epsilon+i\delta)} - \frac{\bar{\Theta}(\vec{k}+\vec{q})}{(\epsilon-i\delta)+\omega_{\vec{q}}-\tilde{\epsilon}_{\vec{k}+\vec{q}}}\right]\times\frac{g^2}{2\omega_{\vec{q}}}$$

$$\left.\frac{\partial\Sigma(\overset{\nearrow 0}{\vec{k}},\epsilon)}{\partial\epsilon}\right|_{\omega\to 0} \simeq -\int_{\vec{q}}\left[\frac{\Theta(\vec{k}+\vec{q})}{\left(\omega_{\vec{q}}+\tilde{\epsilon}_{\vec{k}+\vec{q}}\right)^2} + \frac{\bar{\Theta}(\vec{k}+\vec{q})}{\left(\omega_{\vec{q}}-\tilde{\epsilon}_{\vec{k}+\vec{q}}\right)^2}\right]\frac{g^2}{2\omega_{\vec{q}}}$$

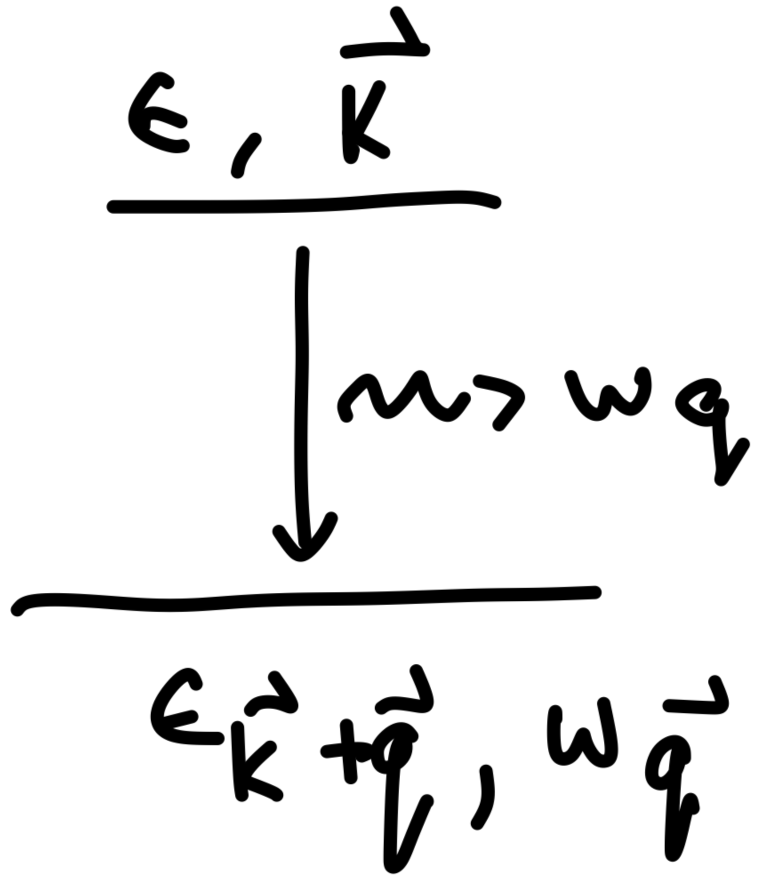
$$\tilde{\epsilon}_{k} = \epsilon_k - \mu$$

I :





$$\delta(\epsilon - \omega_q - \tilde{\epsilon}_{\vec{k}+\vec{q}})\ \theta(\xi_{\vec{k}+\vec{q}})$$

