Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode 14:
Application: z=2 free particle/Strong interacting fixed points of fermions

(A quick wrap up!)

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_{bf}$$

χ : Fermionic (e)

ϕ : Bosonic (2e)

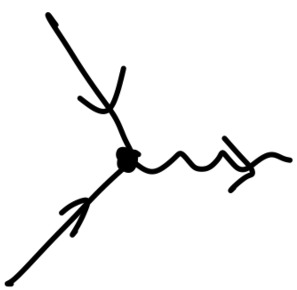
$$\mathcal{L}_f = \chi^*_\alpha \partial_\tau \chi_\alpha - \chi^*_\alpha \frac{\nabla^2}{2} \chi_\alpha - \mu_f \chi^*_\alpha \chi_\alpha$$

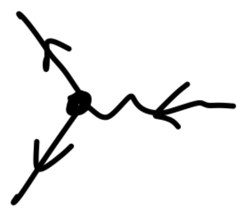
$\alpha = \uparrow, \downarrow$

$$\mathcal{L}_b = \phi^* \partial_\tau \phi - \phi^* \frac{\nabla^2}{4} \phi + (\epsilon_b \underbrace{- 2\mu_f}_{-\mu_b}) \phi^* \phi$$

$$\mathcal{L}_{bF} = g \chi^*_\alpha \sigma_{y,\alpha\beta} \chi^*_\beta \phi + c.c.$$

Anti-Symmetric





Grassmanian field

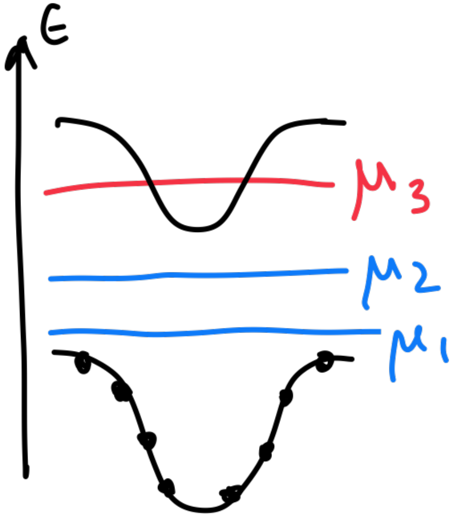
$$\{\chi_{\alpha R}, \chi_{\alpha' R}\} = \{\chi_{\alpha R}, \chi_{\alpha L}\} = 0$$

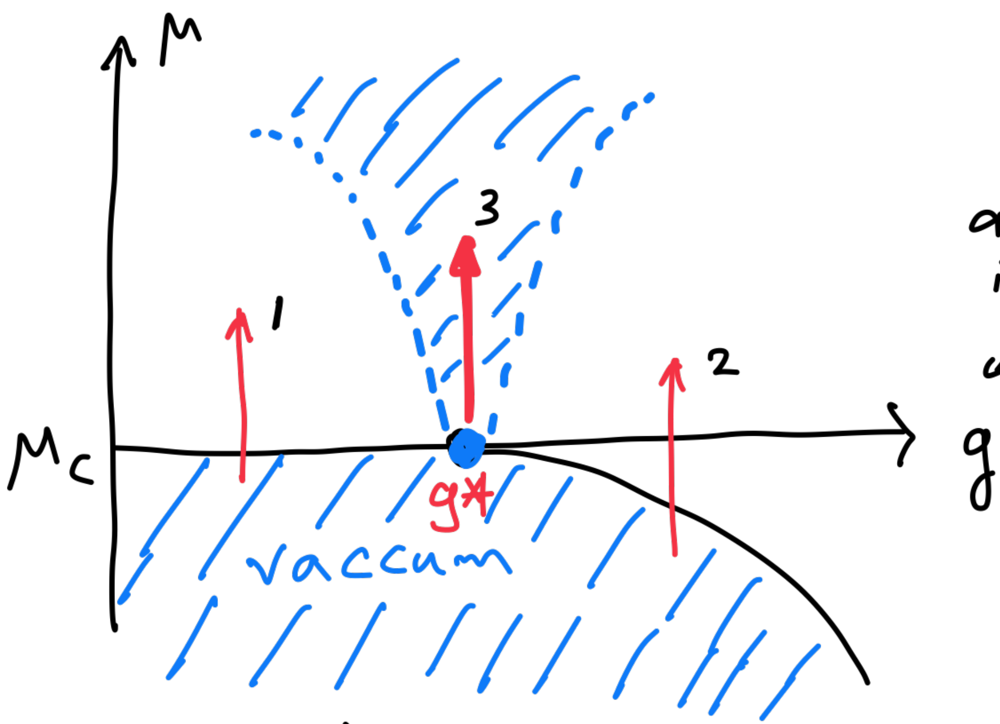
$$\{\chi_{\alpha L}, \chi_{\alpha' L}\} = \{\chi_{\alpha L}, \chi_{\alpha' R}\} = 0$$

