Phys529B: Topics of Quantum Theory

Lecture 10: interacting fermions and Non-Fermi liquid

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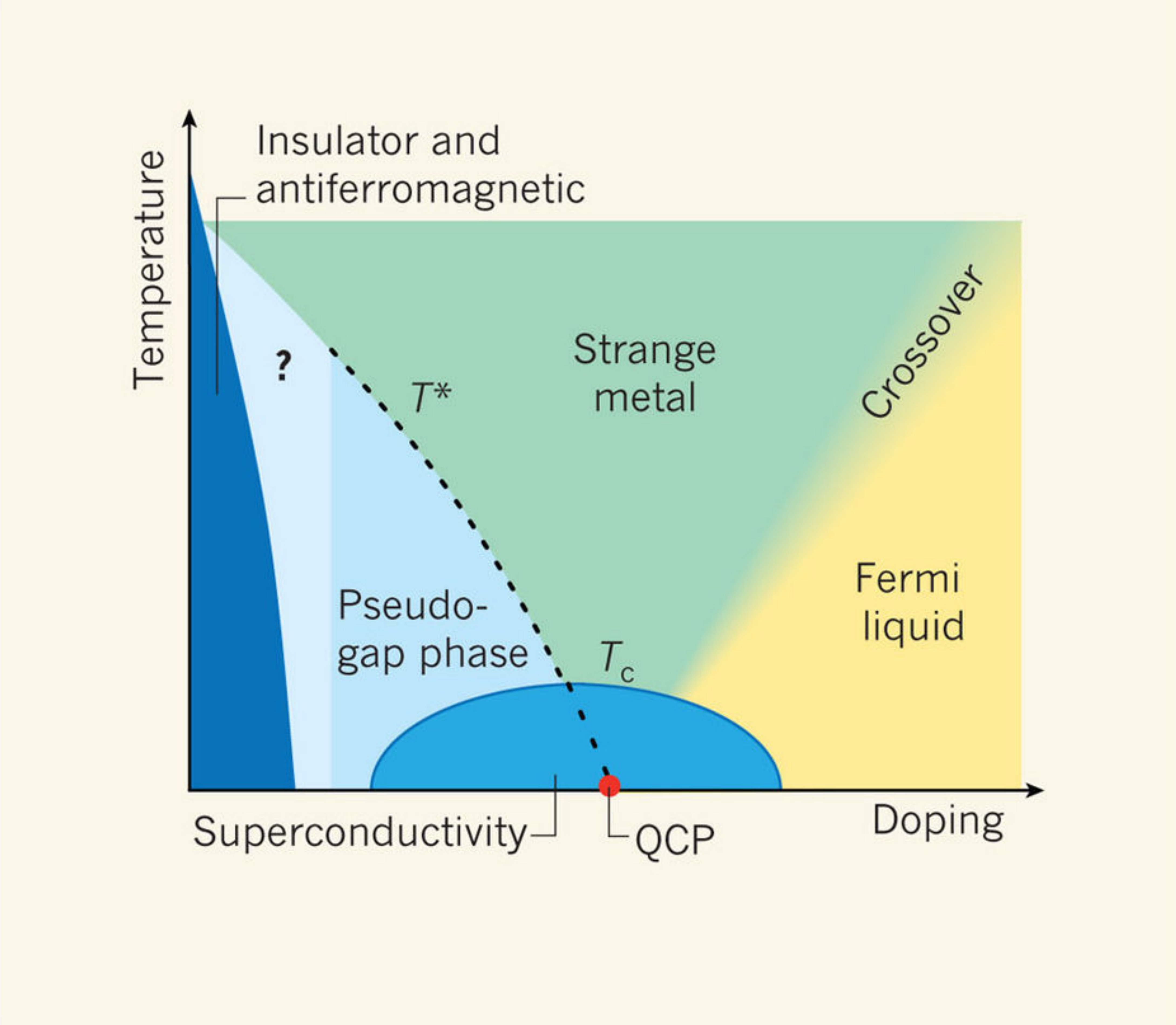
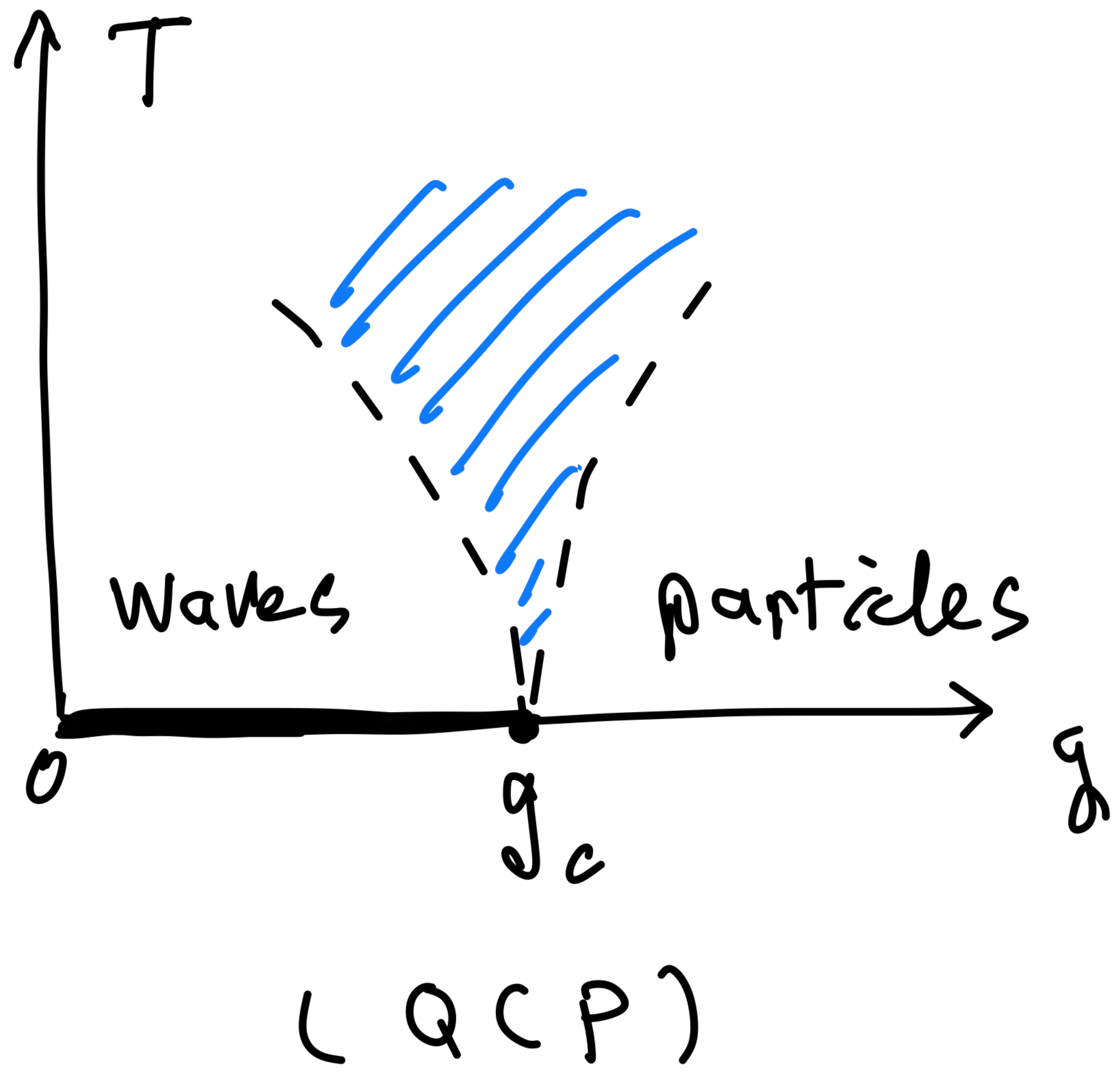


