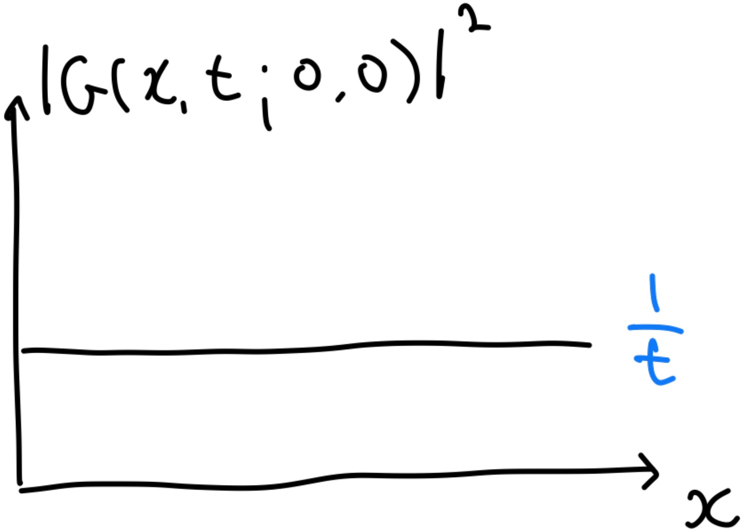
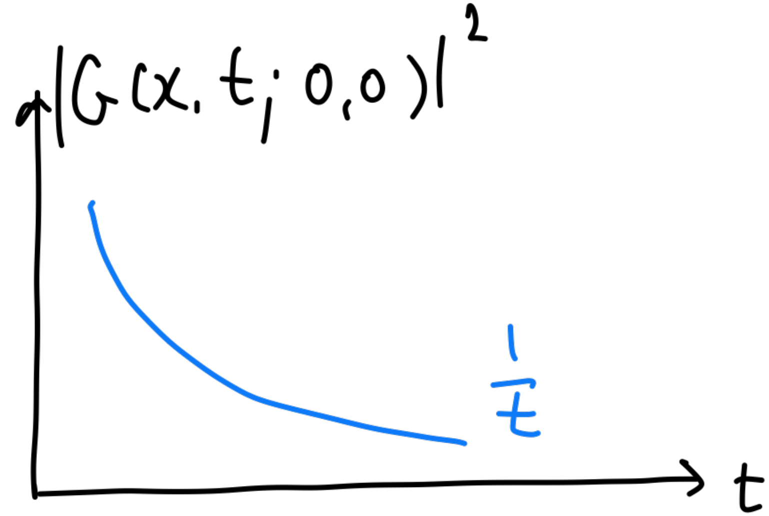
Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

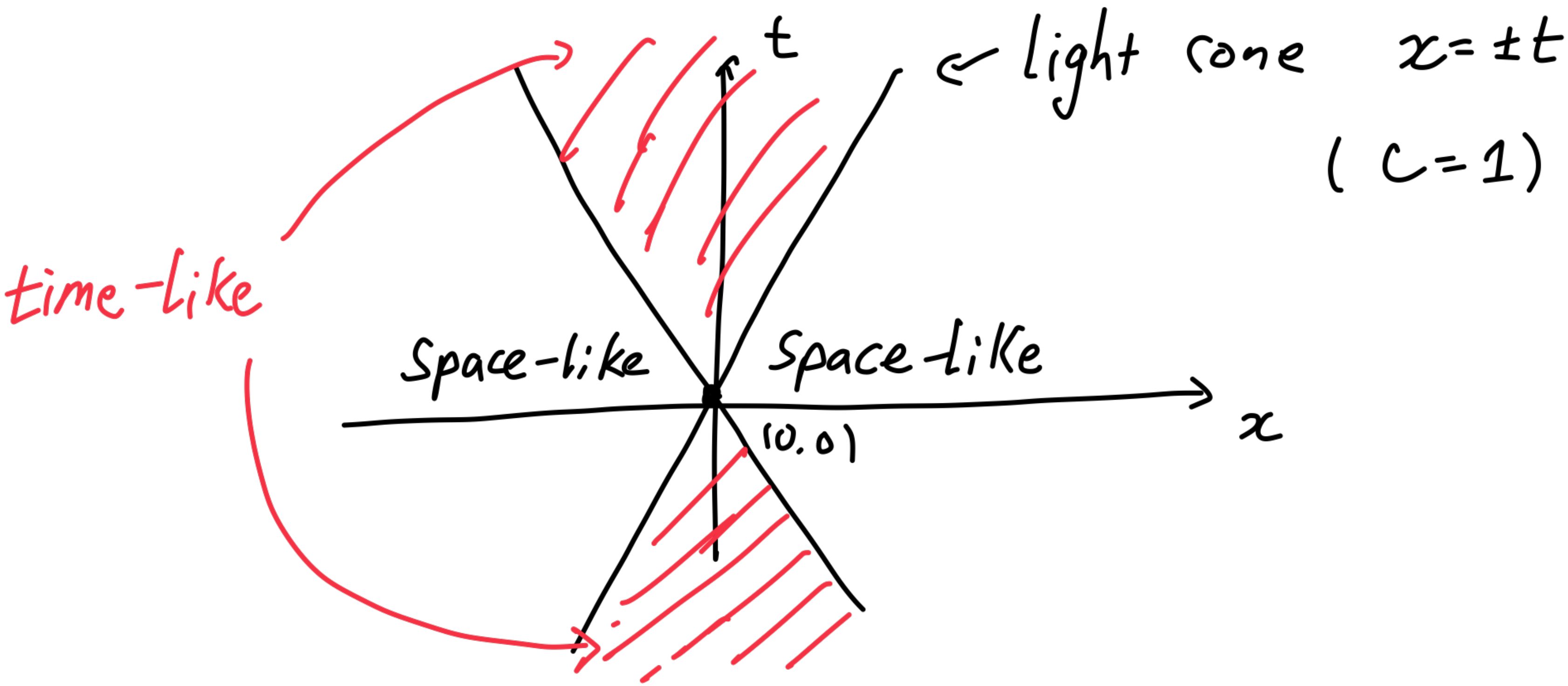
Episode Ten:

