Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode 17:
Application: General comments
(Wrap up)

HW Set III: Superfluid density (See 6B)

$$\rho = \rho(J, u, \ldots) = \rho(J-J_c, u, \ldots)$$

Heuristically, (Proof by the renormalizability of QFT.)

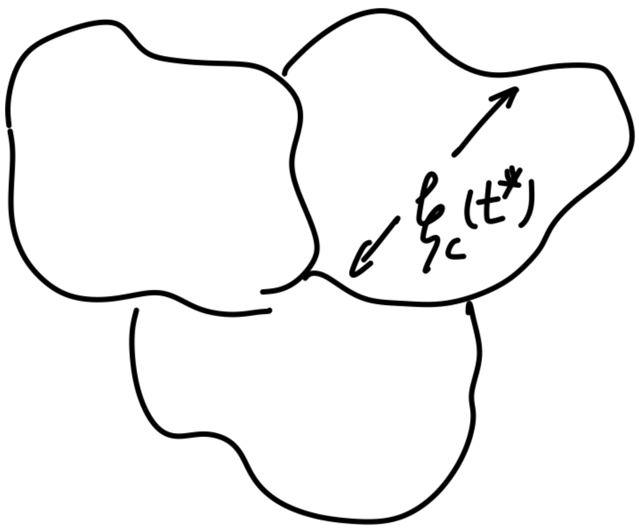
$$\rho \simeq \xi_c^{-d} \underbrace{\tilde{\rho}\left(\frac{J-J_c}{u}\right)}_{\text{Const}}, \qquad \underbrace{\xi_c}_{\text{Obtained by RGE in QFT}} = \xi_c(J-J_c)$$

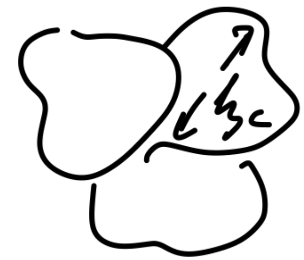
$$\frac{d\,\delta\tilde{m}^2}{dt} = -2\delta\tilde{m}^2 - a_d\tilde{g}^2 + b_d\delta\tilde{m}^2\tilde{g}$$

(See 6A)