$\chi_\alpha = \chi_{\alpha R} + i \chi_{\alpha I}$, $\chi^2_{\alpha R} = \chi^2_{\alpha I} = \frac{1}{2}$,

Related
$z=2$

General fermionic Model (3d)





attractive interaction

1 : free - fermion Universality class

2 : free - boson Universality class

3 : SO(2,1) Conformal field Theory / CFT

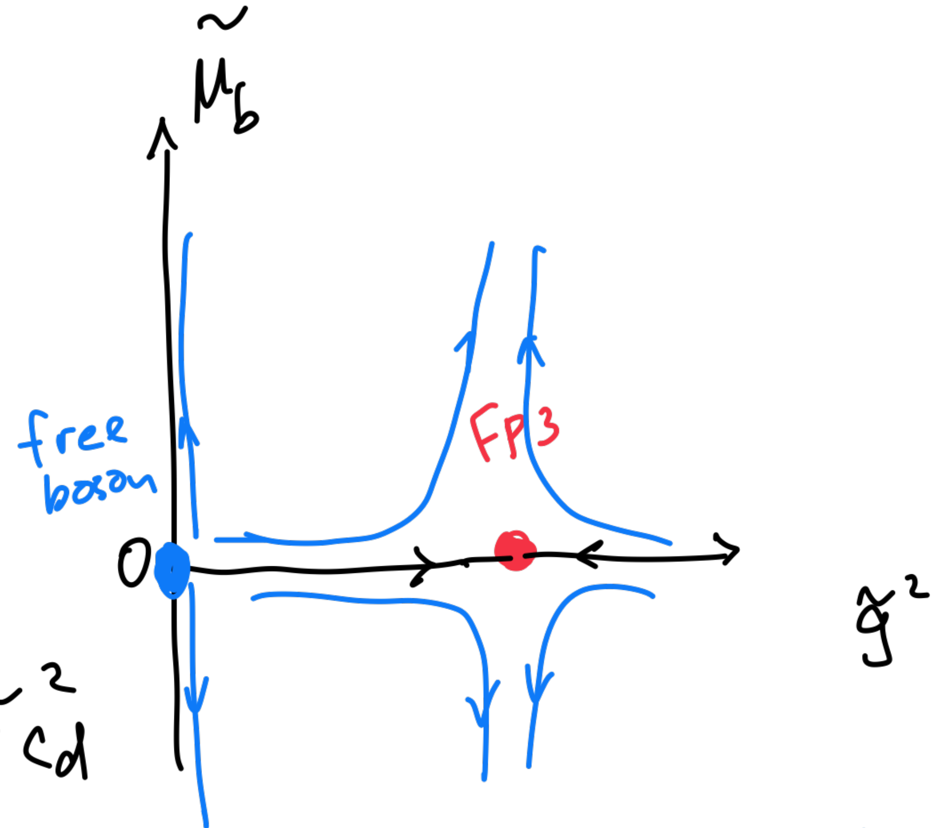
RGE

$$\frac{d\tilde{\mu}_f}{dt} = -2\,\tilde{\mu}_f$$

$$\frac{d\tilde{g}^2}{dt} = (d-4)\,\tilde{g}^2 + \tilde{g}^4$$

$$\frac{d\tilde{\mu}_b}{dt} = -2\,\tilde{\varepsilon}_b + 4\,\tilde{\mu}_f + \tilde{g}^2 c_d$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = \tilde{g}^2$$