Insulator and
antiferromagnetic
Temperature
?
T^*
Strange
metal
Crossover
Pseudo-
gap phase
T_c
Fermi
liquid
Superconductivity
QCP
Doping

II A.



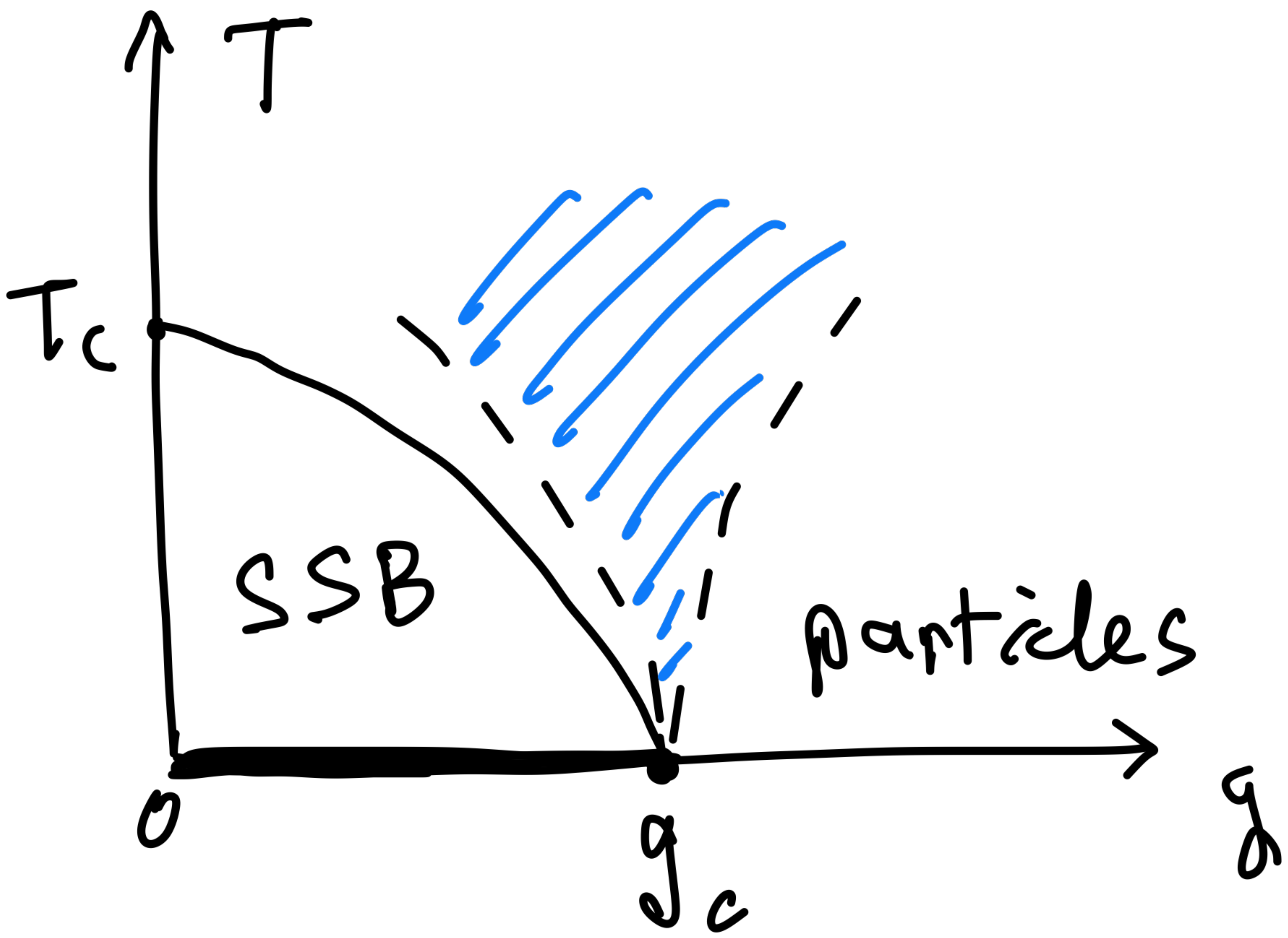
Near a QCP

fermi Surface +

Wilson-Fisher bosons

(Scale / Conformal symmetry)

II A.



(QCP)

Near a QCP

Fermi Surface +

Wilson-Fisher bosons

(Scale / Conformal symmetry)

A Model for Fermi Surface + Wilson-Fisher bosons

$$\mathcal{H} = H_B + H_F + H_{BF}$$

$$H_B = \int [\nabla \varphi(\vec{x})]^2 + \Pi^2(\vec{x}) + m^2 \varphi^2 + \lambda \varphi^4,$$

$$H_F = \int \psi^\dagger(\vec{x}) \left(-\frac{\nabla^2}{2} - \mu\right) \psi(\vec{x}),$$

$$H_{BF} = g \int \varphi(\vec{x})\, \psi^\dagger(\vec{x})\, \psi(\vec{x}) + h.c$$

$$[\varphi(\vec{x}), \Pi(\vec{x}')] = i\delta(\vec{x} - \vec{x}')$$

$$\{\psi(\vec{x}), \psi^\dagger(\vec{x}')\} = \delta(\vec{x} - \vec{x}')$$

Supplementary stuff on scalar fields and WF fixed pt

$$H_{B_0} = \int \pi^2 + (\nabla\varphi)^2 \qquad = \sum_q \hbar\omega_q a_q^\dagger a_q$$

$$\begin{cases} \varphi_q = (a_q^\dagger + a_{-q}) \dfrac{1}{\sqrt{2q}} \\ \pi_q = i(a_q^\dagger - a_{-q}^\dagger)\sqrt{2q} \end{cases}$$

$$[\varphi_q, \pi_{q'}] = i\hbar\delta_{q,q'} \longrightarrow [a_q, a_{q'}^\dagger] = \delta_{q,q'}$$