Cartoons (final state) of Quenched defects

slow Vs fast

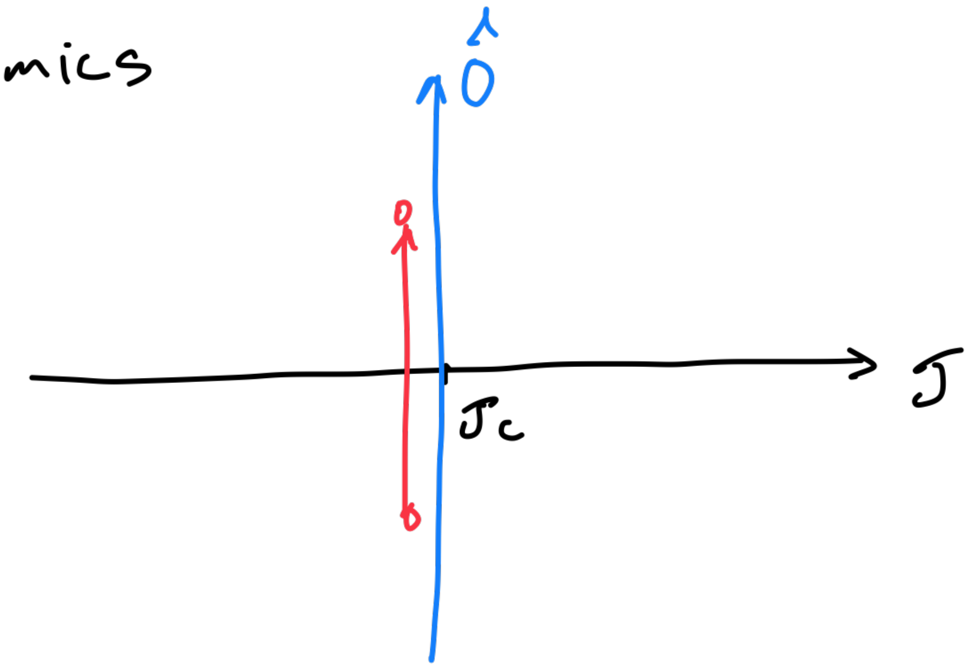




$$n(v) \sim \xi_c(t^*)^{-d} = v^{\frac{\nu d}{z\nu+1}}$$

density of defect excitations

General dynamics



P_L, the probability of finding excitations with "L" size?

$$P_L \approx \left| v \tau_L^2 \langle L | \hat{O} | 0 \rangle \right|^2, \qquad \hat{O} = \int \hat{L}(x)\, d^{(d)}x$$

Only depends on the scaling dimension of $\hat{L}(x)$

General: beyond heuristics $\qquad (\ P_n = |a_n|^2\ )$

$$P_L \approx \left| v\,\tau_L^2 \langle L | \hat{O} | 0 \rangle \right|^2, \qquad \hat{O} = \int \hat{L}(x)\, d^{(d)}x$$

if $x \to e^{\lambda} x'$, $\quad \hat{L}(x) \to \hat{L}'(x') = e^{\eta\lambda} \hat{L}(e^{\lambda} x')$

η the scaling dimension of $\hat{L}(x)$

then

$$P_L \approx \left(v\, L^{2z+d-\eta} \right)^2 + \cdots$$

Scale Transformation Vs Matrix element

A) : $\hat{O} = \int d^{(d)}_x \hat{L}(x)$,

$$\hat{L}(x) \to \hat{L}'(x') = e^{\eta_L \lambda} \hat{L}(e^{\lambda} x')$$

$$\hat{O} \to \hat{O}' = e^{\lambda(d-\eta_L)} \hat{O}$$

Example : (Recall !! See 4B, 5A etc.)

$$\langle \varphi^*(\vec{R}) \varphi(\vec{R}') \rangle \sim \frac{1}{R^{\beta}}$$

($\beta = d-1$ in the MF)

Scale invariance suggests $\varphi(R) \to \varphi'(R') = e^{\eta_\varphi \lambda} \varphi(R' e^{\lambda})$

$$\eta_\varphi = \frac{\beta}{2} = \frac{d-1}{2}$$

$\hat{L}_2(x) = \varphi^*(\vec{R}) \varphi(\vec{R})$, $\eta_{L_2} = 2\eta_\varphi = d-1$

$\hat{L}_4(x) = \left(\varphi^*(\vec{R}) \varphi(\vec{R})\right)^2$, $\eta_{L_4} = 4\eta_\varphi = 2(d-1)$

In general, we can compute "η" by using QFT.

Scale Transformation Vs Matrix element

So that

$$\vec{x} \to e^{\lambda} \vec{x}', \quad \hat{O}(\{L(\vec{x})\}) \to e^{\lambda(d-\eta_L)} \hat{O}(\{L(\vec{x}')\})$$

Example:

$$\hat{O} = \int d\vec{x} \; \varphi^{\dagger}(\vec{x}) \varphi(\vec{x}) \to$$

$$\hat{O} = e^{\lambda(d-\eta_L)} \int \underbrace{d\vec{x}' \; \varphi^{\dagger}(\vec{x}') \; \varphi(\vec{x}')}_{\hat{O}'}$$

B) $\vec{x} \to e^{\lambda} \vec{x}', \quad |L\rangle \to |e^{-\lambda} L\rangle$ At QCP Only

$$\langle L | \hat{O} | 0 \rangle = \langle e^{-\lambda} L | \hat{O} | 0 \rangle \, e^{\lambda(d-\eta_L)}$$

$$\to \langle L | \hat{O} | 0 \rangle \sim L^{(d-\eta_L)}$$