fixed pt 2: $\tilde{\mu}_b^* = 0$, $\tilde{g}^{*2} = 4-d$

$\tilde{\mu}_f^* \to \infty$

$$\left(\beta_{\mu_b} = 0 \;\to\; 2\mu_f = \varepsilon_b(\Lambda) - \frac{\tilde{g}^{*2} c_d}{2}\Lambda\right)$$

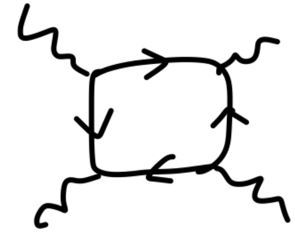
$d < 4$

fixed pt 2:

$\tilde{\mu}_b = 0, \ \tilde{g}^{*2} = 4 - d, \ \tilde{\mu}_f \to \infty$ = Bosons interacting with massive fermions

$\tilde{\mu}_b = 0,$

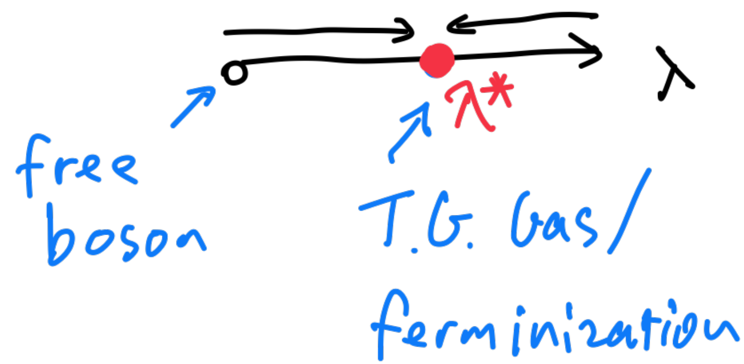
$$\mathcal{L}_{eff} = \varphi^* \left(\partial_t - \frac{\nabla^2}{2} \right) \varphi + \lambda |\varphi^* \varphi|^2 + \dots$$