Time Order Green's function $D(q,\omega)$

$$\langle -i T \varphi_q(t) \varphi_q^+(0) \rangle$$

$$= \frac{1}{2q}\left[-i\Theta(t)\, e^{-i\omega_q t} - i\Theta(-t)\, e^{+i\omega_q t}\right], \quad \omega_q = v_s |\vec{q}| > 0$$

$$D(q,\omega) = \frac{1}{2q}\left[\frac{1}{\omega-\omega_q+i\delta} - \frac{1}{\omega+\omega_q-i\delta}\right]$$

$$\approx \frac{1}{\omega^2-\omega_q^2+i\delta}, \quad \delta > 0$$

Supplementary stuff on RGE

$$H = H(m, \lambda, \ldots; \Lambda) = H(\tilde{m}, \tilde{\lambda}, \ldots, Z(\Lambda); \Lambda)$$

for any given Λ, there shall be a $\tilde{m}(\Lambda)$, $\tilde{\lambda}(\Lambda)$ ...

so that H leads to the same physics.

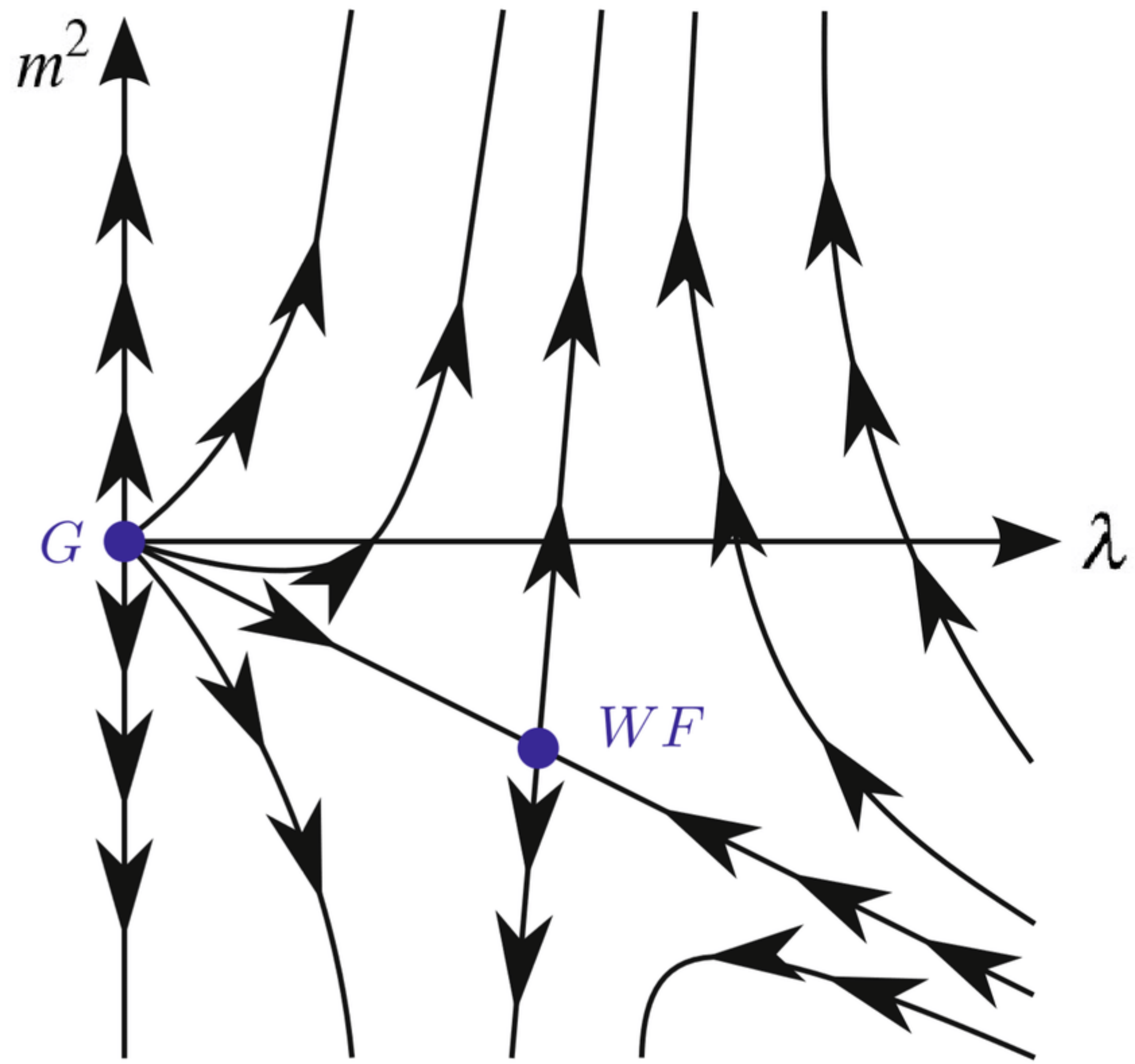
$$\frac{d\tilde{m}}{dt} = \beta_m(\tilde{m}, \tilde{\lambda}, \ldots) \qquad \frac{dZ(\Lambda)}{dt} = \gamma(\tilde{m}, \tilde{\lambda}, Z)$$