Scale transformation and introduction to Renormalization group equations and identifications of scale invariant fixed Points (**SIPT**)





What about $z=1$ Relativistic QCP?



Generally, Non-Relativistic Theory $z=2$
has $SO(2,1)$ conformal symmetry.

$$\langle vac | T\, \hat{O}_{\vec{r}}(t)\, \hat{O}_0^{\dagger}(0) | vac \rangle \sim \frac{1}{t^{2\Delta_0}} e^{i \frac{r^2}{2t}}$$

Δ_0 : Scaling dimension of Operator $\hat{O}^{\dagger}(0)$
"prime"
↑
"N-charge"
Operator

$(\Delta_0 = \frac{1}{2}$ in 1D for $\hat{O}_0 = b_0^{\dagger})$

What does scale symmetry imply at QCPs ?

dynamical critical exponent

Scale dimension of field "ψ"

Scale transformation :

$$\vec{r}\,' = \vec{r}\lambda\,, \quad \tau' = \tau\lambda^{z}\,, \quad \psi'(\vec{r}\,', \tau') = \lambda^{-\eta}\,\psi(\vec{r}, \tau)$$

(for MI-SF QCP at $J = J_c$, $z = 1$, $\eta = \frac{d-1}{2}$ MF fixed pt)

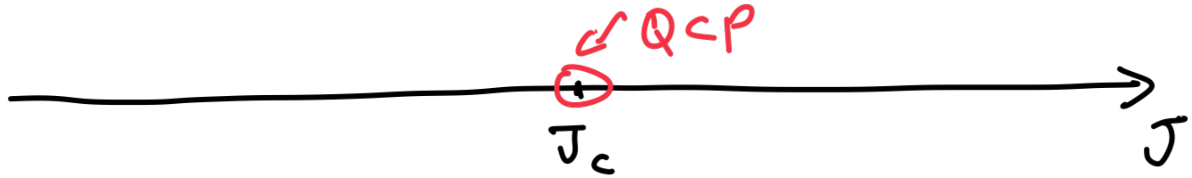
Scale invariance at QCPs

$$S(\{\psi(r,\tau)\}) \longrightarrow S'(\{\psi'(\vec{r}\,',\tau')\}) = S(\{\psi'(\vec{r}\,',\tau')\})$$

or

$$\mathcal{H}(\{b^{+}_{\vec{r}}, b_{\vec{r}}\}) \longrightarrow \mathcal{H}'(\{b'^{+}_{\vec{r}\,'}, b'_{\vec{r}\,'}\}) = \lambda^{-z}\,\mathcal{H}(\{b^{+}_{\vec{r}\,'}, b_{\vec{r}\,'}\})$$

$z = 1$ **Example I.**



$c = 1$,

$$\mathcal{L} = |\partial_\tau \varphi|^2 + |\nabla \varphi|^2 + m^2 |\varphi|^2 + \underbrace{\cdots}_{\text{irrelevant } d>3}$$

$$\mathcal{L}_{QCP} = \mathcal{L}(m=0)$$

$$\tau \to \tau' = \tau\lambda \quad x \to x' = x\lambda, \quad \varphi \to \varphi'(x') = \lambda^{-\frac{d-1}{2}} \varphi(x,\tau)$$

$$\mathcal{L} = \{|\partial_{\tau'} \varphi'|^2 + |\nabla_{\vec{r}'} \varphi'|^2\} \lambda^{+\frac{d-1}{2} \times 2 + 2} + m^2 |\varphi'|^2 \lambda^{+\frac{(d-1)}{2} \times 2}$$

$$S = \int d^{(d)}x \, d\tau \, \mathcal{L} \to S' = \int d^{(d)}x' d\tau'$$ ← breaks scale symmetry

$$\{[|\partial_{\tau'} \varphi'|^2 + |\nabla_{\vec{r}'} \varphi'|^2] + \lambda^{-2} m^2 |\varphi'|^2\}$$

So that $S \to S' = S(\{\varphi'(\vec{r}', \tau')\})$ if $m = 0$ or QCP!

Scale invariant.