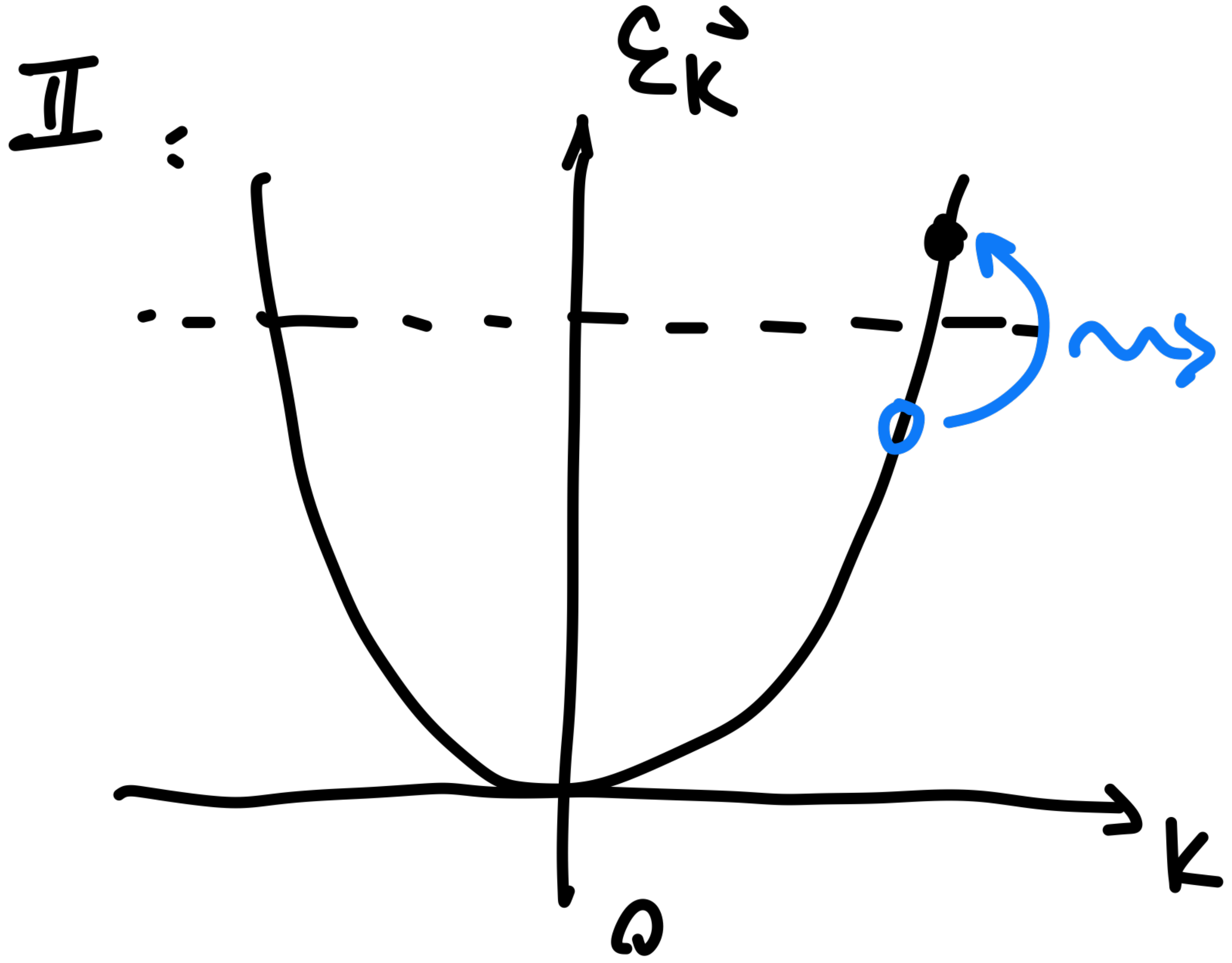


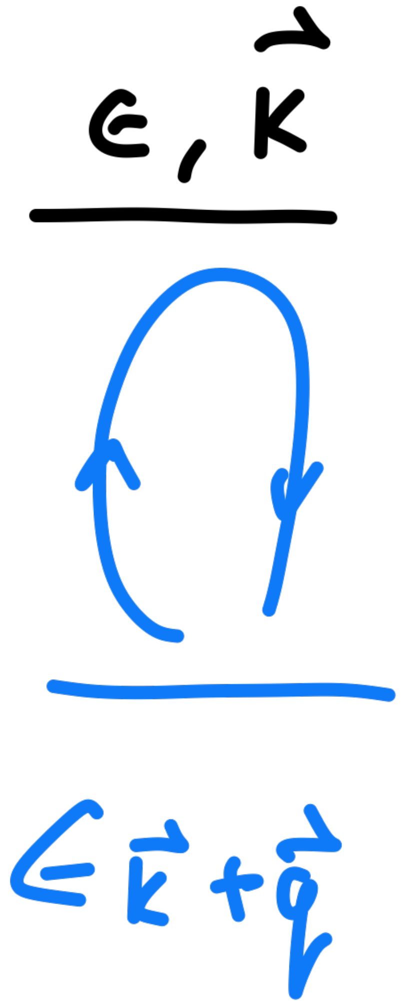
Virtual / $\mathrm{Re}\,\Sigma(\epsilon, \vec{k})$

