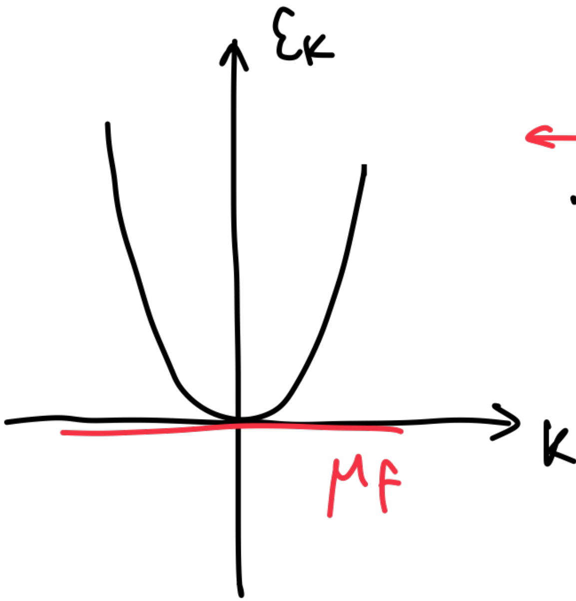
Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

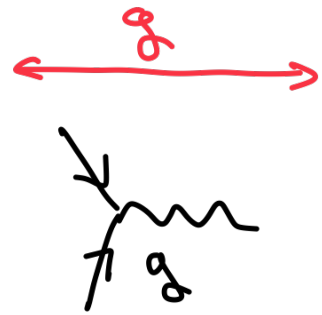
Episode 14:
Application: Sweeping across a QCP:
introduction to quantum dynamics of Kibble-Zurek type

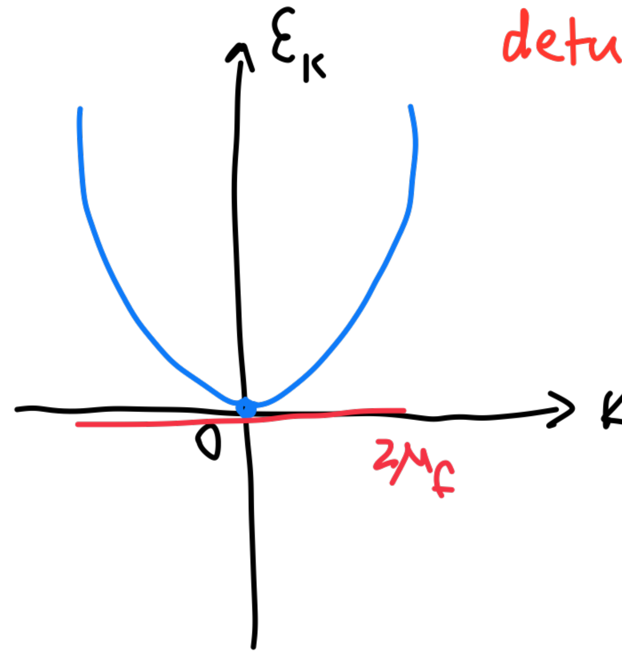
At Feshbach Resonance

$\varepsilon_b^{(R)} = 0$

detuning







Atoms

Feshbach Molecules

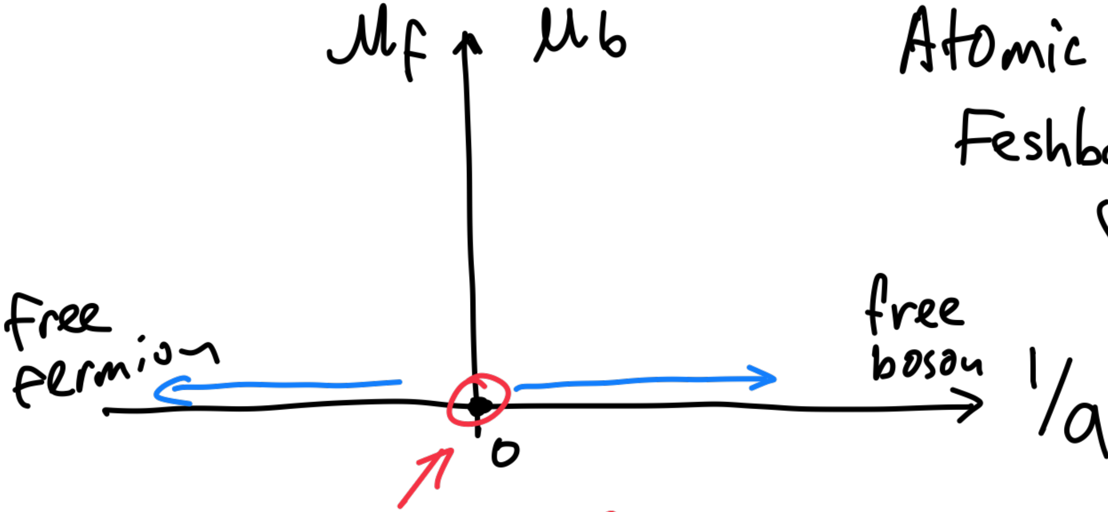
$\mu_F = 0$

$\mu_b = 0$

$$\mu_b = 2\mu_F - \varepsilon_b^{(R)} = 2\mu_F = 0$$

$$\varepsilon_b^{(R)} = \varepsilon_b - \frac{g^2}{2} \Lambda_{uv} C_d = 0$$

Atomic Gas Near Feshbach Resonance
$(d=3)$



$\leftarrow$ Scattering length

Feshbach Resonance
(Strong Coupling fixed pt)

