Phys529B: Topics of Quantum Theory

https://phas.ubc.ca/~feizhou/phys%20529,%202021

Lecture 1: basic introduction to interacting fermions

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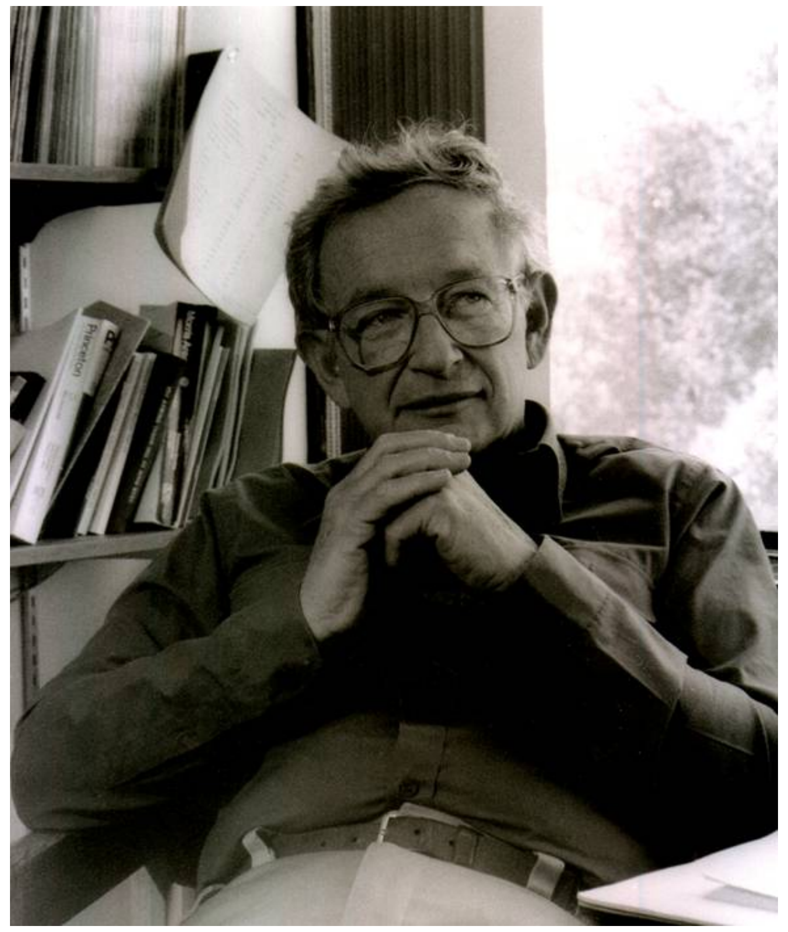
Interacting quantum many-body systems:

"emergent phenomena"



More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted

planation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps

less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it seems to me that one may array the sciences roughly linearly in a hierarchy, according to the idea: The elementary entities of science X obey the laws of science Y.

Large scale quantum phenomena can't be understood as a simple extrapolation of microscopic individual particles. Strong interactions lead more exotic phenomena.