- Two corner stones of theory of QCPs

- **Symmetry groups and the corresponding QFT**

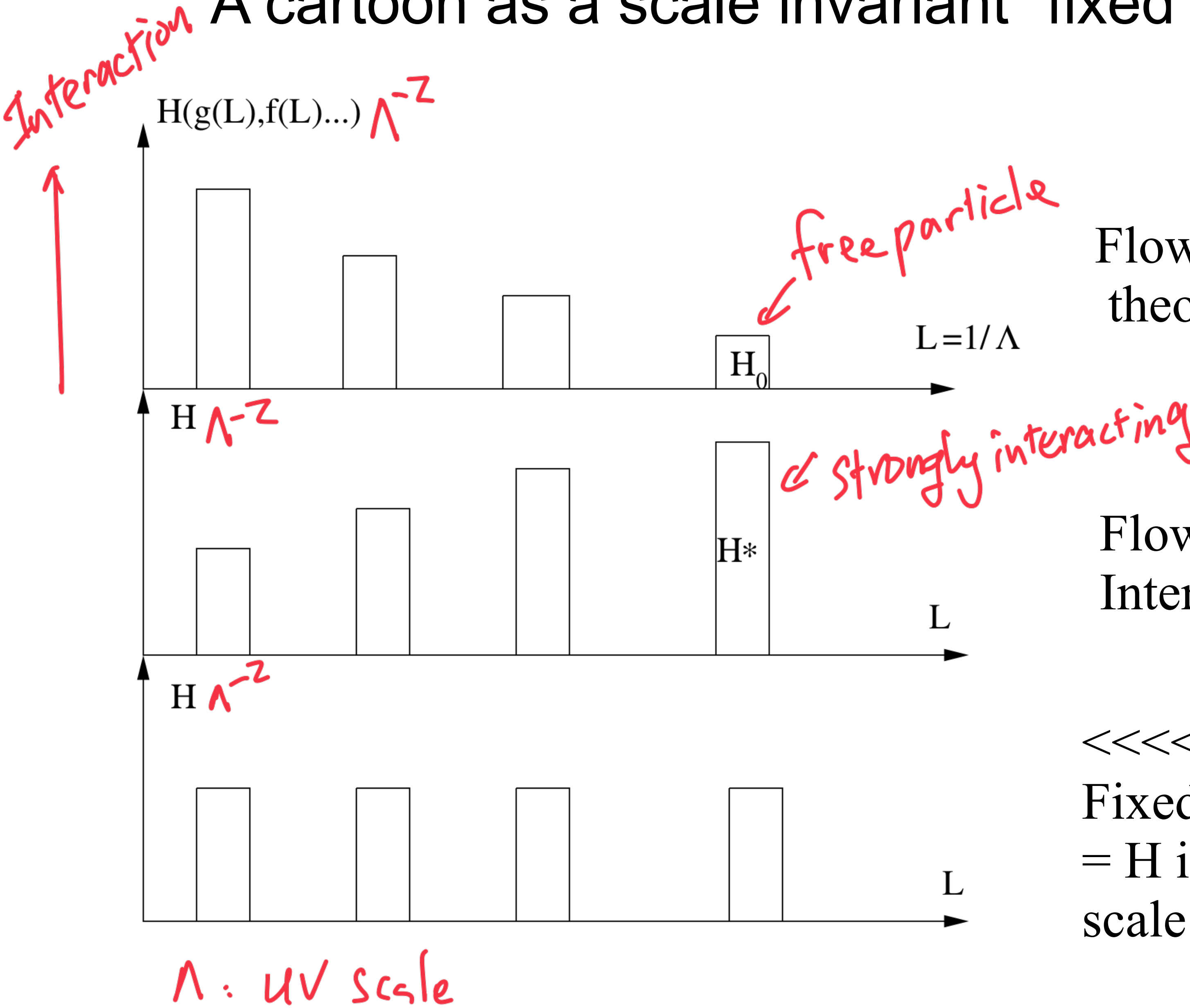
$G = \mathbb{Z}_2, U(1), \mathbb{Z}_N, SU(2), SU(N), S^2/\mathbb{Z}_2, \ldots\ldots$

(However, G may not be enough for modern QCPs!!)

- **Scale Symmetric QFTs or fixed points (SIFPs).**

1. Wilson, K. G. (1971). "Renormalization Group and Critical Phenomena. I. Renormalization Group and the Kadanoff Scaling Picture". *Physical Review B*. **4** (9): 3174–3183. Bibcode:1971PhRvB...4.3174W. doi:10.1103/PhysRevB.4.3174.
2. ^ Wilson, K. (1971). "Renormalization Group and Critical Phenomena. II. Phase-Space Cell Analysis of Critical Behavior". *Physical Review B*. **4** (9), 3184.
3. Wilson, K. (1983). "The renormalization group and critical phenomena". *Reviews of Modern Physics*. **55** (3): 583–600.

Wilson's view of critical points (70s):
A cartoon as a scale invariant "fixed point"



Flows into a free theory

Flows into a strongly Interacting theory

<<<<< Critical point/ Fixed Point = H invariant under scale transformation

Scale transformation: General

$\ln\lambda = t$ or $\lambda = e^{t}$

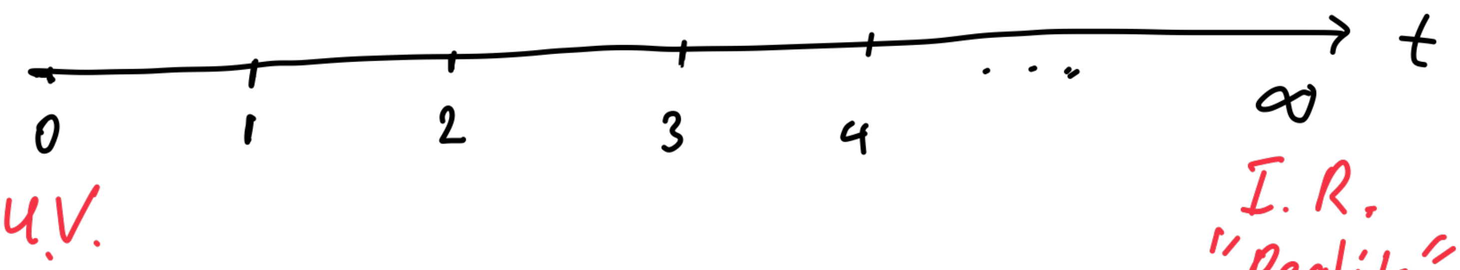
↗ best defined in terms of t Not λ

$$\tau \to \tau' = \tau e^{-tz}$$

$$\vec{x} \to \vec{x}' = \vec{x} e^{-t}$$

$$\varphi(\vec{x},t) \to \varphi'(\vec{x}',\tau') = e^{+\eta t}\varphi(\vec{x},t),$$