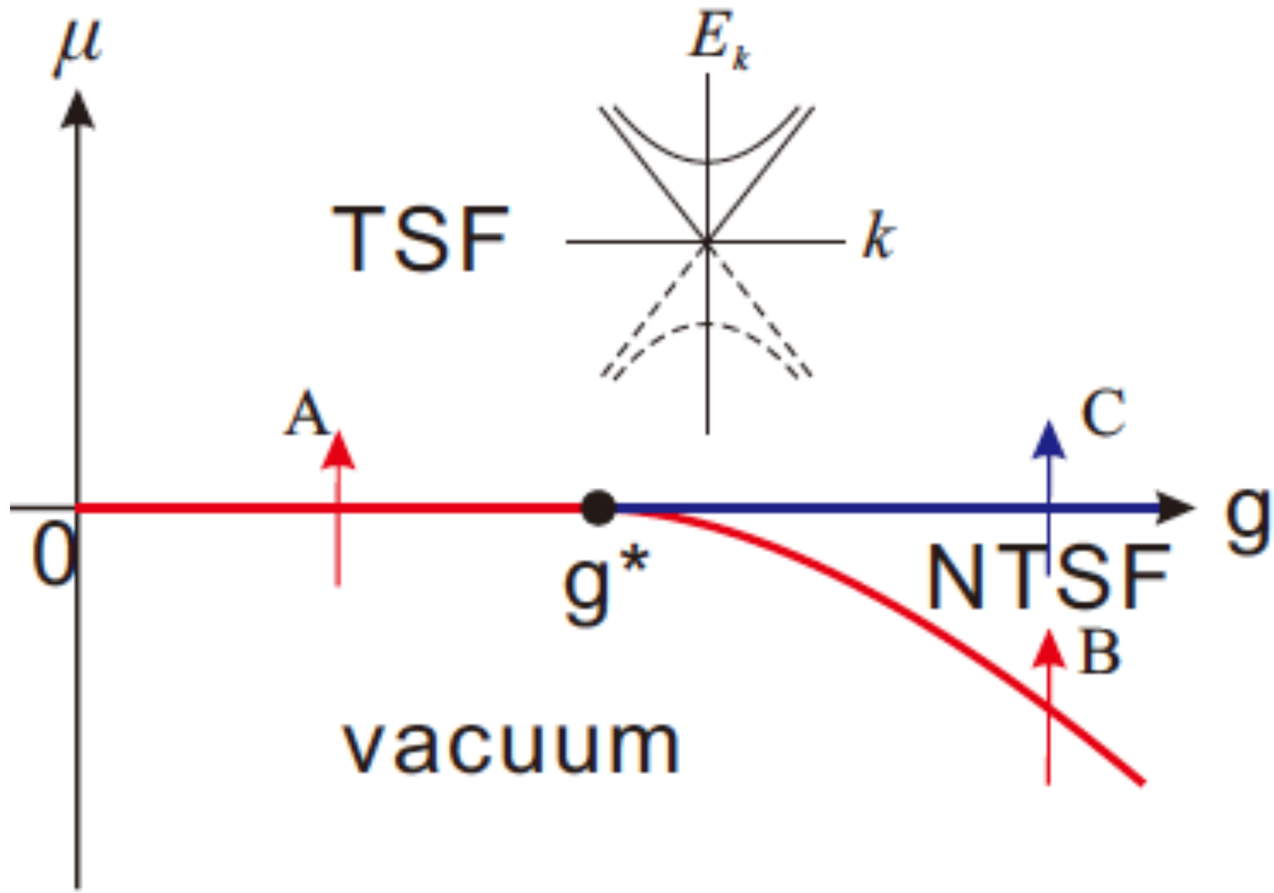
Feshbach Resonance

$= SO(2,1)$ CFT theory

with $Z_b = 0$ $\left(\tilde{g}^{*2} = 4-d\right)$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = \tilde{g}^2 = \tilde{g}^{*2} = 1 \longrightarrow Z_b(L) \sim \frac{1}{L}$$

