Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

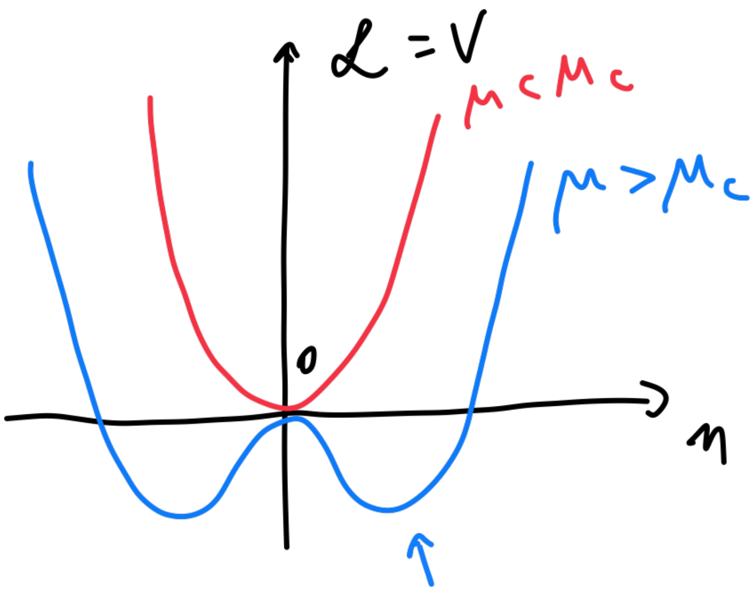
Episode 13:

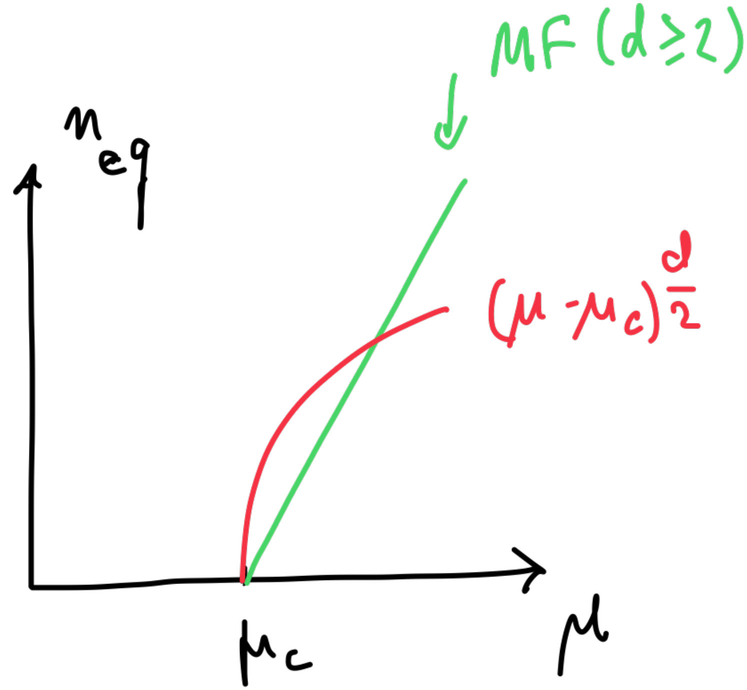
Application: z=2 free particle/Strong interacting fixed points of fermions

BHM

$$\mathcal{L} = \varphi^*\left(\partial_\tau - \frac{\nabla^2}{2}\right)\varphi + (\mu_c - \mu)\varphi^*\varphi + \lambda|\varphi|^4 + \dots$$

$d<2$, strongly interacting fixed pt;