$$\left[\varphi(\vec{x},\tau) = e^{-\eta t}\varphi_t(\vec{x}_t,\tau_t)\right]$$



$$\mathcal{L}_0(\{\varphi(\vec{x},t)\}) \to \mathcal{L}_{t=1}[\{\varphi_1(\vec{x}_1,t_1)\}] \to \mathcal{L}_{t=2}(\{\varphi_2(x_2,t_2)\}) \to \ldots$$

↙ bare field $UV = \Lambda_{UV}$

↙ Renormalized field with $UV = \Lambda_{UV} e^{-1}$

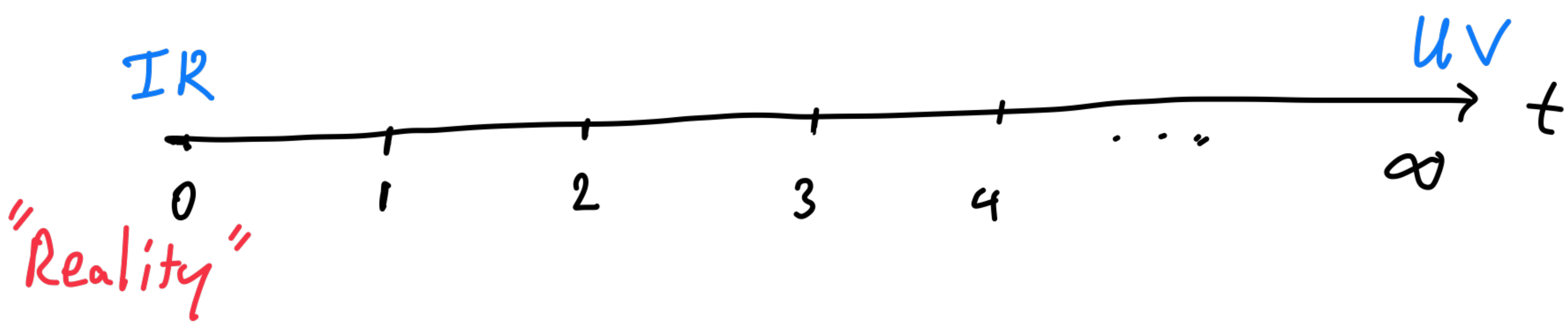
↙ Renormalized field with $UV = \Lambda_{UV} e^{-2}$

Scale transformation: General II; "particle physics" towards unification

$$\tau \to \tau' = \tau e^{tz}$$

$$\vec{x} \to \vec{x}' = \vec{x} e^{t}$$

$$\varphi(\vec{x},t) \to \varphi'(\vec{x}',\tau') = e^{-\eta t}\varphi(\vec{x},t), \qquad \varphi(\vec{x},\tau) = e^{+\eta t}\varphi_t(\vec{x}_t,\tau_t)$$



$$\mathcal{L}_{IR}(\{\varphi(\vec{x},t)\}) \to \mathcal{L}_{t=1}(\{\varphi_1(\vec{x}_1,t_1)\}) \to \mathcal{L}_{t=2}(\{\varphi_2(x_2,t_2)\}) \to \dots$$

Renormalized field (for $\mathcal{L}_{IR}$)

Renormalized field with $UV = \Lambda_{IR} e^{+1}$ (for $\mathcal{L}_{t=1}$)

Renormalized field with $UV = \Lambda_{IR} e^{+2}$ (for $\mathcal{L}_{t=2}$)

Scale transformation and Renormalization Group equations

$0 \longrightarrow \infty$, t (UV $\to$ IR)

$$\Lambda(t) = \Lambda_{UV}\, e^{-t}$$

$$\mathcal{L}_t(\{\varphi(\vec{x},\tau)\};\ m,\ g,\ \dots) = \Lambda(t)^{d+1}\, \tilde{\mathcal{L}}_t(\{\varphi(x,\tau)\};\ \tilde{m},\ \tilde{g},\ \dots)$$

(m: mass; $g, \dots$: interactions; $\mathcal{L} \propto S$)

$\mathcal{L}_t$: define at $\Lambda(t) = \Lambda_{UV}\, e^{-t}$

or a running scale

$$\tilde{m}(t) = m(t)\,\Lambda(t)^{-z}, \quad \tilde{g}(t) = g(t)\,\Lambda(t)^{d-3} \quad \dots$$