$\tilde{\lambda} \propto \tilde{g}^{*4}$

$d = 2, 3$

$d = 1$



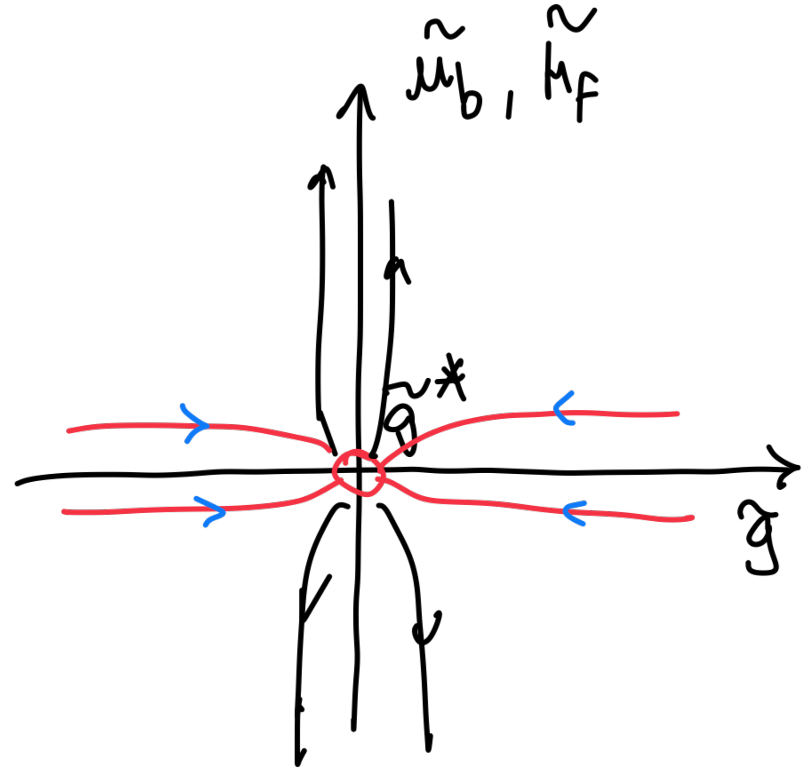
RGE

$$\frac{d\tilde{\mu}_f}{dt} = -2\tilde{\mu}_f$$

$$\frac{d\tilde{g}^2}{dt} = (d-4)\tilde{g}^2 + \tilde{g}^4$$

$$\frac{d\tilde{\mu}_b}{dt} = -2\tilde{\varepsilon}_b + 4\tilde{\mu}_f + \tilde{g}^2 c_d$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = +\tilde{g}^2$$