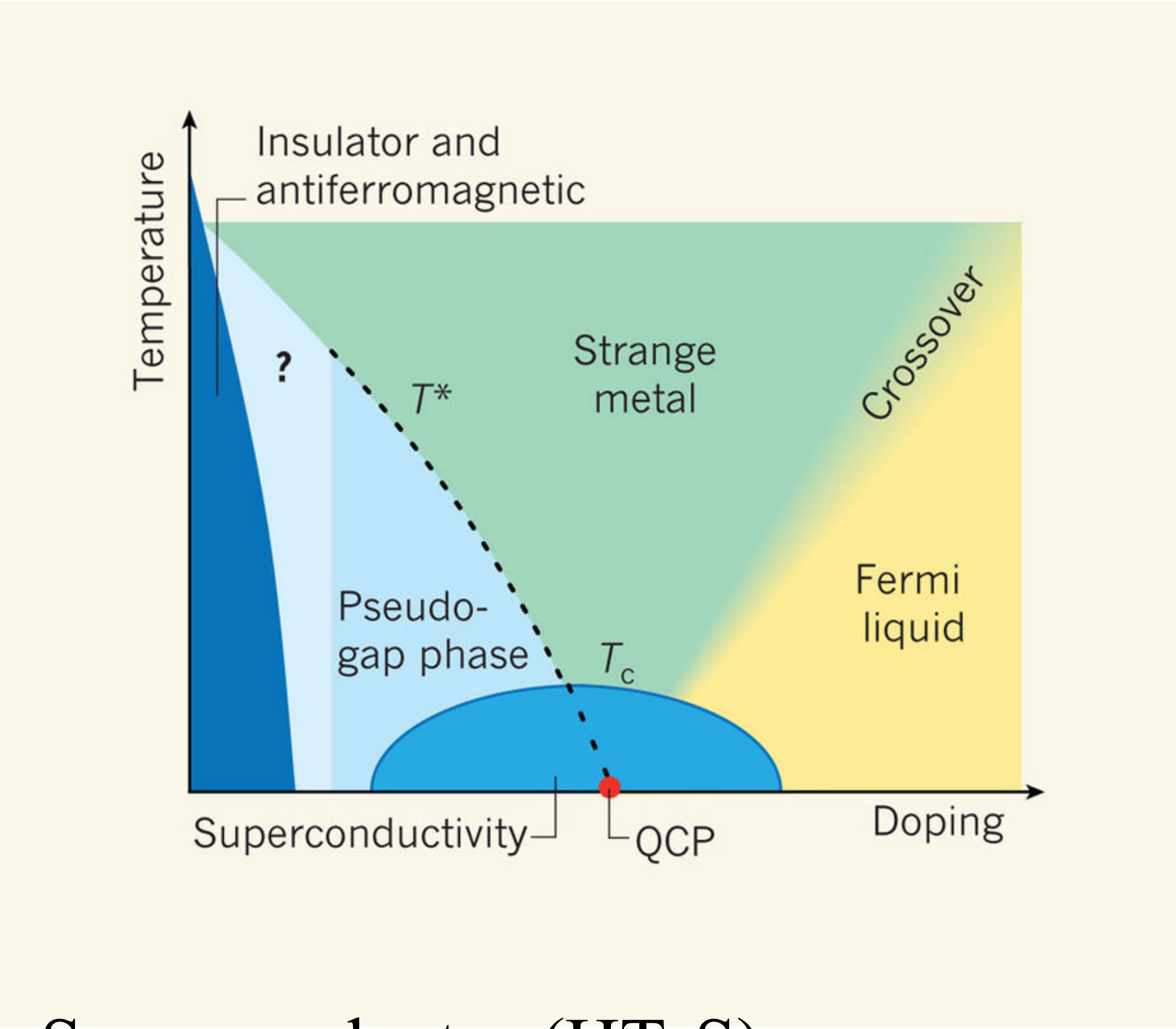
“emergent phenomena” and
why they are surprising?
More is different!

A) Quantum matter can break the symmetries of the microscopic interactions (Superconductor/superfluid, ferromagnetic-anti-ferromagnetic etc)