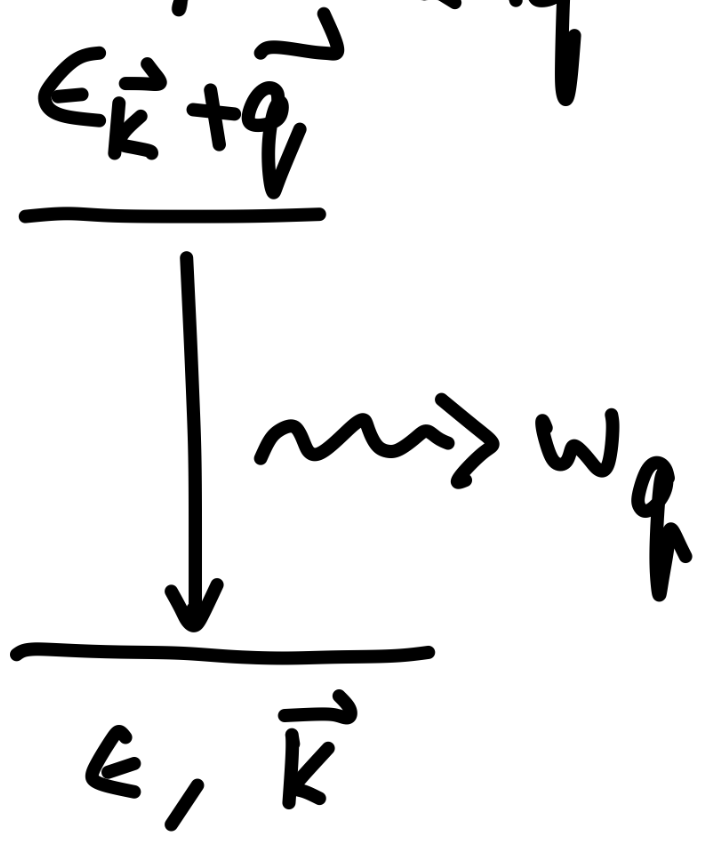
Real / $\mathrm{Im}\,\Sigma(\epsilon, \vec{k})$

$$\delta(\epsilon + \omega_q - \tilde{\epsilon}_{\vec{k}+\vec{q}})\ \theta(-\xi_{\vec{k}+\vec{q}})$$

II :