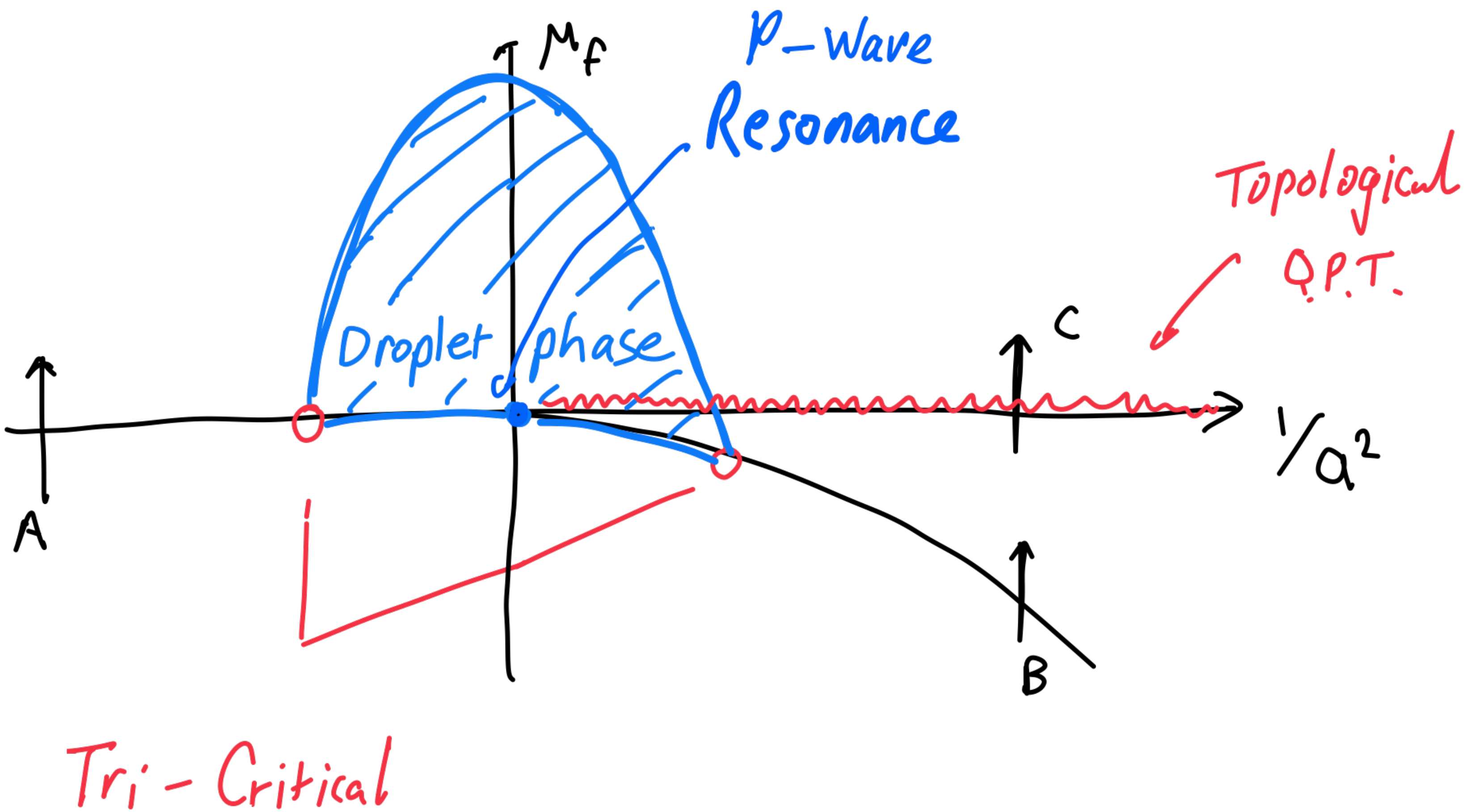
Example: Phase diagram of p+ip spinless SF



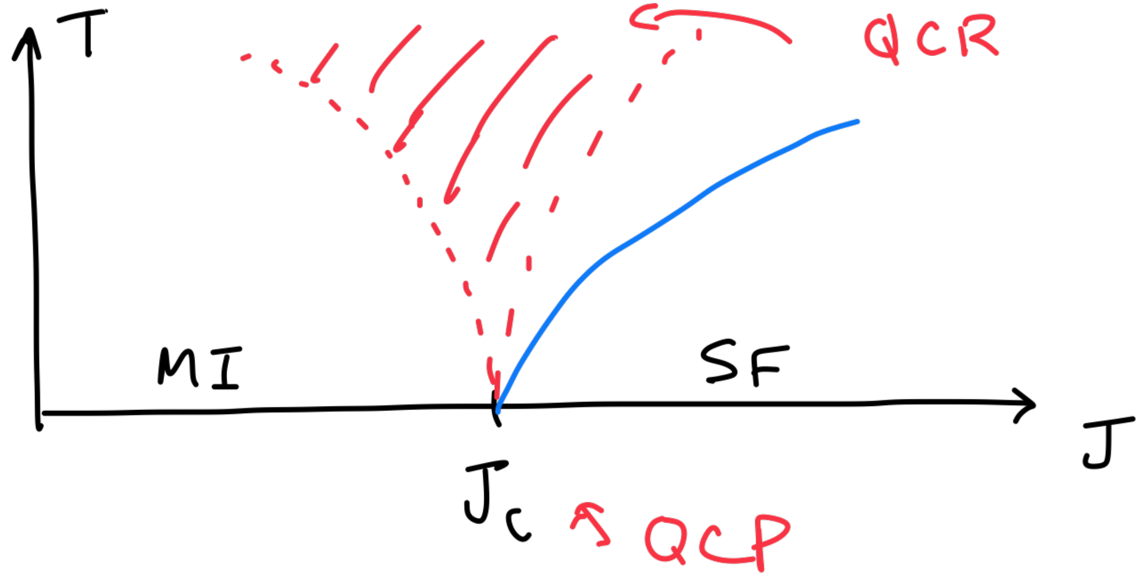
A: Free Fermion; B: Wilson-Fisher/Free boson; C: Majorana class
g*: QCP of SO(2,1) CFT.