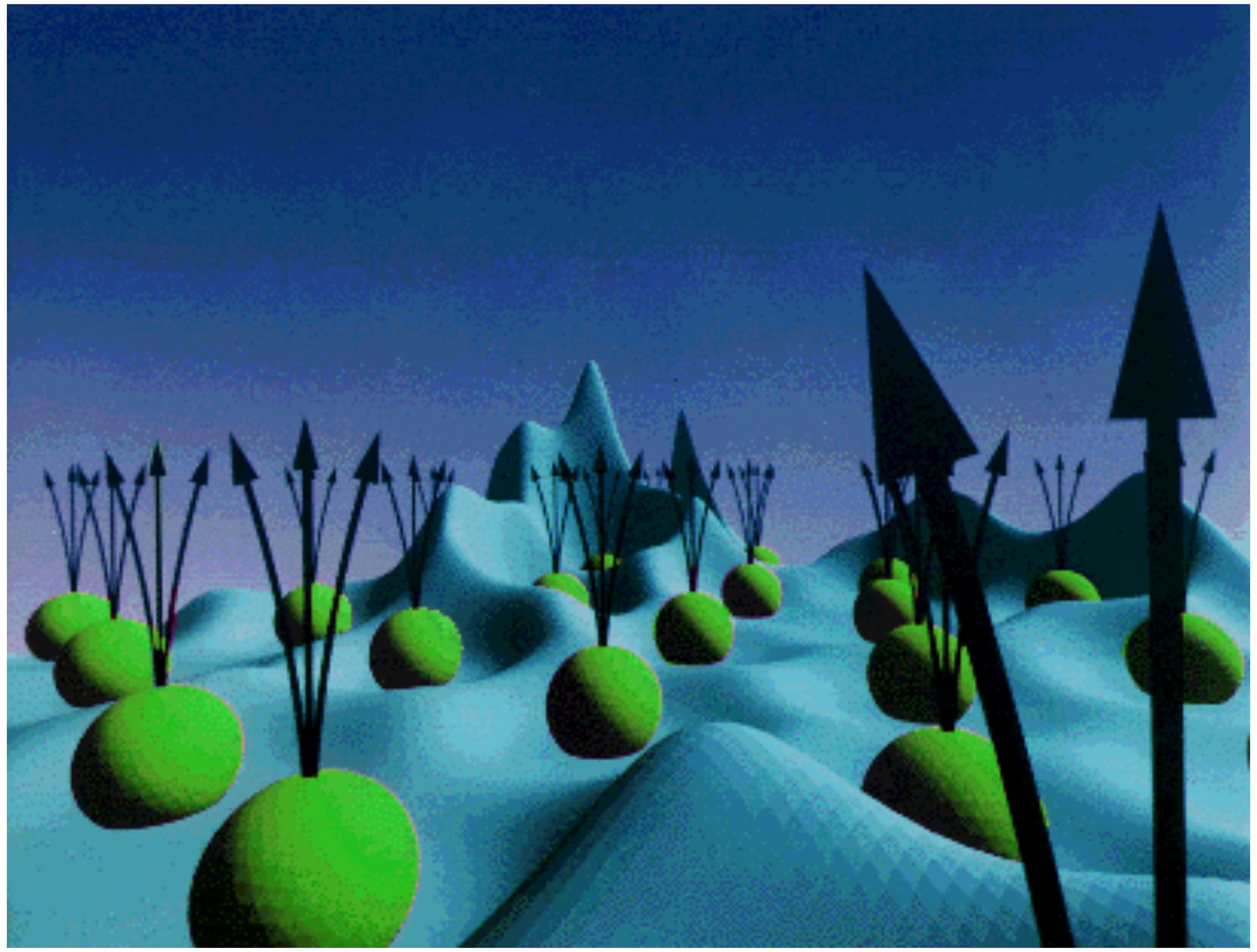
B) Strong interactions +high “degeneracy” due to either quantum or classical configurations; leading to fractionalization in Non-Fermi-liquids or even non-local ”topological order” (i.e.FQHs, spin liquids etc); an exciting frontier.

C) Strong interactions but fine tuned to certain points where scale (and/or conformal) symmetries emerge as a large scale quantum phenomena.

Non-Fermi liquids in Emergent Quantum Phenomena as " More is different"



Superconductor (HTcS)
1) electrons can condense and super-flow!
2) strongly interacting electrons form a strange metal or Non-Fermi liquid above high T.



Fractional Quantum Hall (FQH) as an anyon liquid: Laughlin state with 1/3 electron per flux in B-field Quasi- particles carrying 1/3 of electron charges.

- Possible quantum states of fermions (Without spontaneously breaking symmetries)
- 1) Fermi Gases; ——> 2) Femi Liquids; ——> 3) non-Fermi liquids;
- 4) incompressible QH/FQH liquids (in magnetic fields); 5) Mott insulators (in lattices)…

- Possible quantum states of fermions (With broken symmetries)
- 1) Superconductors; 2) charge density waves/ Spin density waves (in lattices);…

- Possible quantum states of fermions (Without spontaneously breaking symmetries)

