Phys529 Homework Set 2 (Posted March 22, 5pm; Due April 8, 5pm)
[*Q2 may be subjected to further updates.*]

One dimensional non-Fermi liquid phenomena are very special because of higher symmetries compared with high dimension ones. First all, there are no particle-hole continuum in 1D when the dispersion is linearized near the Fermi momenta. This is in striking contrast to 2D and 3D we talked about before, leading to the concept of bosonization. Second, because of absence of a Fermi surface (i.e. instead, there are Fermi points), dynamics near Fermi points have relativistic features which can further lead to relativistic conformal symmetry, or CFT states. In this homework set, we are going to work out a few details or gaps that we have omitted during my lectures on 1D physics.

Q1: (See lecture notes L16, L17 and L18)
a) Verify that the current algebras we apply to construct the low energy theories in 1D.

$$[J_R(x), J_R(x')] = -\frac{i}{2\pi}\partial_x \delta(x-x'); [J_L(x), J_L(x')] = +\frac{i}{2\pi}\partial_x \delta(x-x');$$

b) In the presence of a periodic potential due to an underlying lattice, fermions are further coupled to an external scalar potential $H_{ext} = \int \rho(x) V_0(x), V_0(x) = \cos(2k_F x)$. Find out the effect of this term on excitation spectrum using the Dirac fermion formulation.

c) How does the interaction in b) look like in the bosonized form? Can you verify your conclusion in b) using in the bosonized form? Hint: How does the local density operator looks like in terms of boson fields?

d) (Optional) Using the basic results in the bosonization approach, find out the effect of Umklapp interaction term on excitations in 1D Fermi gas.

$$H_u = V_u \int \psi_R^\dagger \psi_R^\dagger \psi_L \psi_L$$

Q2: (See notes L19-L22) Consider massless Dirac fermions coupled with non-interacting relativistic massless real scalar field with a Yukawa form.

$$H_{Yukawa} = g_{YK}\int d\mathbf{r}\phi(\mathbf{r})\psi^{\dagger}(\mathbf{r})\psi(\mathbf{r})$$

As in the Wilson-Fisher theory, free massless scalar fields are unstable but at moment we ignore this issue and assume we fine tune to this limit. It is possible to further turn on scalar field interactions but part a) and b) you need to work on are actually not affected. So we work in a simplified limit. And the speed of fermions is ***different*** from that of bosons so the Lorentz symmetry is also absent in this model.

a) Obtain the RGE for Yukawa interactions using the one-loop approximation. Show that there appears to be a strong coupling fixed point for Yukawa coupling in dimensions lower than (3+1)D. Express your results as a function of dimension D, assuming space-time dimension D=4-epsilon.

b) Using the result in a), study the renormalization effect on fermion speed and boson speed via RGEs. Show that in the limit of infrared, there can be an emergent Lorentz symmetry. This emergent symmetry also appears near Wilson-Fisher fixed points of strongly interacting massless bosons.