QFT/RGE Applications to p-wave interactions:

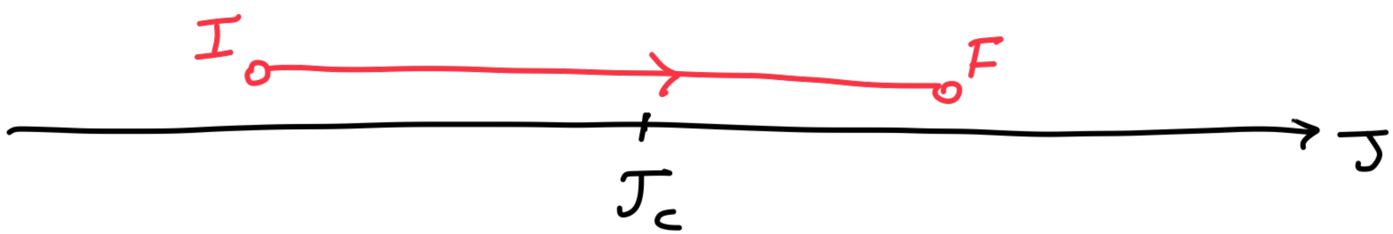
Tricritical Physics in Two-Dimensional p-Wave Superfluids, Fan Yang, Shao-Jian Jiang, and Fei Zhou, Phys. Rev. Lett. 124, 225701 (2020).



BHM

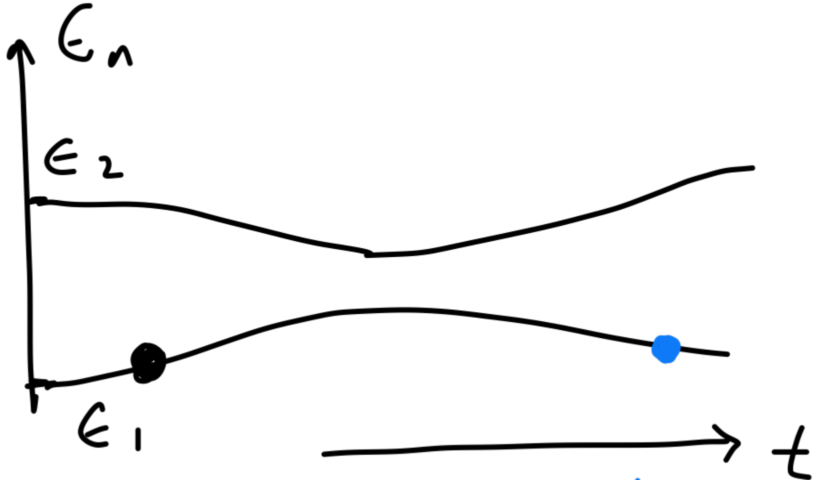


Universal transport dynamics in QCR near QCP
(See References provided in L5B)

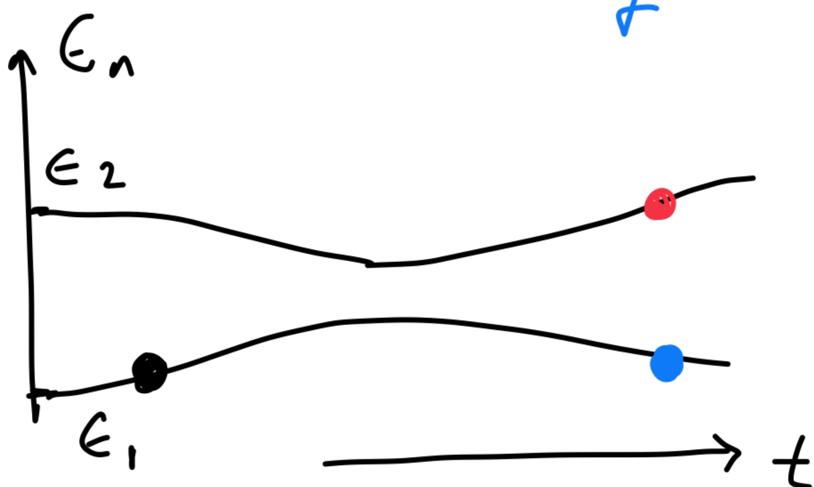


QM 100

Review of QM adiabatic theory, $T e^{-i\int_0^t H(t')dt'}$



adiabatically



Non-adiabatically

instaneous eigen-states

$$H(t)\,|n\rangle = \epsilon_n(t)\,|n\rangle$$

$n = 1, 2, \ldots$

$$\Gamma_n(t) = \int_{-\infty}^{t} \epsilon_n(t')\,dt'$$

$$\psi(t) = \sum_{n=1}^{\infty} a_n(t)\, e^{-i\Gamma_n(t)}\, |n\rangle$$

$$\begin{cases} |a_1(t)|^2 \neq 100\% \\ |a_n(t)|^2 \neq 0, \quad n = 2, 3, \ldots \end{cases}$$