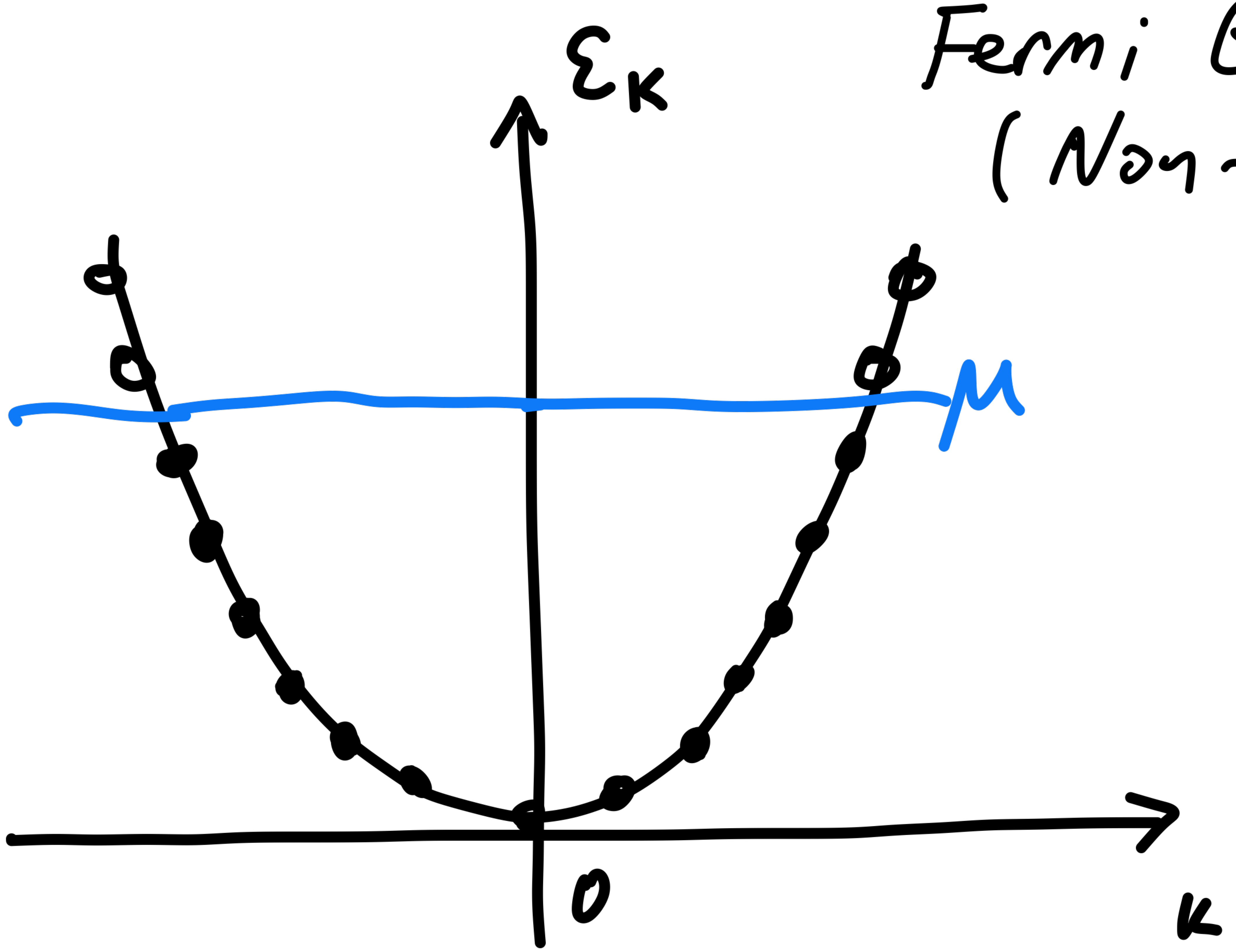
- 1) Fermi Gases; ——> 2) Femi Liquids; ——> 3) non-Fermi liquids;