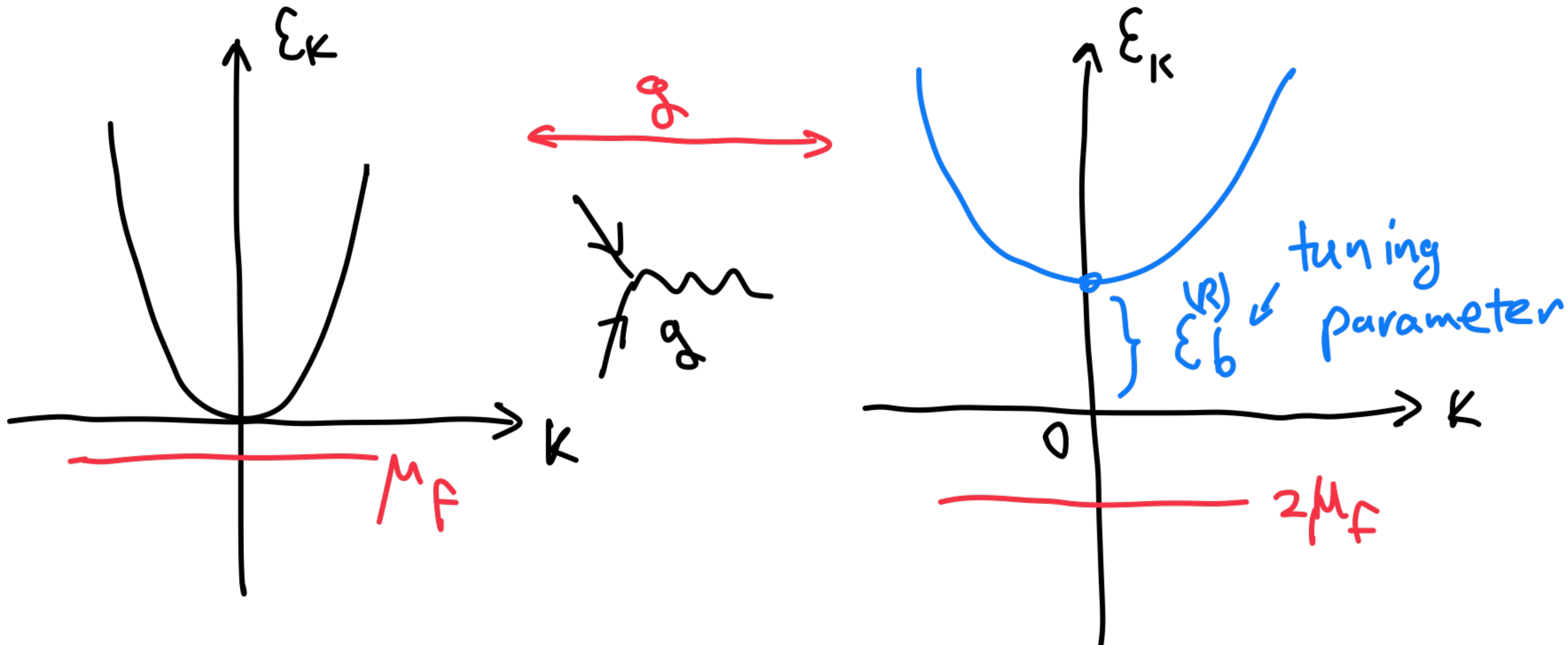
IR stable

fixed pt 3: $\tilde{\mu}_f = \tilde{\mu}_b = Z_b = 0$

$\tilde{g}^2 = 4-d = \tilde{g}^{2*}$

"No-boson" QFT

Connections to Feshbach Resonance



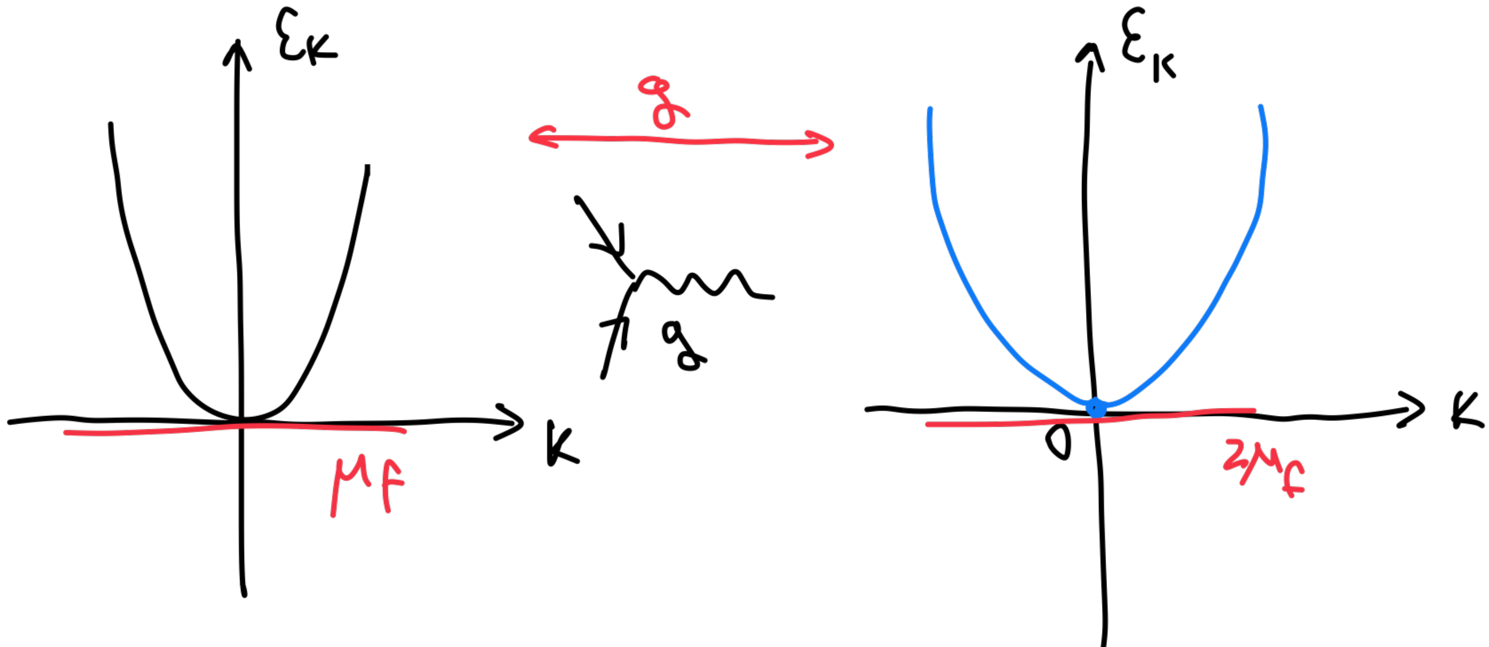
Atoms

Feshbach Molecules

$$\mu_b = 2\mu_f - \varepsilon_b^{(R)}$$

$$\varepsilon_b^{(R)} = \varepsilon_b - \frac{g^2}{2}\Lambda_{uv}\, C_d$$