QM 100

$$\psi(t) = \sum_{n=1}^{\infty} a_n(t)\, e^{-i\Gamma_n(t)} \, |n(t)\rangle,$$

$$\int_{-\infty}^{t'} \left(\epsilon_n(t'') - \varepsilon_1(t'')\right) dt''$$

$$a_n(t) \approx \int_{-\infty}^{t} \frac{\langle n | \frac{\partial H(t')}{\partial t'} | 1 \rangle}{\epsilon_n(t') - \varepsilon_1(t')} \, e^{i\left(\Gamma_n(t') - \Gamma(t')\right)} + \ldots$$

Gaped

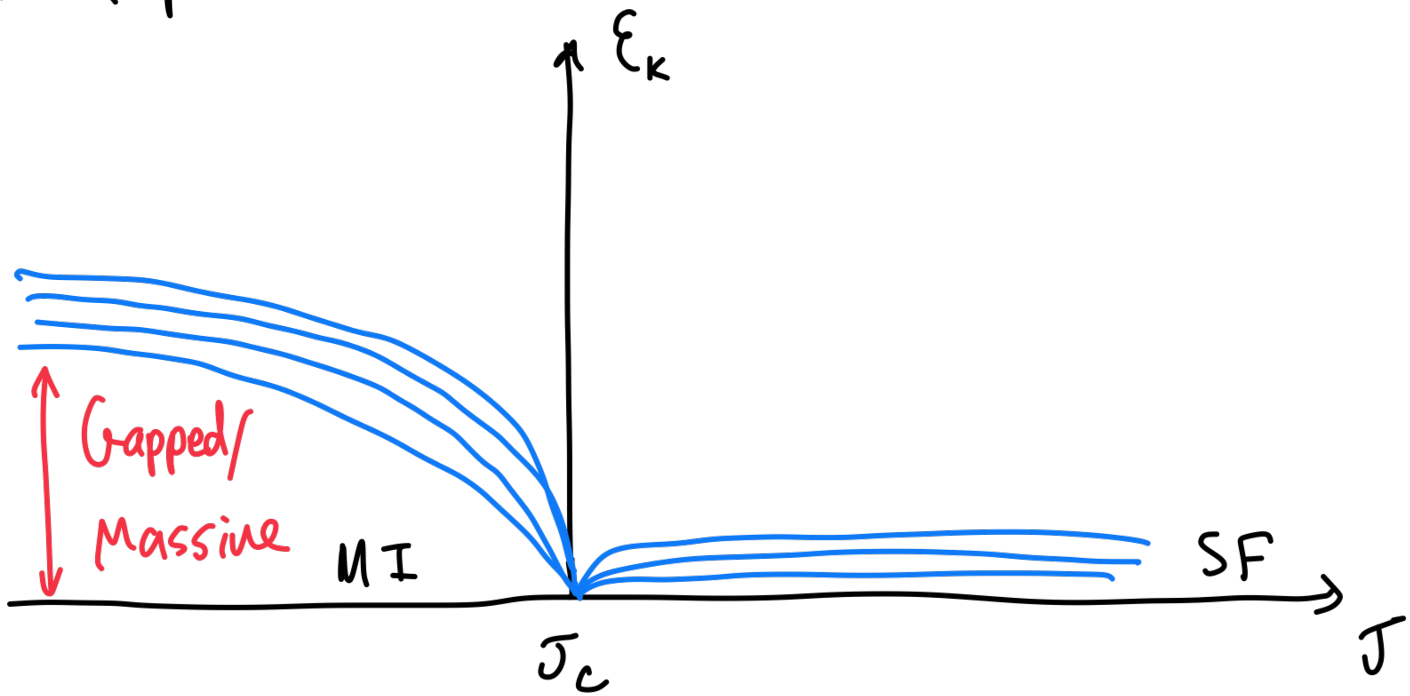
$$\sim \langle n | \tau_n^2 \frac{\partial H(t')}{\partial t'} | \underline{1} \rangle, \quad \tau_n \sim (\epsilon_n - \varepsilon_1)^{-1}$$

typical value

$$\frac{\partial H(t)}{\partial t} = v\, \hat{O}, \quad a_n \sim \tau_n^2 v \, \langle n | \hat{o} | \underline{1} \rangle$$

Many-body spectrum

Near QCP



What happens if we vary J as $J(t) = J_c + vt$?