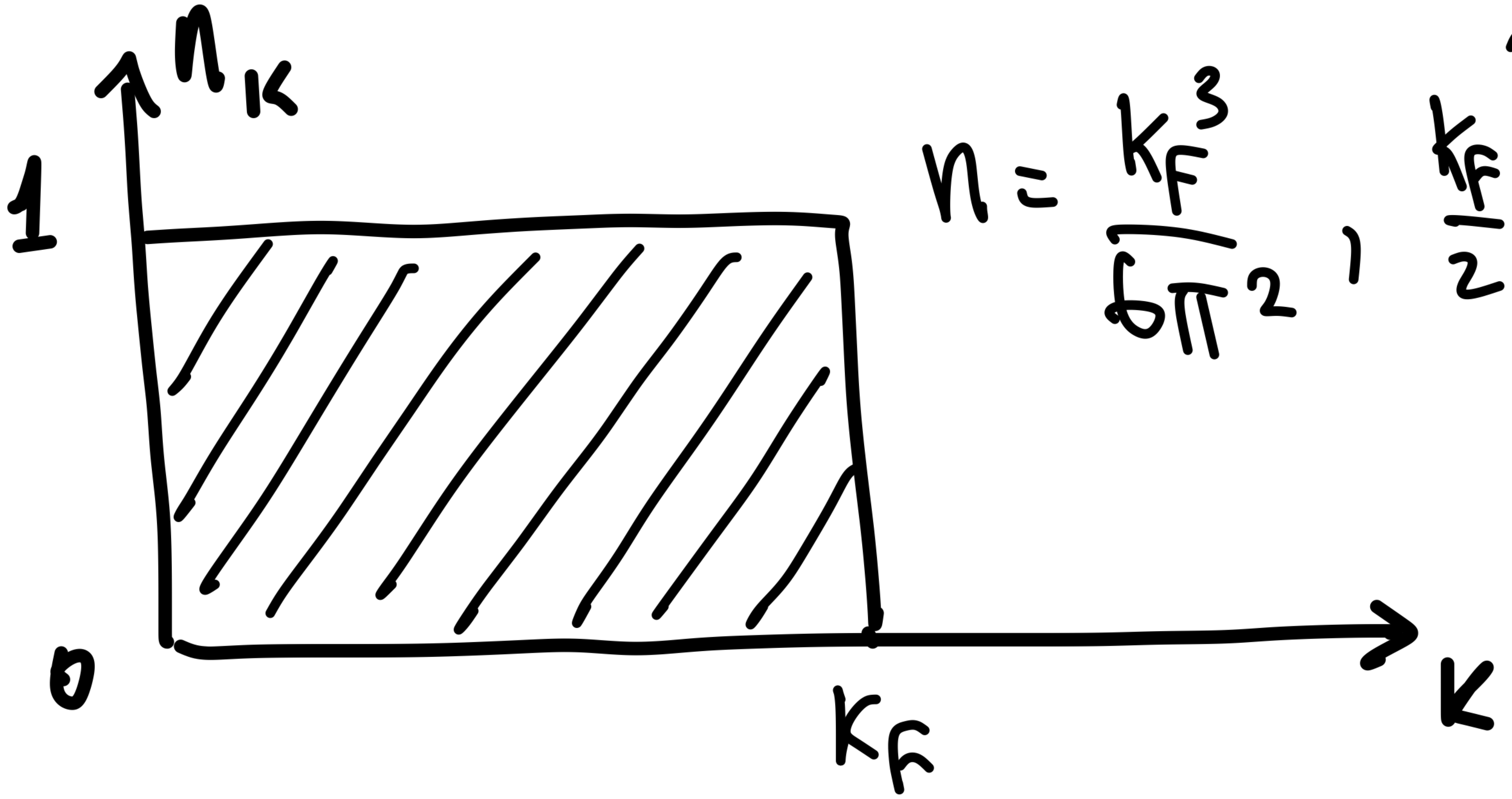
- 1) Fermi Gases (non-interacting): all low energy excitations are fermions.

- 2) Femi Liquids (interacting): low energy excitations are emergent fermionic quasi-particles with spin-1/2 and with renormallzed properties; moreover there are emergent bosonic excitations.