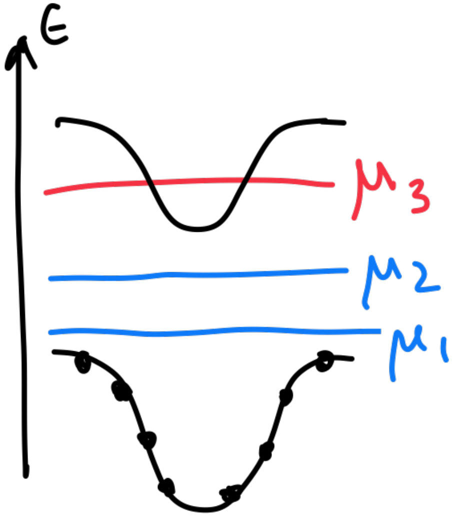


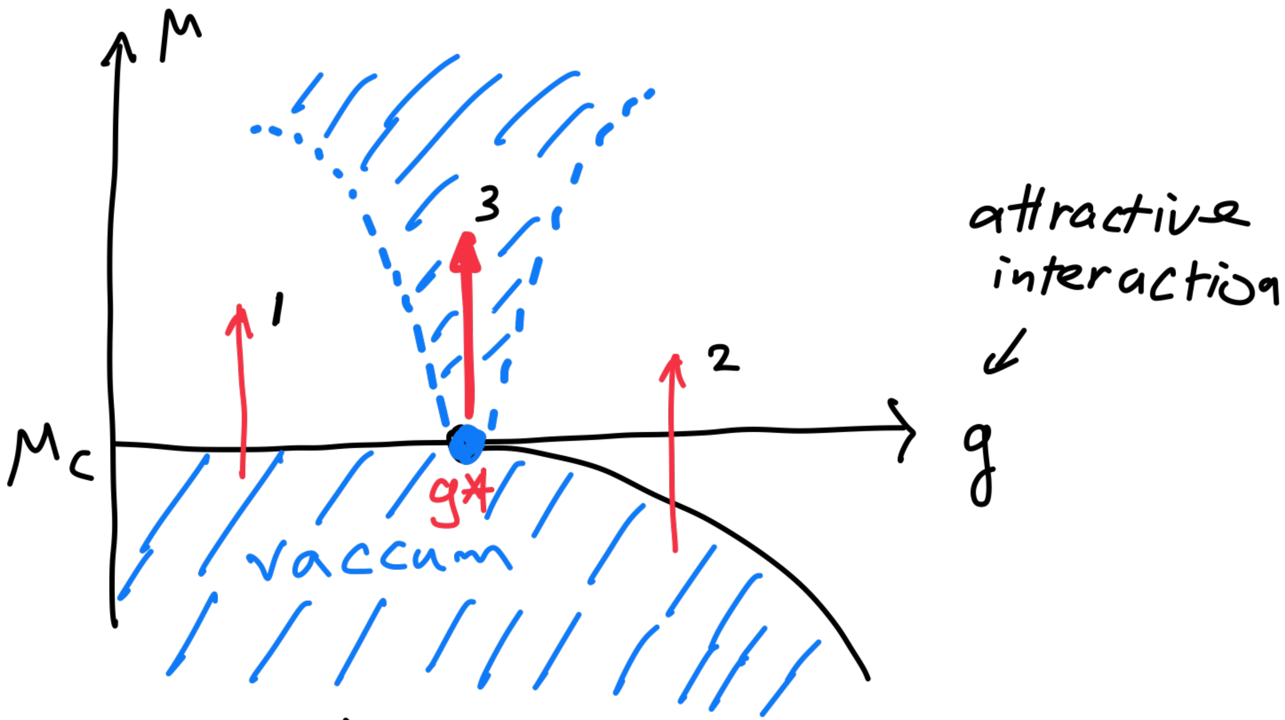
$n_{eq} \sim (\mu - \mu_c)^{\alpha}$, $\alpha = \frac{d}{2}$

($d=1$, $\alpha = \frac{1}{2}$ instead of 1)

General fermionic Model (3d)

Related
$z=2$





1 : free - fermion Universality class

2 : free - boson Universality class

3 : $SO(2,1)$ Conformal field Theory / CFT

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_{bf}$$

χ : Fermionic (e)

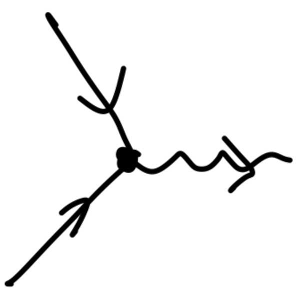
ϕ : Bosonic (2e)