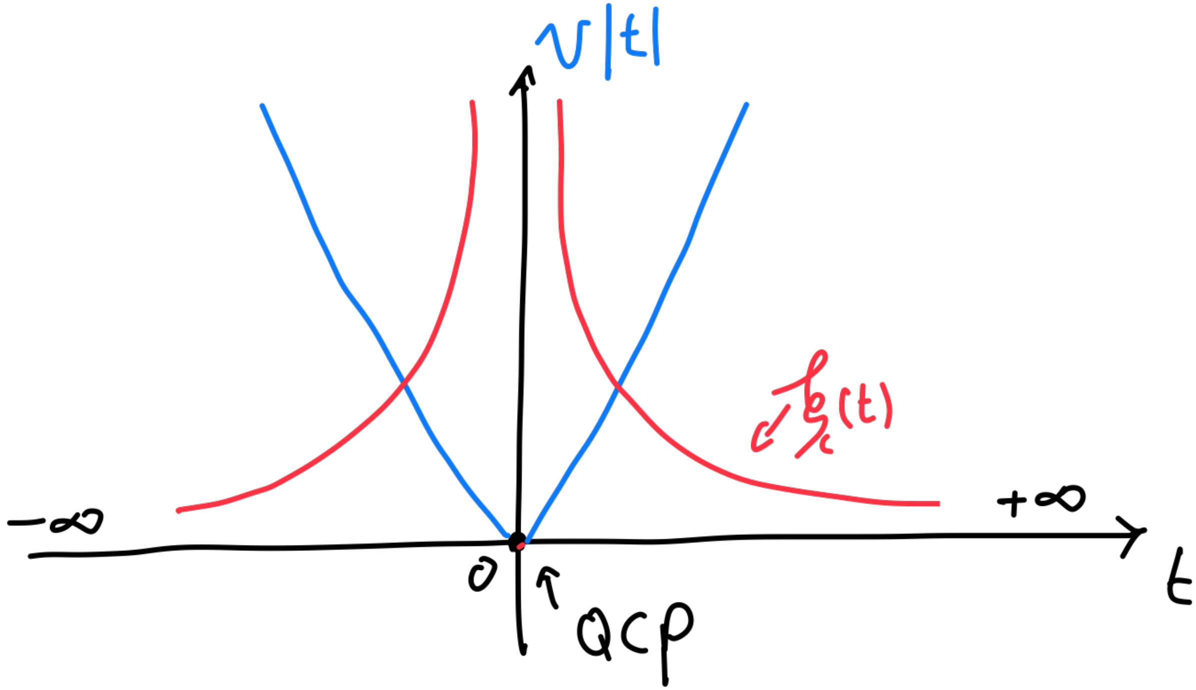
Universal Far-away-from equilibrium dynamics