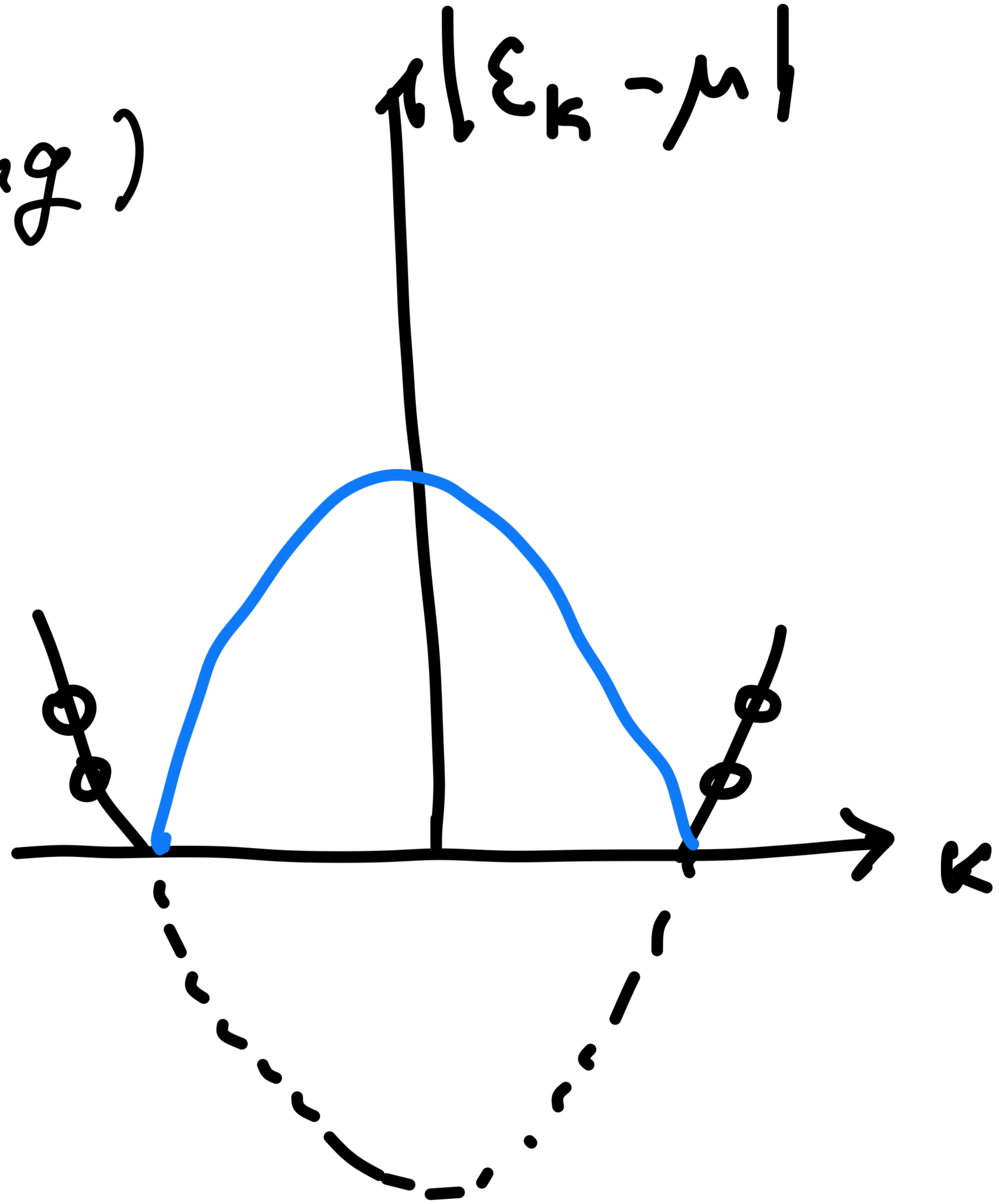
- 3) non-Fermi liquids;: a) NO fermionic excitations at all at low energy sectors; fully bosonized; b) NO well-defined fermions at fermion surfaces (like a molasses); c) only anyons in low energy sectors (abelian or non-abelian); [fermionic quasi particles but no spin or chagres so that fermions are fractionalized]

Fermi Gas
(Non-interacting)