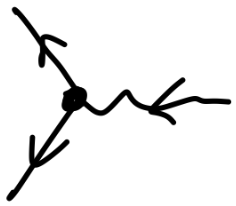
$$\mathcal{L}_f = \chi^*_\alpha \partial_\tau \chi_\alpha - \chi^*_\alpha \frac{\nabla^2}{2} \chi_\alpha - \mu_f \chi^*_\alpha \chi_\alpha \qquad \alpha = \uparrow, \downarrow$$

$$\mathcal{L}_b = \phi^* \partial_\tau \phi - \phi^* \frac{\nabla^2}{4} \phi + (\epsilon_b \underbrace{- 2\mu_f}_{-\mu_b}) \phi^* \phi$$

$$\mathcal{L}_{bF} = g \chi^*_\alpha \sigma_{y,\alpha\beta} \chi^*_\beta \phi + \text{c.c.}$$

Anti-Symmetric

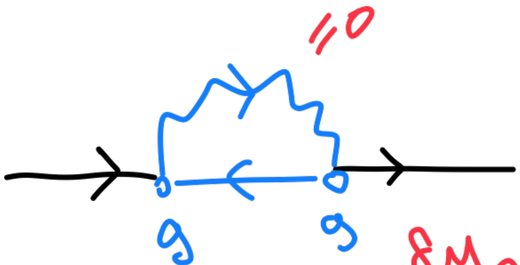




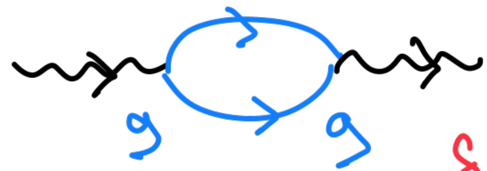
Grassmanian field

$$\{\chi_{\alpha R}, \chi_{\alpha' R}\} = \{\chi_{\alpha R}, \chi_{\alpha L}\} = 0$$

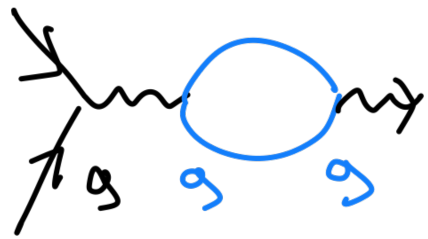
$$\chi_\alpha = \chi_{\alpha R} + i \chi_{\alpha I}, \quad \chi^2_{\alpha R} = \chi^2_{\alpha I} = \frac{1}{2}, \quad \{\chi_{\alpha L}, \chi_{\alpha' L}\} = \{\chi_{\alpha L}, \chi_{\alpha' R}\} = 0$$



δM_F

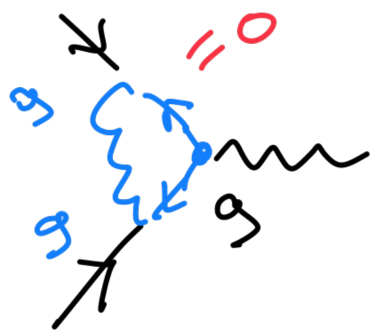


$\delta \epsilon_b$



δg

δg



δg

RGE

① $$\frac{d\tilde{\mu}_f}{dt} = -2\tilde{\mu}_f$$

② $$\frac{d\tilde{g}^2}{dt} = (d-4)\tilde{g}^2 + \tilde{g}^4 c_d$$

③ $$\frac{d\tilde{\mu}_b}{dt} = -2\tilde{\varepsilon}_b + 4\tilde{\mu}_f + \tilde{g}^2 c_d$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = +\tilde{g}^2$$

$$\left(\frac{d\tilde{\varepsilon}_b}{dt} = -2\tilde{\varepsilon}_b + \tilde{g}^2\right)$$

Upper critical dimension

$d_c = 4$

$d \geq 4$, $\tilde{g}^* = 0$ is the only fixed pt for $\beta_g = 0$.