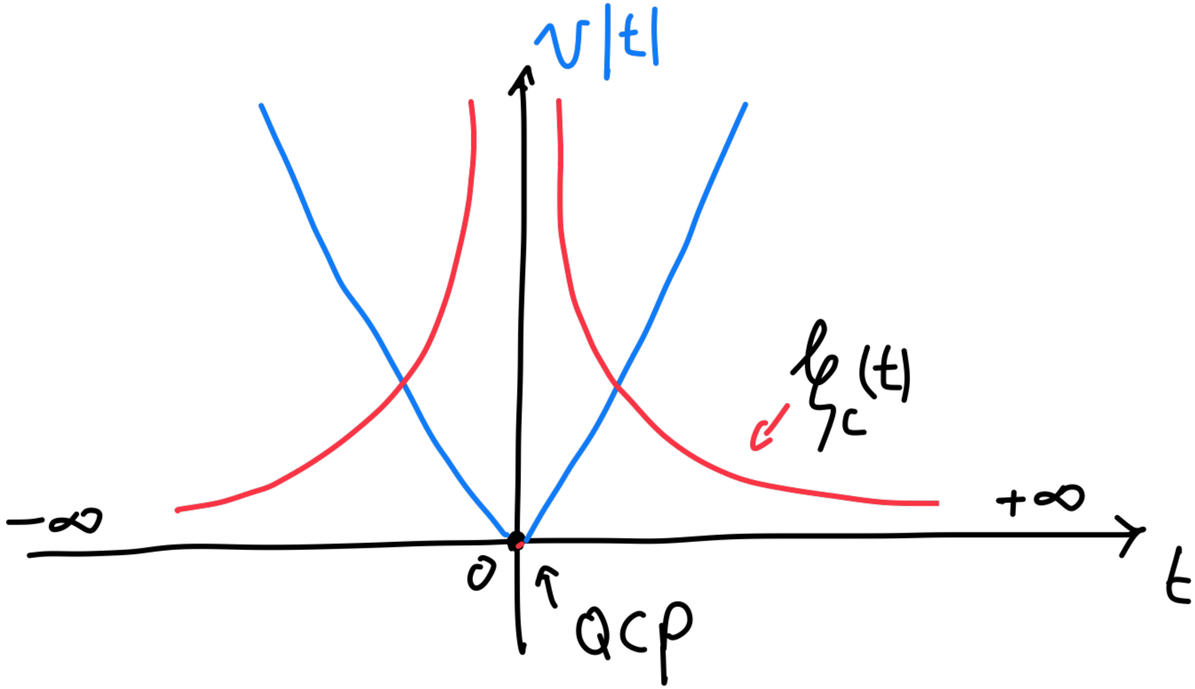
Heurist Arguments/statements

$$H(t) = H_{QCP} + (J(t) - J_c)\,\hat{O}$$

$\hat{O} = \int \phi^\dagger(\vec{r})\,\phi(\vec{r})$

$\delta H(t) = v t\,\hat{O}$





$$H(t) = H_{QCP} + \underbrace{vt}_{J(t)-J_c}\,\hat{O}$$

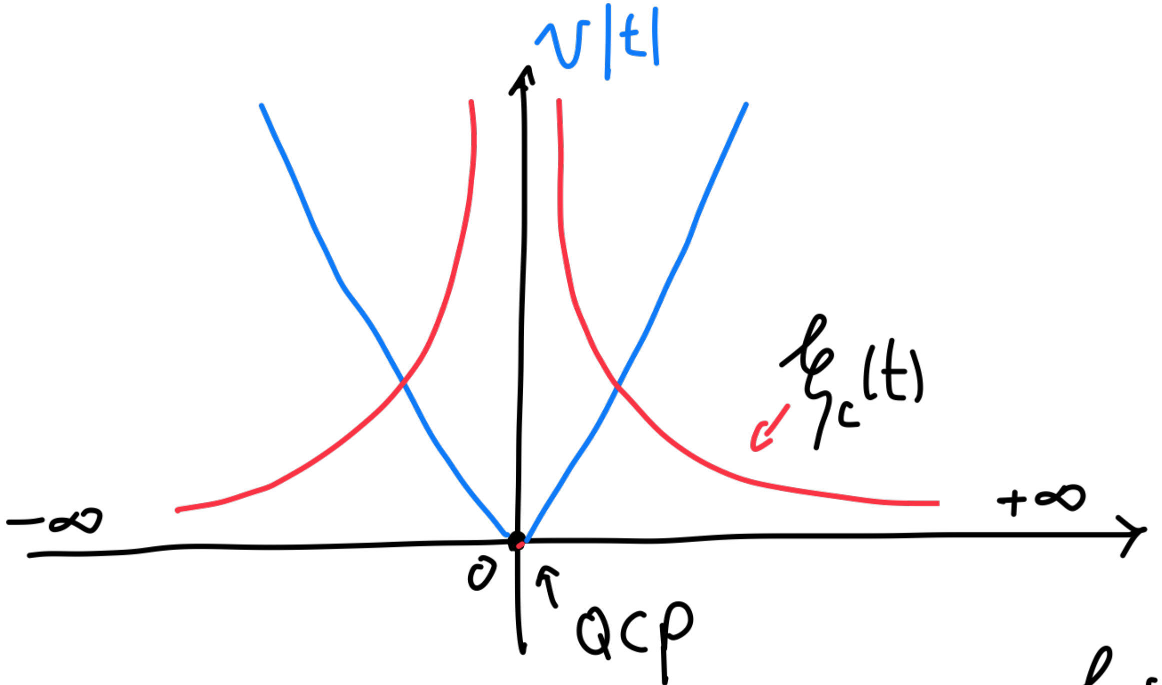
$$\xi_c \sim |J - J_c|^{-\nu}$$

$$\tau_c \sim \xi_c^{\,z}$$

$$\tau(t)\,\frac{d}{dt}\big(J(t)-J_c\big) \sim \big(J(t)-J_c\big) \quad \leftarrow \text{adiabatic breakdown}$$

$$\xi_c^{\,z}\, v \sim \xi_c^{-\frac{1}{\nu}} \qquad \text{or} \qquad \xi_c(t) \sim v^{-\frac{\nu}{z\nu+1}}$$

($z=1$ for QCP at $J=J_c$; $\nu = 1/2$ for $d \geq 3$)

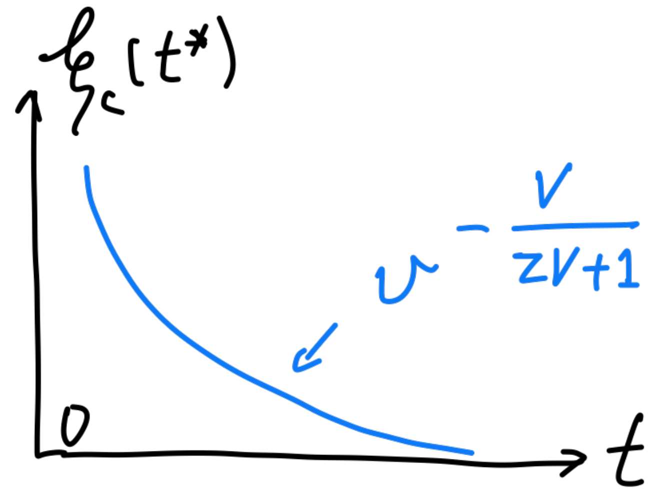


$$\xi_c \sim |J - J_c|^{-\nu}$$

$$\tau_c \sim \xi_c^{\,z}$$

$$\xi_c \sim (v t)^{-\nu}$$

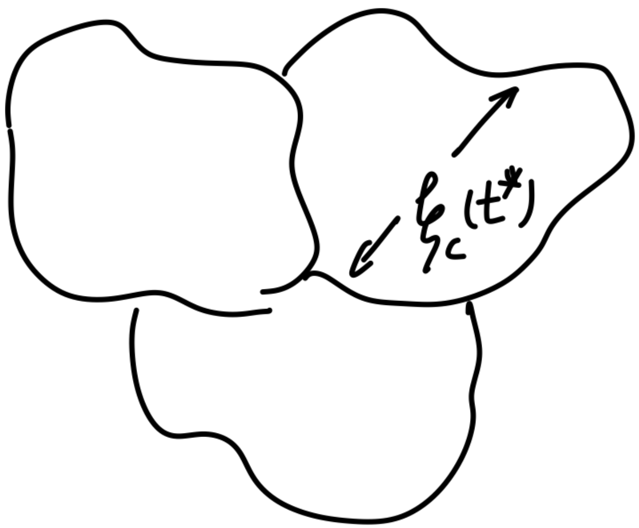
$$\xi_c(t^*) \sim v^{-\frac{\nu}{z\nu+1}}$$