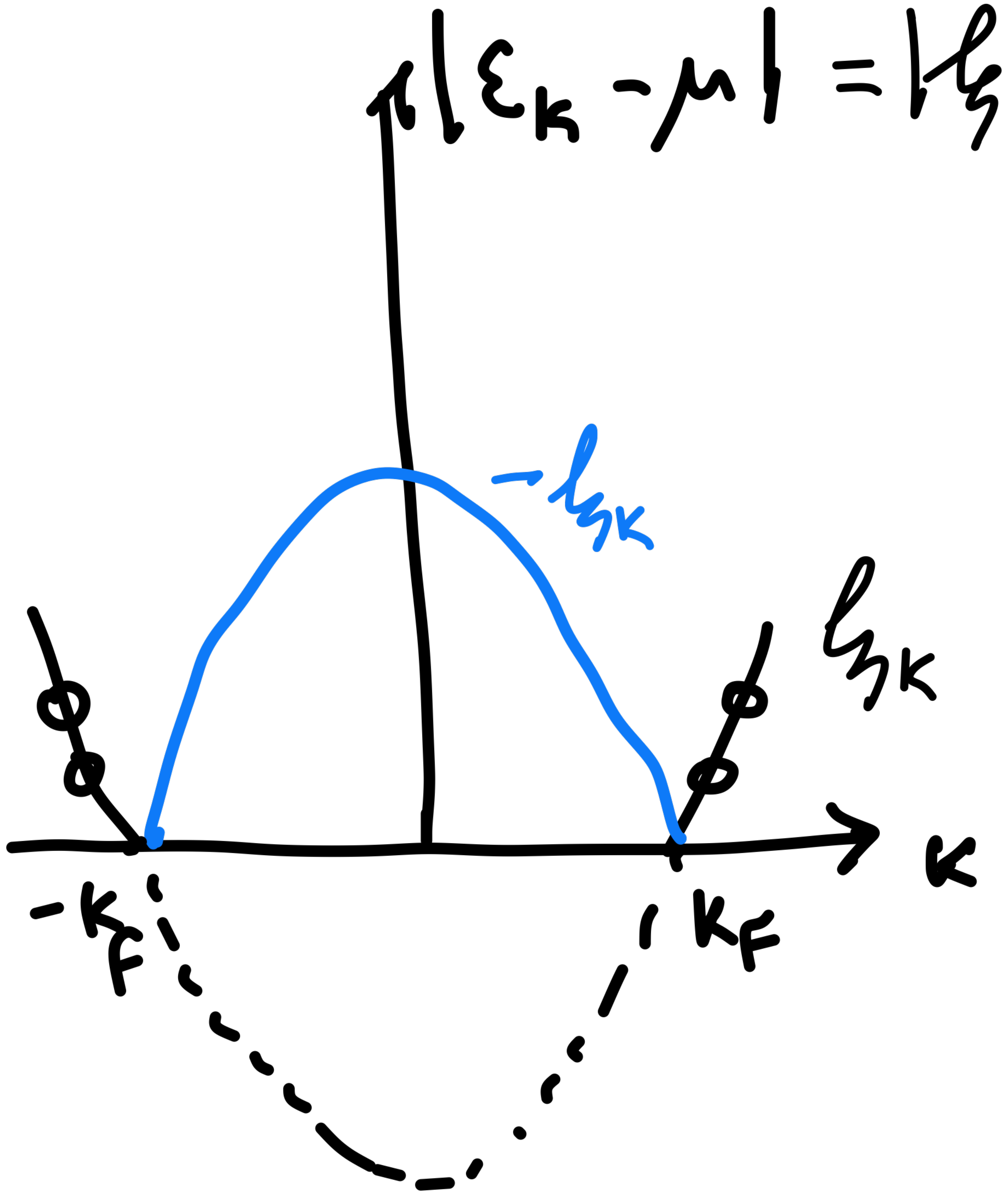




$$n = \frac{k_F^3}{6\pi^2}, \quad \frac{k_F^2}{2} = \mu$$

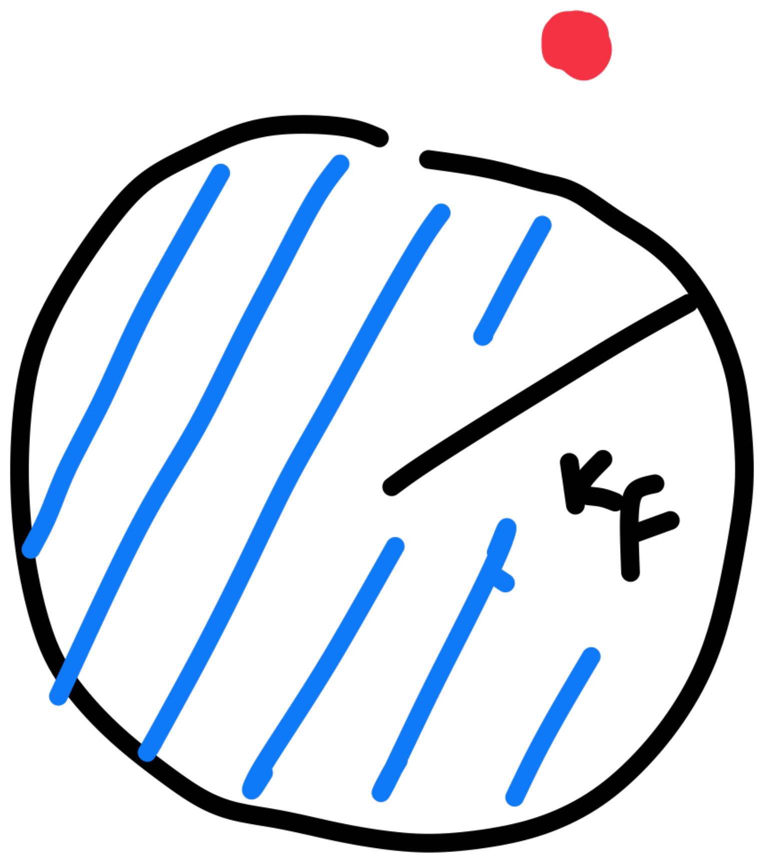


"Excitation Spectrum"



Fermonic Excitation Spectrum

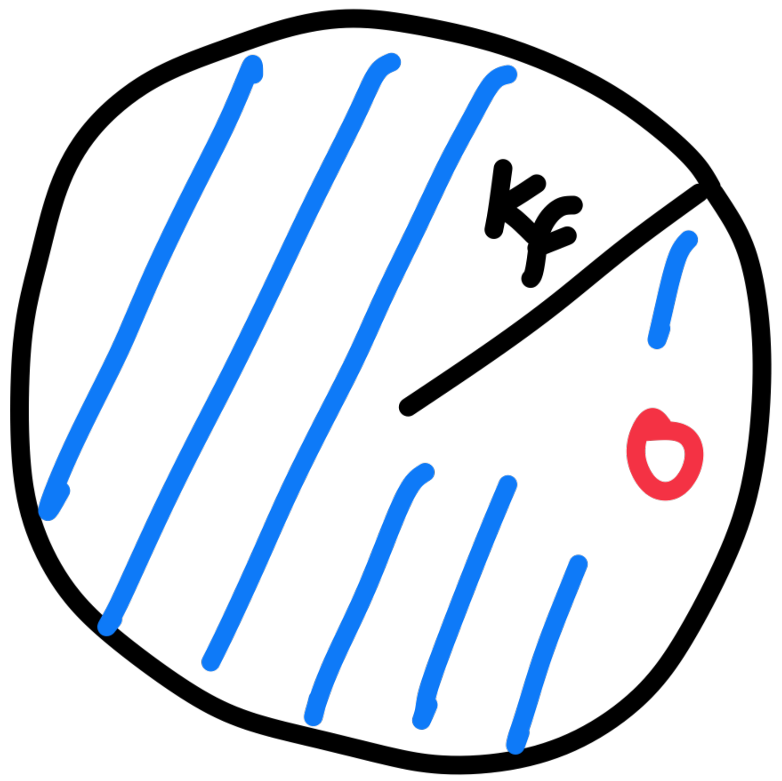
$$\xi_k = \epsilon_k - \mu$$



Particle-like excitations

$$\xi_k = \varepsilon_k - \mu > 0$$

Forward propagating



Hole-like excitations

$$-\xi_k = \mu - \varepsilon_k > 0$$

Backward propagating

Dynamics (Time Ordered Green's function)

$$G(k,t) = -i \langle g.s.| T \psi_k(t) \psi_k^+(0) | g.s. \rangle$$

$$T \psi_k(t) \psi_k^+(0) = \psi_k(t) \psi_k^+(0)\, \theta(t) - \psi_k^+(0) \psi_k(t)\, \theta(-t)$$

$$G(k,t) = -i e^{-i\xi_k t} \theta(k-k_F)\, \theta(t) + i e^{+i\xi_k t} \theta(k_F-k)\, \theta(-t)$$

$$\left(\xi_k = \epsilon_k - \mu, \quad \mu = \frac{k_F^2}{2} \right)$$

$$G(k,\omega) = \frac{1}{\omega - \xi_k + i\eta_k}, \quad \eta_k = \delta \operatorname{sign} \xi_k \quad (\delta > 0)$$

$$\text{or } \delta \operatorname{sign} \omega$$