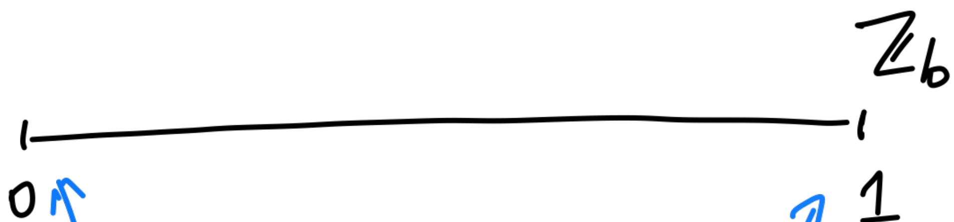
but there are three fixed pt solutions all together.

Following HW set II, $G(t,0;\vec{q},\vec{q}) = \langle vac|\, T\, b_{\vec{q}}(t)\, b^{\dagger}_{\vec{q}}(0)\,|vac\rangle$

$$G(t>0,0;\vec{q},\vec{q}) = \sum_{\vec{q}} \langle vac|\, b_{\vec{q}}(t)\, \underbrace{|\vec{q}\rangle\langle\vec{q}|}_{\text{complete of QFT states with momentum } \vec{q}.}\, b^{\dagger}_{\vec{q}}(0)\,|vac\rangle$$

$$Z_b = \langle\vec{q}|\, b^{\dagger}_{\vec{q}}\,|vac\rangle$$

exact QFT eigen states with $\vec{q}$-Momentum



No Bosons (or Completely fermonic)

Non-interacting "free boson"

$d \geq 4$, there are 3 fixed pts, all with $\tilde{g}^* = 0$

fixed pt 1.

① $\tilde{\mu}_f = 0$, $\tilde{\mu}_b \neq 0$

"free fermion"

② $\tilde{\mu}_b = -2\tilde{\varepsilon}_b + 4\tilde{\mu}_f = 0$

$\tilde{\mu}_f \neq 0$, $Z_b = 1$

"free boson"