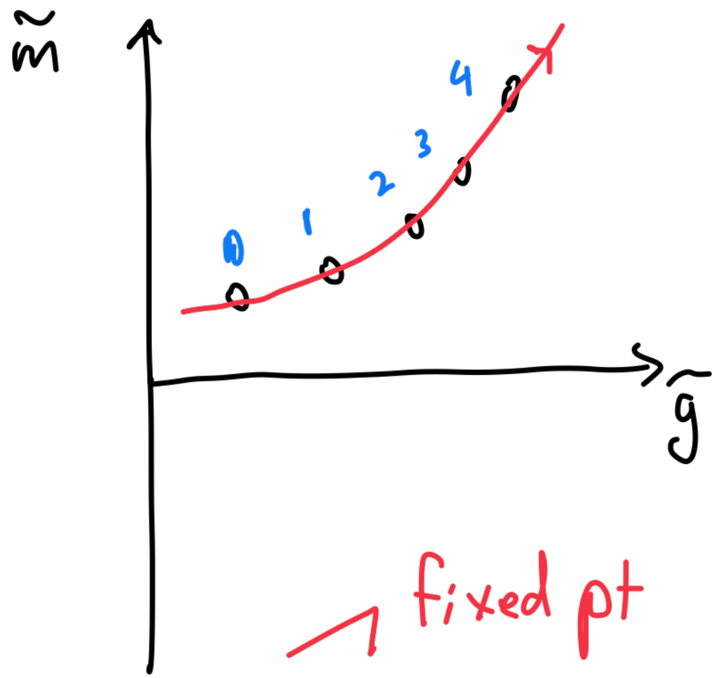
transforms in unique way under Scale Transformation

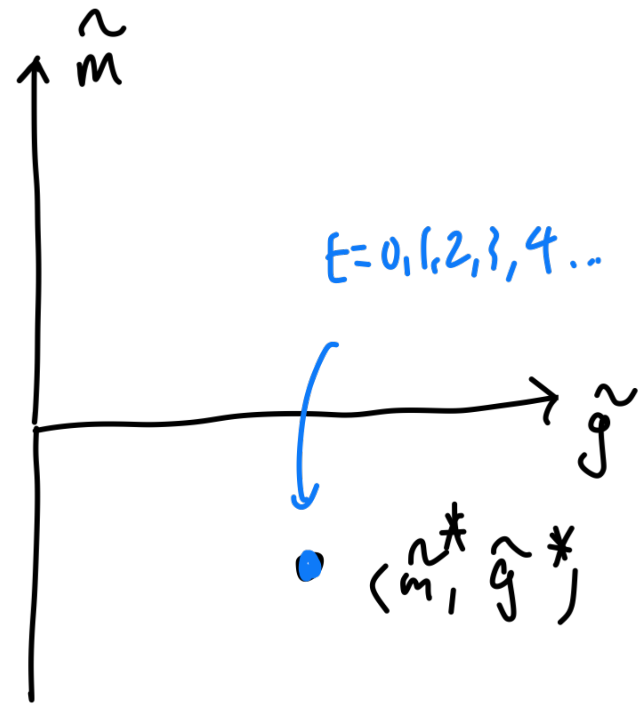
follows a set of PDE the called "R.G.E."

RGE

$$\frac{d\tilde{m}}{dt} = \beta_m(\tilde{m}, \tilde{g}, \dots), \qquad \frac{d\tilde{g}}{dt} = \beta_g(\tilde{m}, \tilde{g}, \dots)$$

A) Generically, $\beta_m \neq \beta_g \neq 0$. and $\tilde{m}(t)$, $\tilde{g}(t)$ are running.