At Feshbach Resonance $\quad \varepsilon_b^{(R)} = 0$



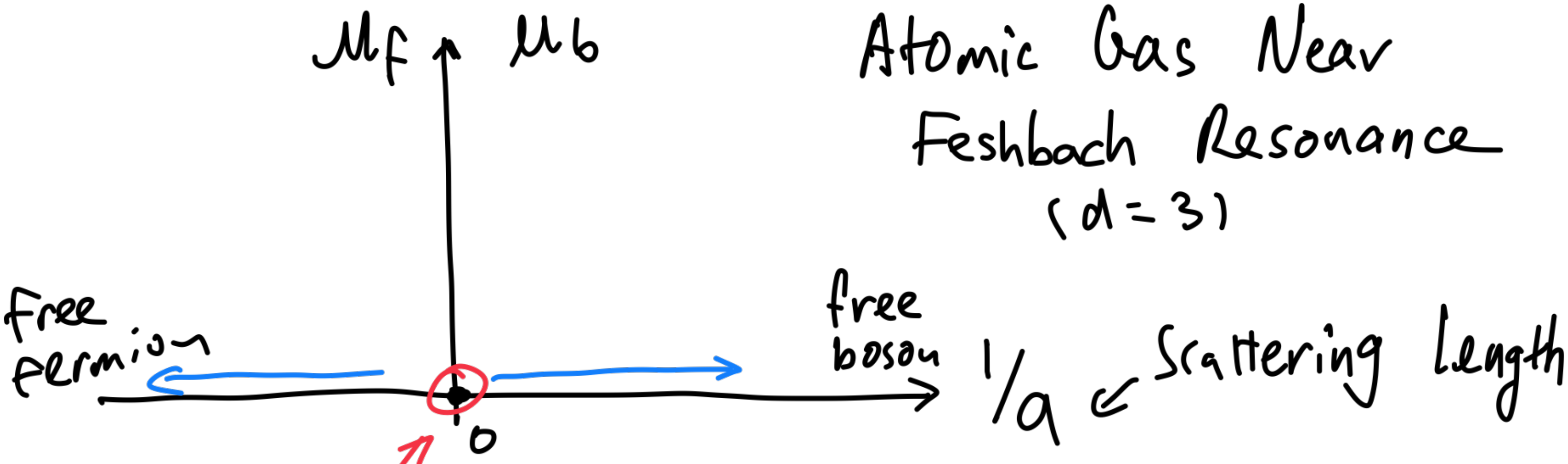
Atoms $\qquad\qquad$ **Feshbach Molecules**

$\mu_F = 0$ $\qquad\qquad$ $\mu_b = 0$

$$\mu_b = 2\mu_F - \varepsilon_b^{(R)} = 2\mu_F = 0$$

$$\varepsilon_b^{(R)} = \varepsilon_b - \frac{g^2}{2}\Lambda_{uv}C_d = 0$$

Atomic Gas Near Feshbach Resonance
(d = 3)



Feshbach Resonance
(Strong Coupling fixed pt)