③ $\tilde{\mu}_f = \tilde{\mu}_b = 0$ $Z_b = 1$

free-fermion-boson theory

RGE

$$\frac{d\tilde{\mu}_f}{dt} = -2\tilde{\mu}_f \qquad ①$$

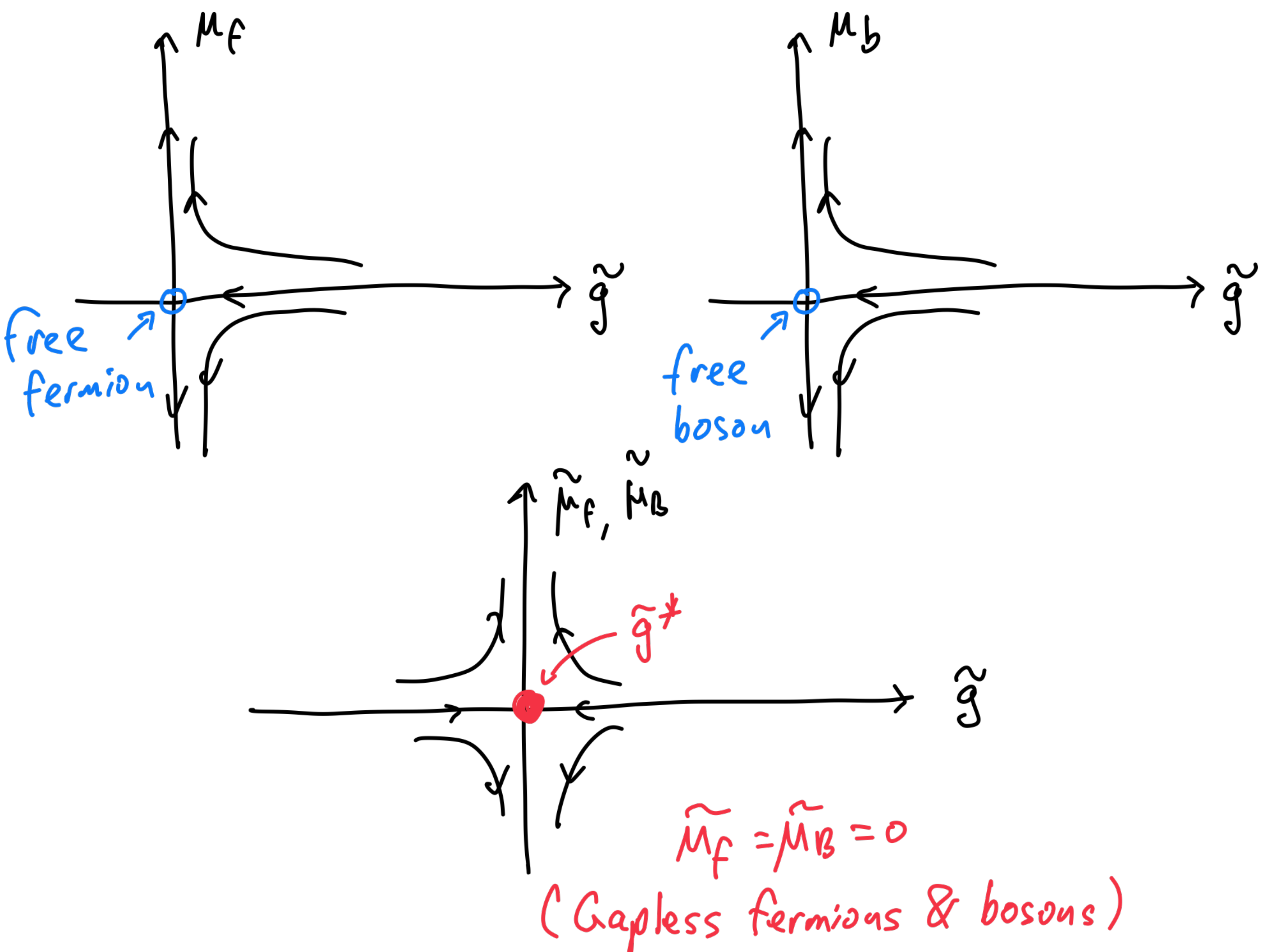
$$\frac{d\tilde{g}^2}{dt} = (d-4)\tilde{g}^2 + \tilde{g}^4 \qquad ②$$

$$\frac{d\tilde{\mu}_b}{dt} = -2\tilde{\varepsilon}_b + 4\tilde{\mu}_f + \tilde{g}^2 C_d$$

$$\frac{d\tilde{\varepsilon}_b}{dt} = -2\tilde{\varepsilon}_b + \tilde{g}^2 \qquad ③$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = \tilde{g}^2$$

$d \geq 4$



$\tilde{\mu}_f$
$\tilde{g}$
free fermion
$\tilde{\mu}_b$
$\tilde{g}$
free boson
$\tilde{\mu}_f, \tilde{\mu}_B$
$\tilde{g}^*$
$\tilde{g}$
$\tilde{\mu}_f = \tilde{\mu}_B = 0$
(Gapless fermions & bosons)