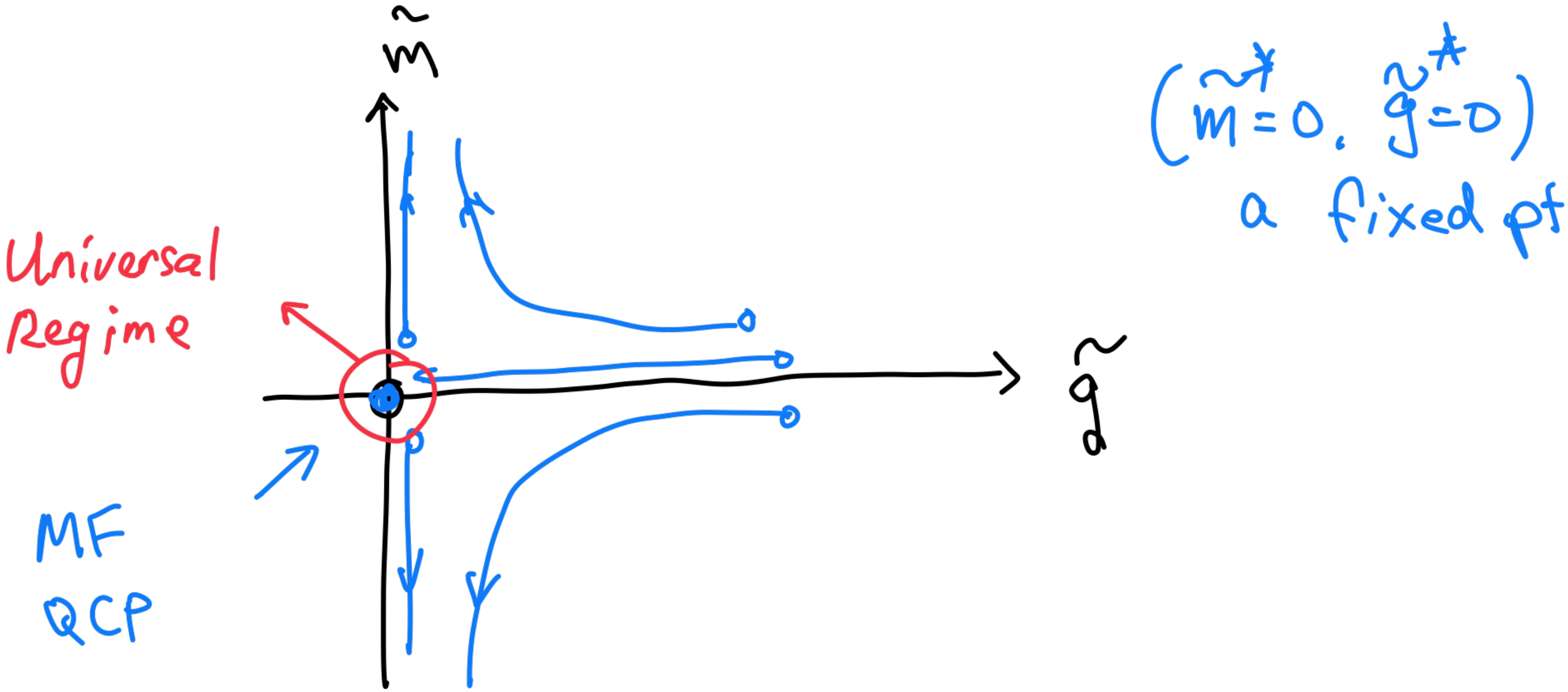




fixed pt

B) if $\beta_m(\tilde{m}^*, \tilde{g}^* \dots) = \beta_g(\tilde{m}^*, \tilde{g}^*)$, $\tilde{m}(t)$, $\tilde{g}(t)$ Not Running

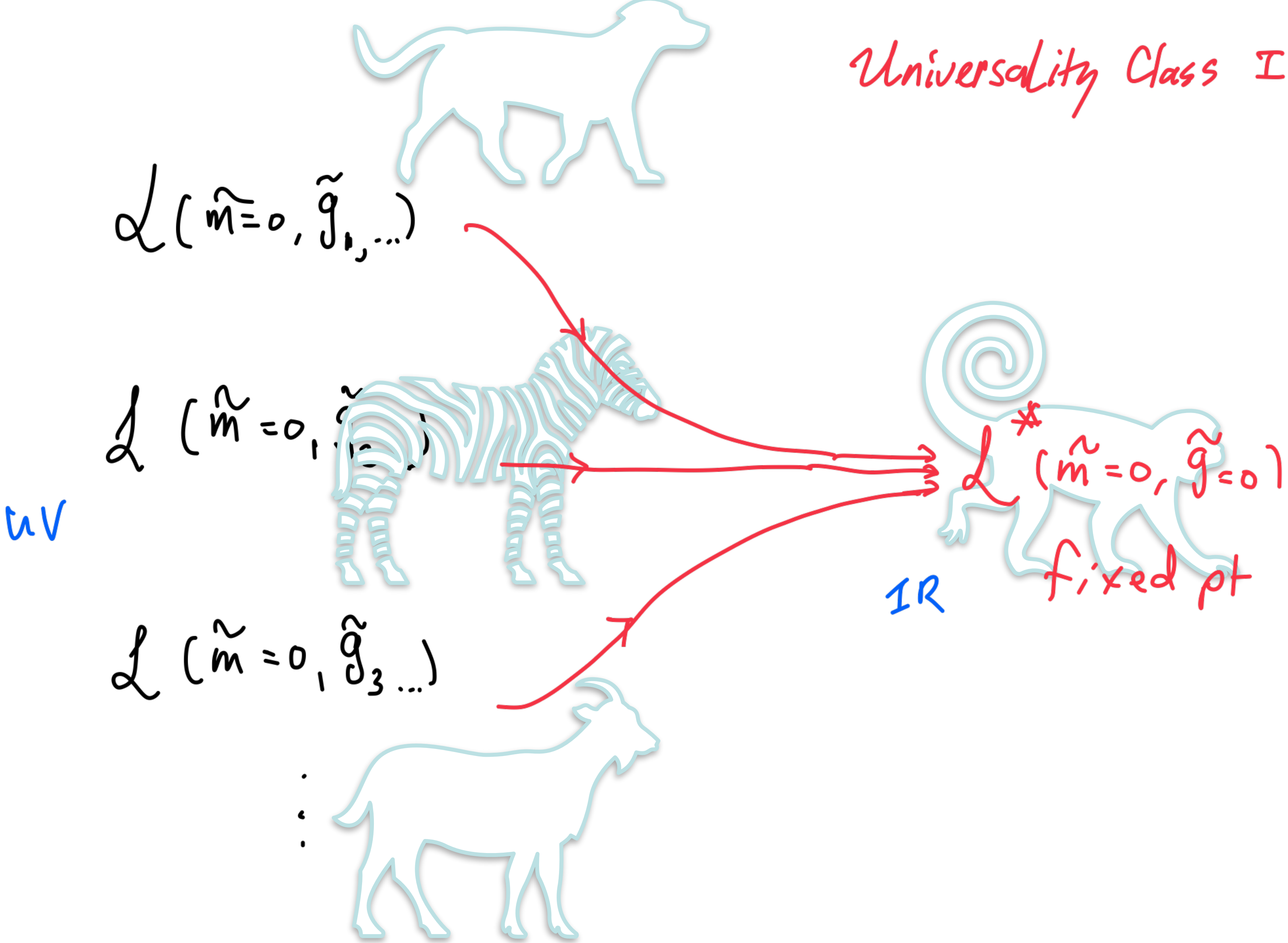
$d>3$, QCP for MI-SF transition at $J=J_c$