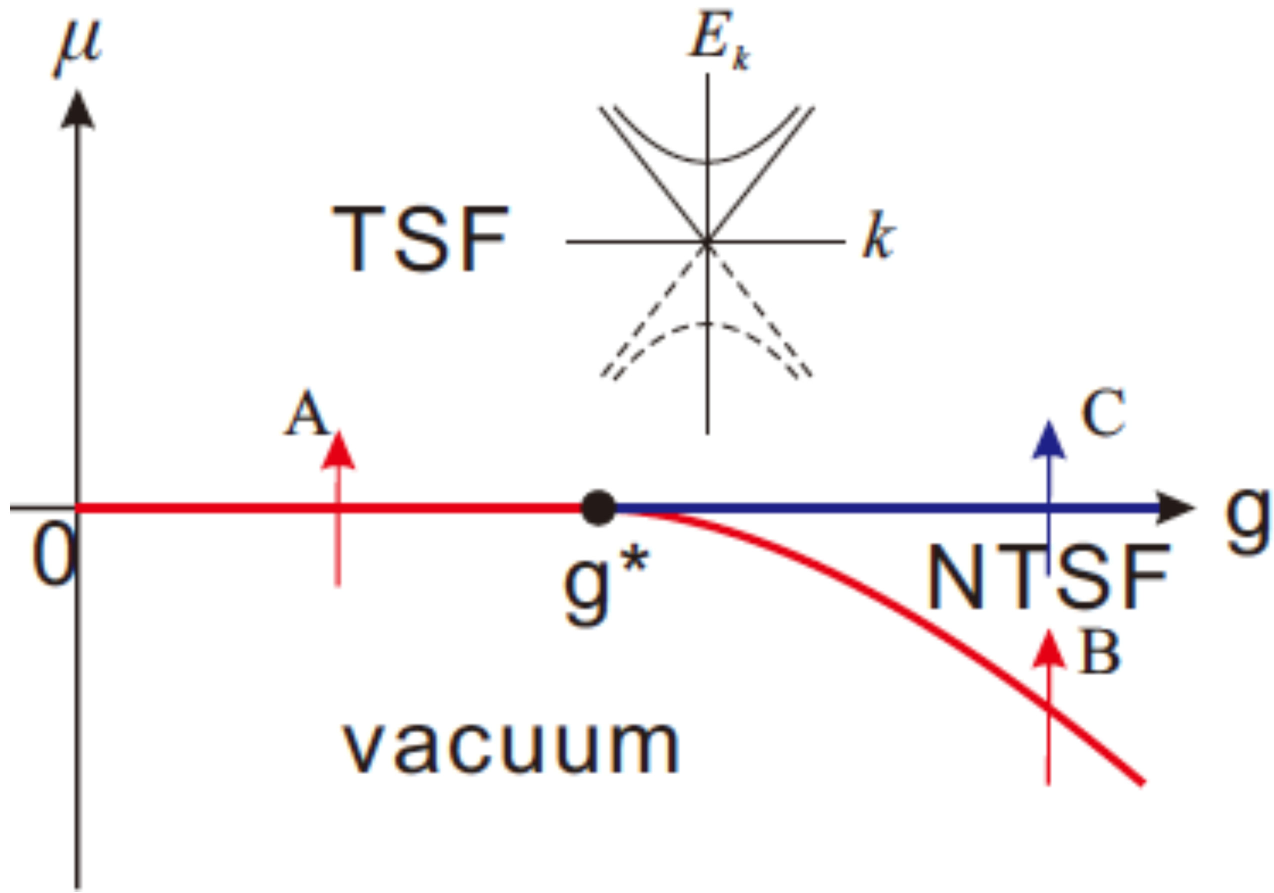
Feshbach Resonance

= SO(2,1) CFT theory

with $Z_b = 0$ $\left(\tilde{g}^{*2} = 4-d\right)$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = \tilde{g}^2 = \tilde{g}^{*2} = 1 \longrightarrow Z_b(L) \sim \frac{1}{L}$$

Example: Phase diagram of p+ip spinless SF ??



A: Free Fermion; B: Wilson-Fisher/Free boson; C: Majorana class
g*: QCP of SO(2,1) CFT.

QFT/RGE Applications to p-wave interactions:

Tricritical Physics in Two-Dimensional p-Wave Superfluids, Fan Yang, Shao-Jian Jiang, and Fei Zhou, Phys. Rev. Lett. 124, 225701 (2020).