$d < 4$

RGE

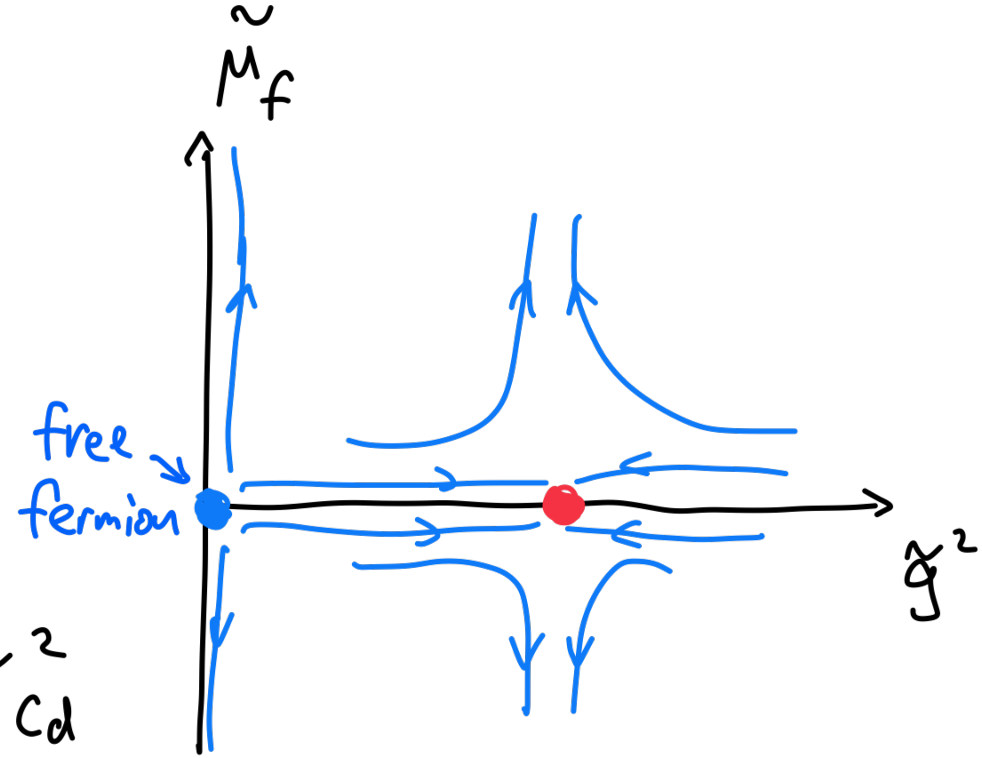
$$\frac{d\tilde{\mu}_f}{dt} = -2\,\tilde{\mu}_f$$

$$\frac{d\tilde{g}^2}{dt} = (d-4)\tilde{g}^2 + \tilde{g}^4$$

$$\frac{d\tilde{\mu}_b}{dt} = -2\,\tilde{\varepsilon}_b + 2\,\tilde{\mu}_f + \tilde{g}^2 c_d$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = +\tilde{g}^2$$

$$\left(\frac{d\tilde{\varepsilon}_b}{dt} = -2\tilde{\varepsilon}_b + \tilde{g}^2\right)$$



fixed pt 1:

$\tilde{\mu}_f = 0$, $\tilde{g}^{*2} = 4-d$, $\tilde{\mu}_b \to \infty$

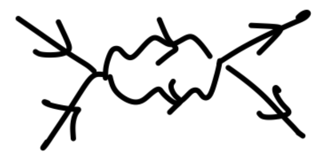
$d < 4$

fixed pt 1: $\tilde{\mu}_f = 0,\ \tilde{g}^{*2} = 4-d,\ \tilde{\mu}_b \to \infty$ = fermions interacting with massive bosons

$\tilde{\mu}_f = 0$

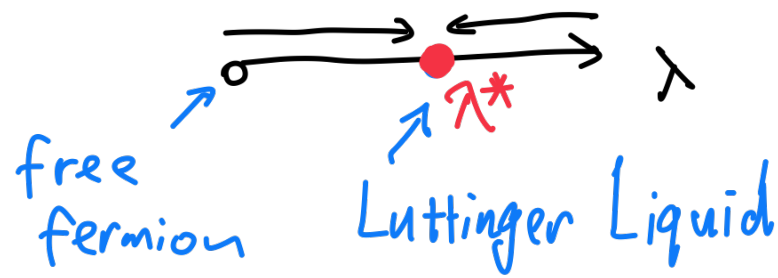
$$\mathcal{L}_{eff} = \chi^\dagger\left(\partial_t - \frac{\nabla^2}{2}\right)\chi + \lambda\, \chi^\dagger \sigma_y \chi^\dagger \chi \sigma_y \chi + \dots$$