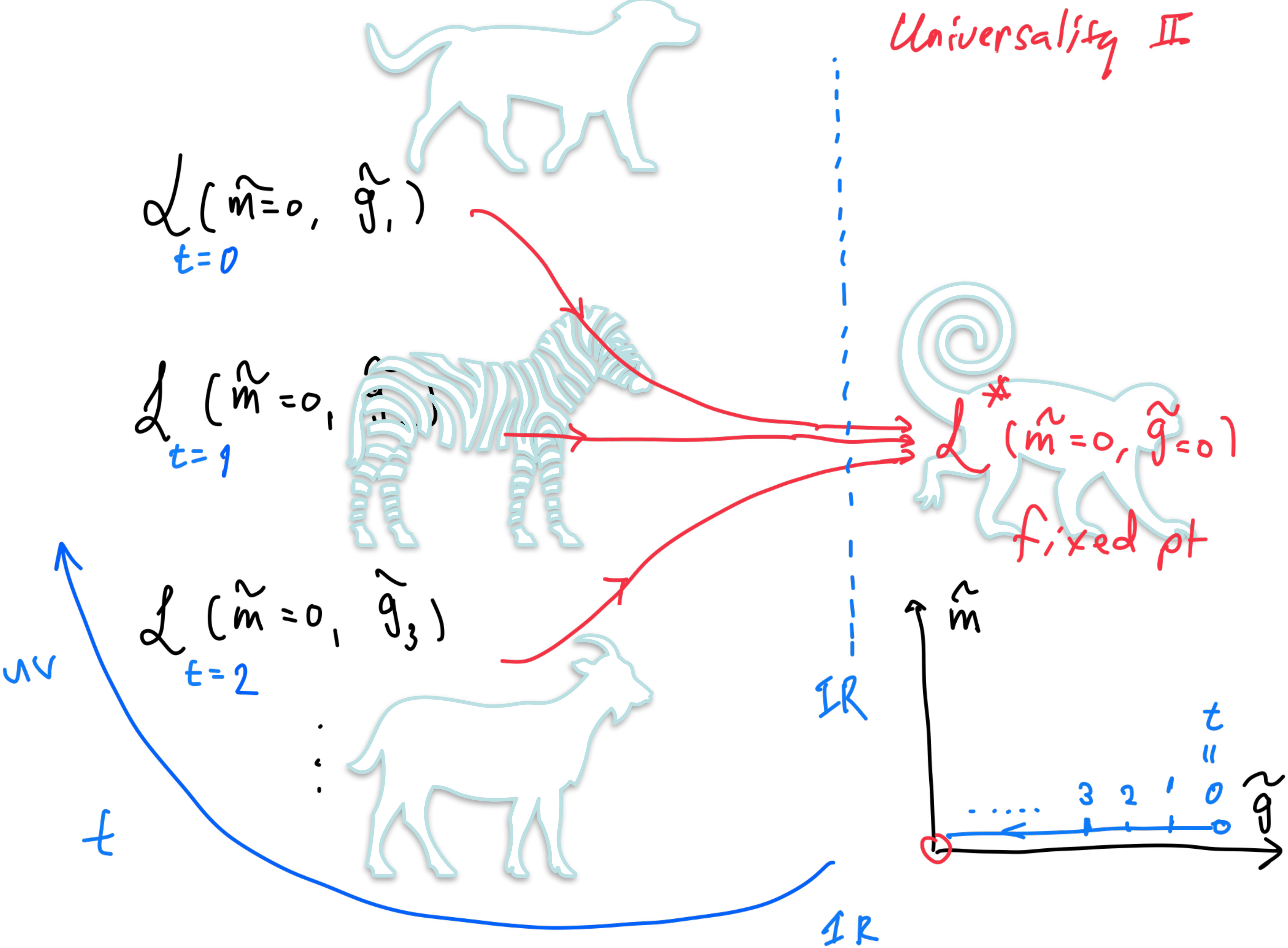
$$\mathcal{L} = |\partial_t \varphi|^2 + |\vec{\nabla}\varphi|^2 + m^2|\varphi|^2 + g|\varphi|^4 + \cdots$$

$$\mathcal{L}(m=0, g\neq 0, \ldots) \xrightarrow{\text{Flow into}} \mathcal{L}^*(m=0, g=0, 0, 0, \ldots)\ \mathcal{L}^*$$