$$\frac{d\tilde{\lambda}}{dt} = \beta_{\tilde{\lambda}}(\tilde{m}, \tilde{\lambda}, \ldots) \qquad t = \ln\frac{\Lambda}{\Lambda_{uv}}$$

Renormalization Group Equations

Running scale

RG flows (i.e. scale transformation) of a scalar model :
Scale Symmetric Wilson-Fisher F.P. (d+1<4)



perturbative Limit (partial story)

Ⓐ $\lambda = 0$, Wilson-Fisher field = Gaussian field

(unstable $d+1 < 4$)

Ⓑ $g \to 0$, weak Coupling between F.S. and WF field

$$\Sigma(\vec{k}, \omega) = \tilde{g}^2 \Sigma(\tilde{k}, \tilde{\omega}), \qquad \tilde{g} = g k_F^{d-2}, \quad \tilde{k} = \frac{k}{k_F} \text{ etc.}$$

Self-energy

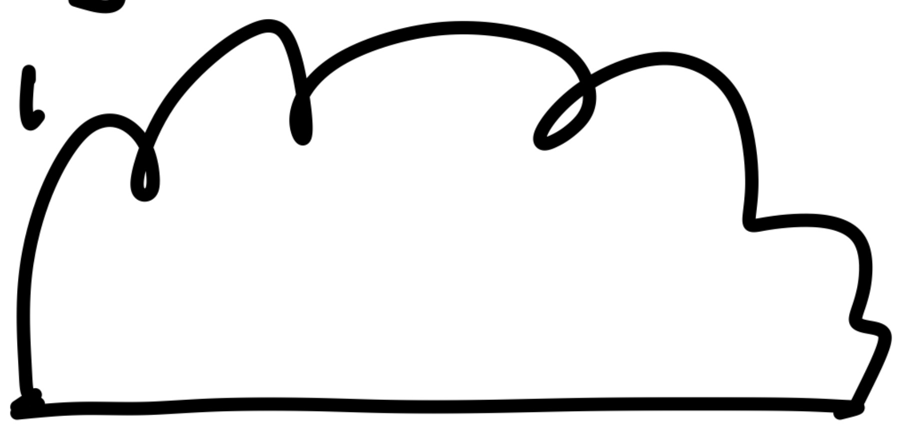
Generally $\quad G = G_0 + G_0 \Sigma G \quad$ or $\quad G^{-1} = G_0^{-1} - \Sigma$

perturbation theory :

$$G_k = \langle 0 | -i T \, \psi_k(0) \, \psi_k^{+}(0) | 0 \rangle$$

$$\delta G = G_0 \Sigma^{(2)} G_0 , \quad \Sigma^{(2)}(\vec{k}, \epsilon) = i \int D_0(\vec{q}, \Omega) \, G_0(\vec{k}+\vec{q}, \epsilon+\Omega) \times g^2$$

q, Ω



$\vec{k}+q, \epsilon+\Omega$

$$D_0(\vec{q}, \Omega) = \frac{1}{\Omega^2 - w_{\vec{q}}^2 + i\delta}$$

$$G_0(\vec{k}, \epsilon) = \frac{1}{\epsilon - (\epsilon_{\vec{k}} - \mu) + i\delta_{\vec{k}}}$$

Putting things togather for 3D

$$\frac{\partial \, \mathrm{Re}\Sigma(k_F, \epsilon)}{\partial \epsilon} \sim -g^2 \ln \frac{\Lambda}{|\epsilon|} \, ,$$

$$\mathrm{Im}\,\Sigma(k_F, \epsilon) \sim g^2 |\epsilon| \, \pi \quad (\text{even function})$$

$$Z(k_F) \simeq \left. \frac{1}{1 + \frac{g^2}{4\pi^2} \ln \frac{\Lambda}{|\epsilon|}} \right|_{|\epsilon| \to 0} \to 0$$