$$\tilde{\lambda} \propto -\tilde{g}^{*2}$$



$d = 2, 3$

$d = 1$



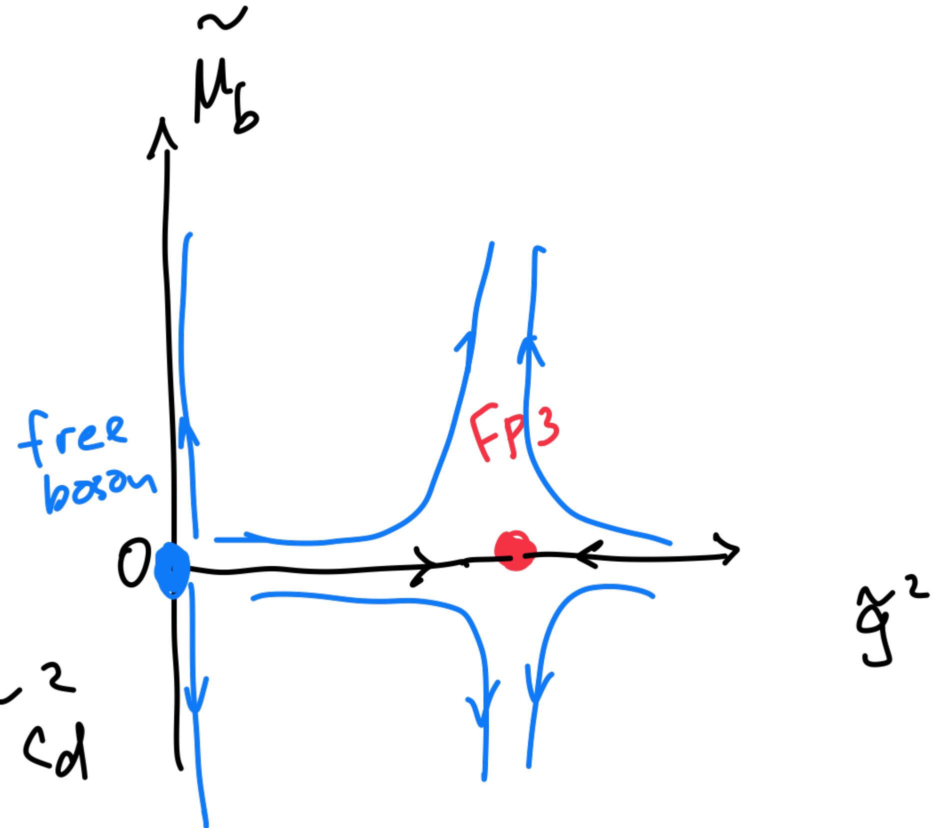
RGE

$$\frac{d\tilde{m}_f}{dt} = -2\,\tilde{m}_f$$

$$\frac{d\tilde{g}^2}{dt} = (d-4)\tilde{g}^2 + \tilde{g}^4$$

$$\frac{d\tilde{\mu}_b}{dt} = -2\,\tilde{\varepsilon}_b + 4\,\tilde{\mu}_f + \tilde{g}^2 c_d$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = \tilde{g}^2$$



fixed pt 2: $\tilde{\mu}_b^* = 0$, $\tilde{g}^* = 4-d$

$(2\tilde{\mu}_f - \tilde{\varepsilon}_b + \tilde{g}^{*2} c_d = 0)$

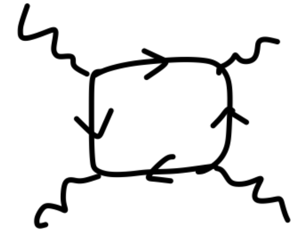
$d < 4$

fixed pt 2:

$\tilde{\mu}_b = 0,\ \tilde{g}^{*2} = 4-d,\ \tilde{\mu}_f \to \infty$ = Bosons interacting with massive fermions

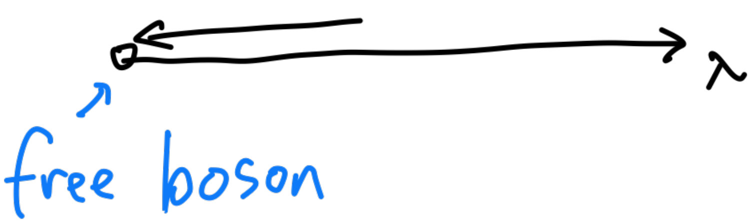
$\tilde{\mu}_b = 0,$

$$\mathcal{L}_{eff} = \varphi^*\left(\partial_t - \frac{\nabla^2}{2}\right)\varphi + \lambda|\varphi^*\varphi|^2 + \dots$$