fixed pt

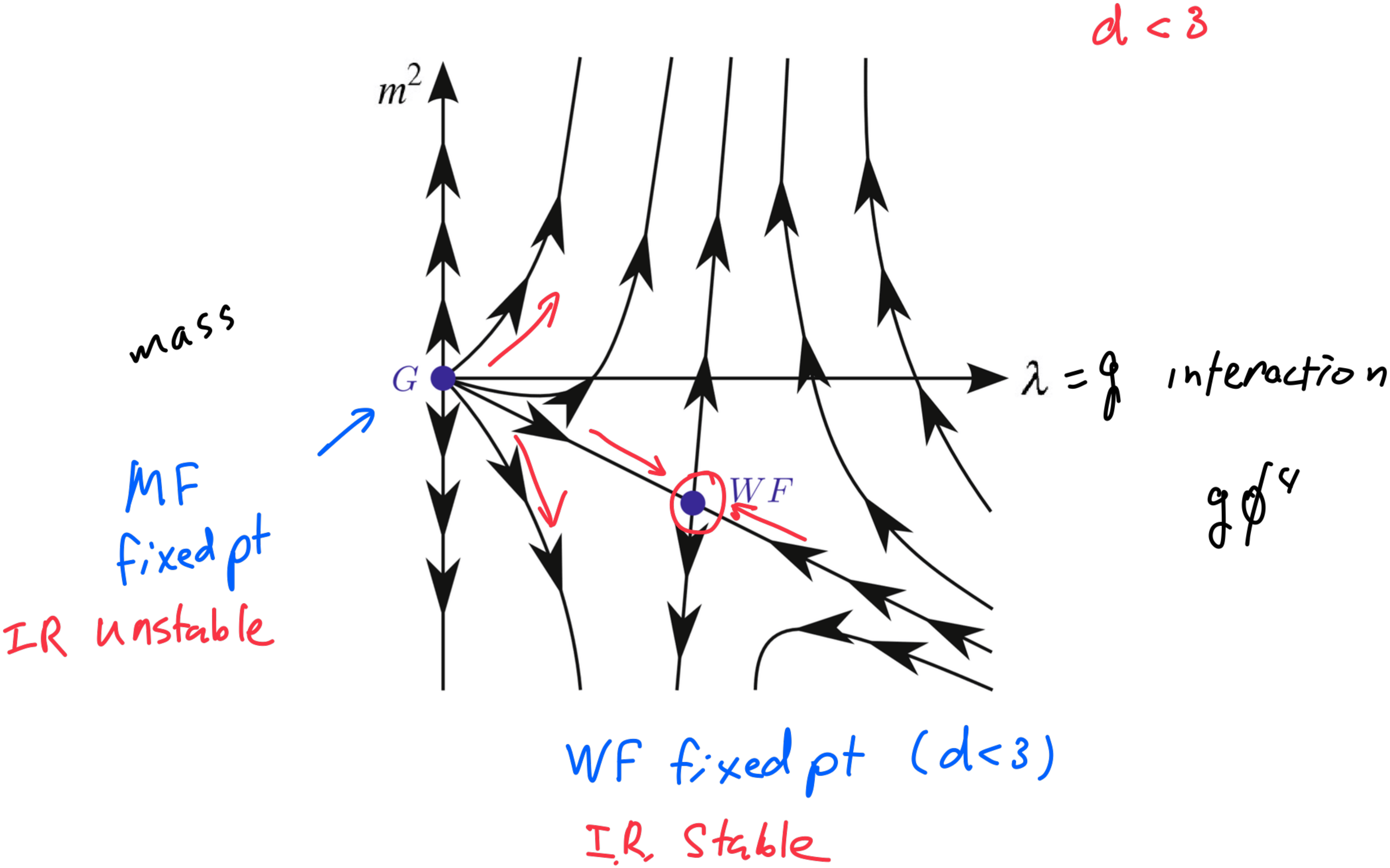


Universality Class I
$\mathcal{L}(\tilde{m}=0, \tilde{g}_1, \ldots)$
$\mathcal{L}(\tilde{m}=0, \ldots)$
$\mathcal{L}(\tilde{m}=0, \tilde{g}_3 \ldots)$
UV
IR
$\mathcal{L}^*(\tilde{m}=0, \tilde{g}=0)$
fixed pt



Universality II
$\mathcal{L}(\tilde{m}=0,\ \tilde{g}_1)$
t=0
$\mathcal{L}(\tilde{m}=0,$
t=1
$\mathcal{L}(\tilde{m}=0,\ \tilde{g}_3)$
t=2
$\mathcal{L}^*(\tilde{m}=0,\ \tilde{g}=0)$
fixed pt
UV
t
IR
IR
$\tilde{m}$
$\tilde{g}$
t
=
3 2 1 0

RG flows (i.e. scale transformation) of a scalar model : Scale Symmetric Wilson-Fisher F.P. (d+1<4)



Scale invariance and Phase Transitions

1) Effective QFT or EFT constructed out of symmetry and other general considerations.

1) Critical point is identified as a scale invariant QFT (or CFT if z=1,2) or a fixed point Hamiltonian understand scale transformation.

3) Microscopic information (which appears in EFT) wiped out in the course of renormalization leading to universalities.

Midterm Presentation (the week of March 1-5)

Far away-Non-equilibrium dynamics near QPT:

Colloquium: Nonequilibrium dynamics of closed interacting quantum systems
Anatoli Polkovnikov, Krishnendu Sengupta, Alessandro Silva, and Mukund Vengalattore
Rev. Mod. Phys. 83, 863.

Zurek, W. H.; Dorner, U.; Zoller, P. (2005). "Dynamics of a Quantum Phase Transition". *Phys. Rev. Lett*. **95** (10): 105701.

Transport Dynamics near QCPs and some modern applications:

Nonzero-temperature transport near quantum critical points,
Kedar Damle and Subir Sachdev, Phys. Rev. B 56, 8714 (1997)

Legros et al., L. Taillefer, and C. Proust, "Universal T-linear resistivity and Planckian dissipation in overdoped cuprates, Nature Physics 15, 142 (2018),

Jan Zaanen, "minimal viscosity and the transport in cuprate strange metals", SciPost Phys. 6, 061 (2019).

Aavishkar A. Patel and Subir Sachdev, Theory of strange metals,