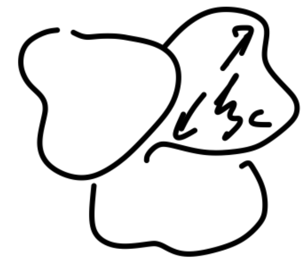


$\boxed{\text{Only the excitations of size larger than } \xi_c(t^*) \text{ will be excited.}}$

Cartoons (final state) of Quenched defects

slow Vs fast

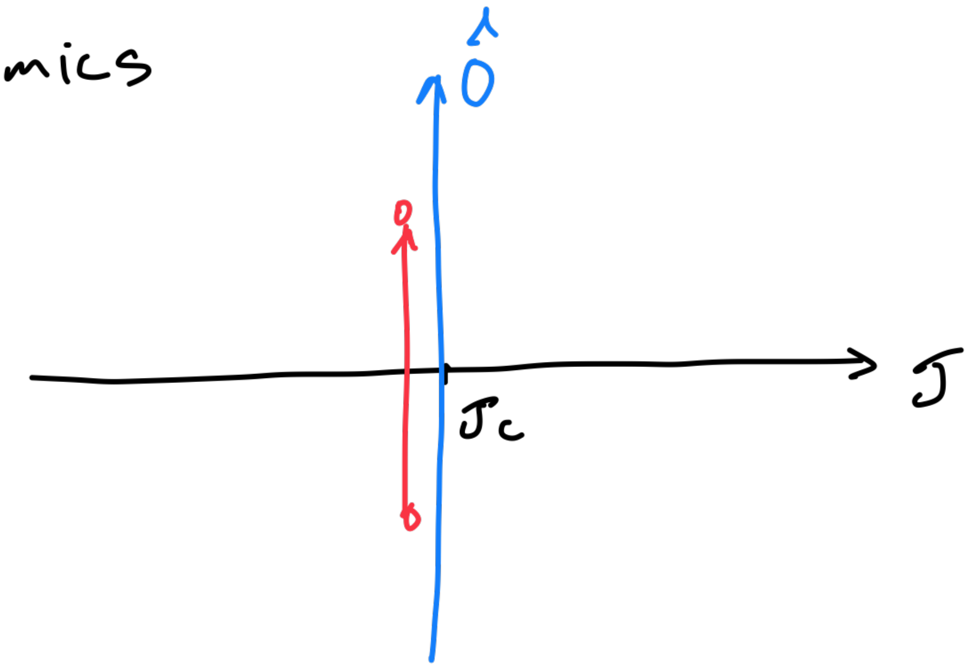




$$n(v) \sim \xi_c(t^*)^{-d} = v^{\frac{\nu d}{z\nu+1}}$$

density of defect excitations

General dynamics



P_L, the probability of finding excitations with "L" size?

$$P_L \approx \left| v \tau_L^2 \langle L | \hat{O} | 0 \rangle \right|^2 , \qquad \hat{O} = \int \hat{L}(x)\, d x^{(d)}$$

Only depends on the scaling dimension of $\hat{L}(x)$

General: beyond heuristics $\quad (P_n = |a_n|^2)$

$$P_L \approx \left| v \tau_L^2 \langle L | \hat{O} | 0 \rangle \right|^2, \qquad \hat{O} = \int \hat{L}(x)\, dx^{(d)}$$

if $x \to e^{\lambda} x'$, $\quad \hat{L}(x) \to \hat{L}'(x') = e^{\eta\lambda} \hat{L}(e^{\lambda} x')$

η the scaling dimension of $\hat{L}(x)$

then

$$P_L \approx \left(v L^{2z+d-\eta} \right)^2 + \cdots$$

Most likely excitations $L > L_c = v^{-\frac{1}{2z+d-\eta}}$.

(for $\hat{L}(x) = \varphi^{+}(x)\varphi(x)$, one can show that $z+d-\eta = \frac{1}{\nu}$.)

$\xi_c \sim |J - J_c|^{-\nu}$

then
$$P_L \approx \left(v\, L^{2z+d-\eta}\right)^2 + \cdots$$

(for $\hat{L}(x) = \varphi^{+}(x)\varphi(x)$, one can show that $z+d-\eta = \frac{1}{\nu}$.)

$$\hat{O} = \int d^d x \, \hat{L}(x), \qquad \hat{L}(x) \to \hat{L}'(x') = e^{\eta\lambda}\hat{L}(e^{\lambda}x')$$

$$\hat{O} \to \hat{O}' = e^{\lambda(d-\eta)}\hat{O}$$

$$\langle e^{\lambda}L|\hat{O}|0\rangle = \langle L|\hat{O}|0\rangle e^{\lambda(d-\eta)}$$

$$\longrightarrow \langle L|\hat{O}|0\rangle \sim L^{(d-\eta)}$$