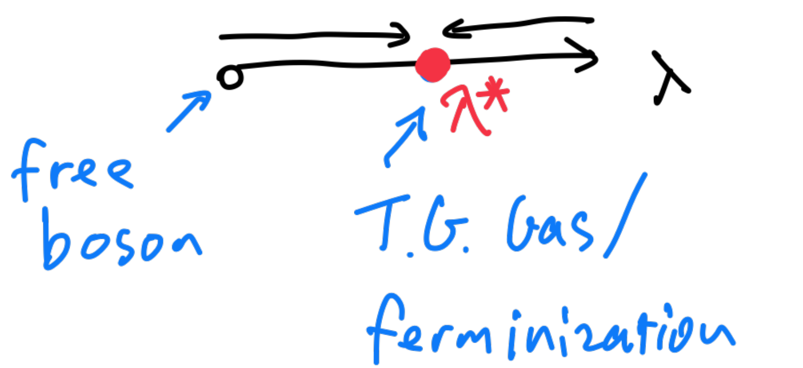


$$\tilde{\lambda} \propto \tilde{g}^{*4}$$

$d=2, 3$



$d = 1$



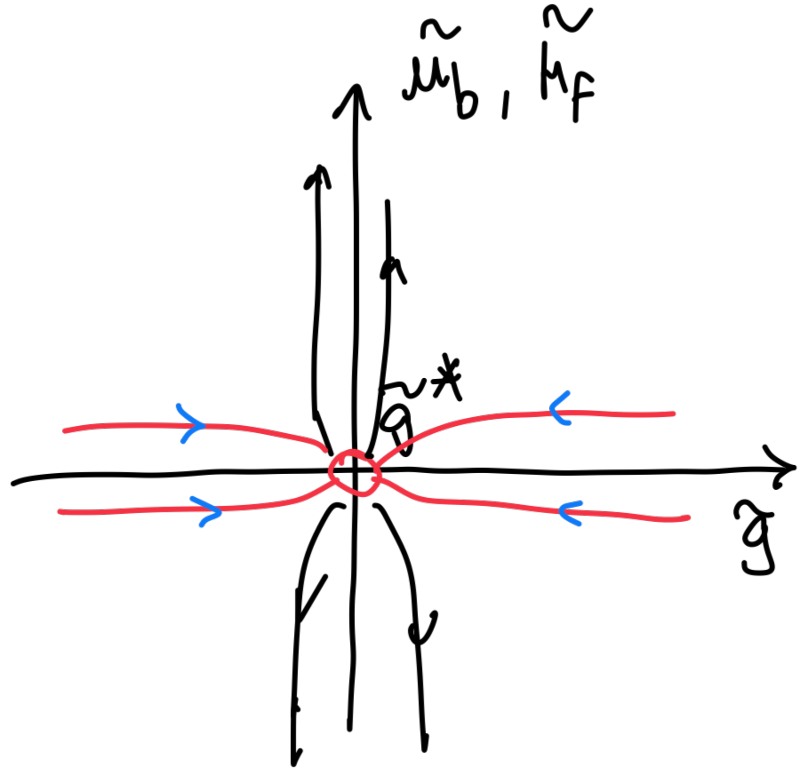
RGE

$$\frac{d\tilde{\mu}_f}{dt} = -2\,\tilde{\mu}_f$$

$$\frac{d\tilde{g}^2}{dt} = (d-4)\tilde{g}^2 + \tilde{g}^4$$

$$\frac{d\tilde{\mu}_b}{dt} = -2\,\tilde{\varepsilon}_b + 4\,\tilde{\mu}_f + \tilde{g}^2$$

$$\frac{1}{Z_b}\frac{dZ_b}{dt} = +\tilde{g}^2$$



IR stable

fixed pt 3 : $\tilde{\mu}_f = \tilde{\mu}_b = Z_b = 0$

$\tilde{g}^2 = 4-d = \tilde{g}^{2*}$

"No-boson" QFT