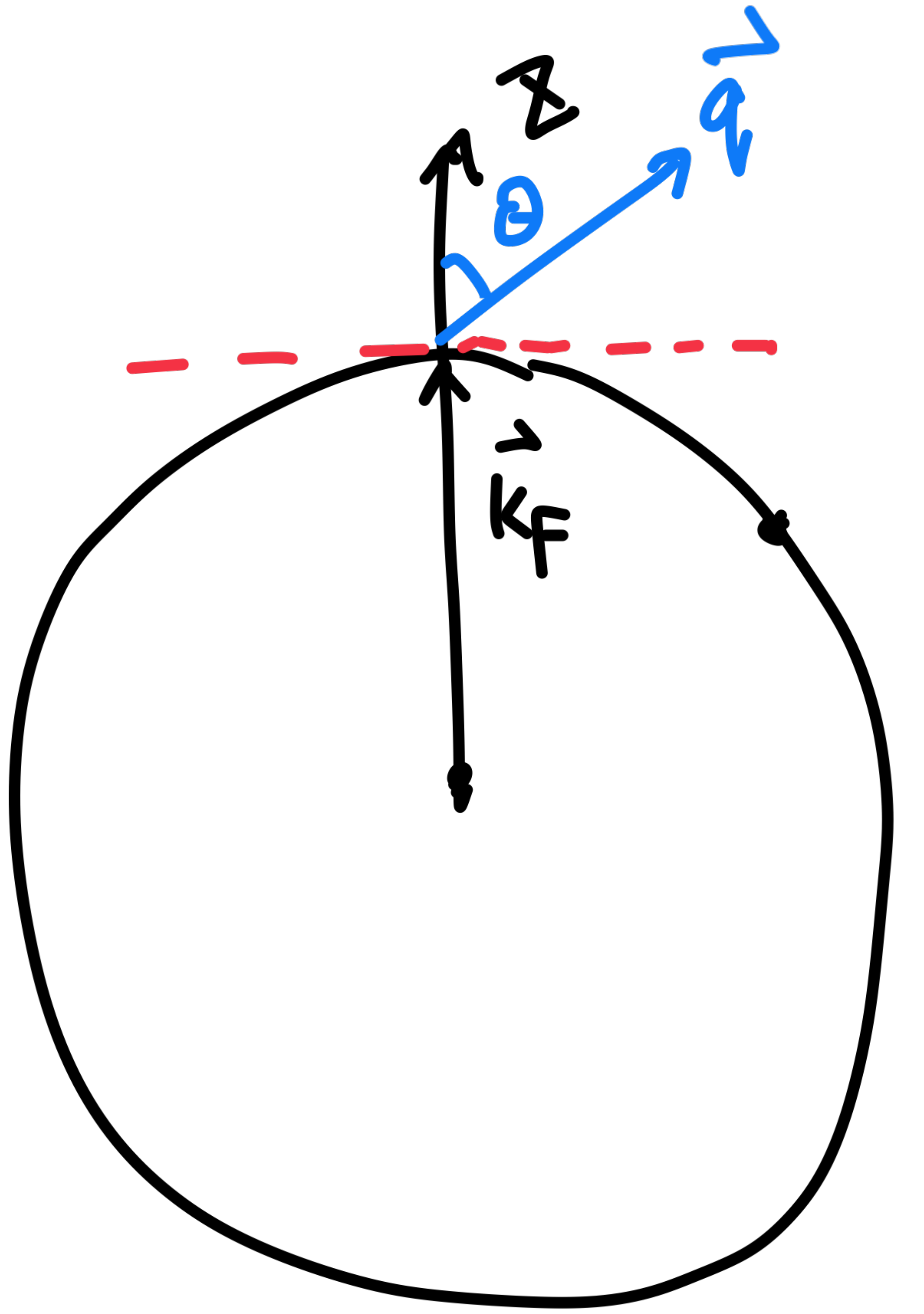






$$-\int \frac{\theta(\vec{k}+\vec{q})}{2\omega_q\left[\omega_q^2 + v_F^2 q^2 \cos^2\theta\right]}\frac{d^3\vec{q}}{(2\pi)^3} = -\int \frac{q^2\,dq}{2|q|^3} \underbrace{\int_0^1 dx\,\frac{1}{v_s^2+v_F^2 x^2}}_{\boxed{\frac{1}{v_s v_F}\arctan\left(\frac{v_F}{v_s}\right)}}$$

$\rightarrow \frac{1}{v_s v_F}\cdot\frac{\pi}{2}$ if $v_F \gg v_s$



$$Z^{-1}(\vec{k}=k_F) = 1 - \frac{\partial\Sigma(\vec{k},\omega)}{\partial\omega}\Big|_{\omega\to 0}$$

$$= 1 + g^2 \ln\frac{\Lambda_{UV}}{\omega}$$

(singular near $k=k_F$)

$$\Sigma(\vec{k},\epsilon) = \int_{\vec{q}} -\left[\underbrace{\frac{\Theta(\vec{k}+\vec{q})}{\omega_{\vec{q}} + \tilde{\varepsilon}_{\vec{k}+\vec{q}} - (\epsilon + i\delta)}}_{\Sigma_{I}} - \underbrace{\frac{\bar{\Theta}(\vec{k}+\vec{q}|}{(\epsilon - i\delta) + \omega_{q} - \tilde{\varepsilon}_{\vec{k}+\vec{q}}}}_{\Sigma_{II}} \right] \times \frac{g^2}{2\omega_q}$$

3D $\quad \parallel \vec{k}_F$

$$\Sigma_{I}(\vec{k},\epsilon) = \frac{g^2}{8\pi^2}\int dq\, q \underbrace{\int_0^1 dx \frac{-1}{q v_s + v_F q x - (\epsilon + i\delta)}}_{I}$$

$$I = \frac{-1}{q}\left[\ln\left|\frac{q(v_s+v_F) - \epsilon}{q v_s - \epsilon}\right| + i\pi\, \Theta(\epsilon - q v_s)\left((v_F+v_s)q - \epsilon\right) \right]$$

$$\Theta(\epsilon)$$

Putting things togather for 3D

$$\frac{\partial \, \mathrm{Re}\Sigma(k_F, \epsilon)}{\partial \epsilon} \sim -g^2 \ln \frac{\Lambda}{|\epsilon|} ,$$

$$\mathrm{Im}\,\Sigma(k_F, \epsilon) \sim g^2 |\epsilon| \pi \quad (\text{even function})$$

$$Z(k_F) \simeq \left. \frac{1}{1 + \frac{g^2}{4\pi^2} \ln \frac{\Lambda}{|\epsilon|}} \right|_{|\epsilon| \to 0} \to 0$$