Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode Eight: More On **Time Ordering** via “ Z”:

Space-time temporal correlations and introduction to scale symmetry at QCPs

TIME ORDERING via "Z"

$\tau > 0$

$$\langle \underset{b_{\vec r}}{\phi(\vec r,\tau)}\ \underset{b_0^{\dagger}}{\phi^*(0,0)} \rangle_Z \overset{T\to\infty}{=} \langle g.s.|\, \underbrace{e^{H\tau}\, b_{\vec r}\, e^{-H\tau}}_{b(\vec r,\tau)}\, b_0^{\dagger(0)} |g.s.\rangle$$

"classical like" $\longleftarrow$ "Z" $\longrightarrow$ "Quantum evolution"

$\tau < 0$

$$\langle \underset{b_{\vec r}}{\phi(\vec r,\tau)}\ \underset{b_0^{\dagger}}{\phi^*(0,0)} \rangle_Z \overset{T\to\infty}{=} \langle g.s.|\, b_0^{\dagger}\, \underbrace{e^{H\tau}\, b_{\vec r}\, e^{-H\tau}}_{b(\vec r,\tau)} |g.s.\rangle$$

TIME ORDERING

$\tau > 0$

$$\langle \underset{b_{\vec r}}{\phi}(\vec r,\tau)\,\phi^{*}(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\,\underbrace{e^{H\tau}\,b_{\vec r}\,e^{-H\tau}}_{b(\vec r,\tau)}\,b_0^{(0)\dagger}|g.s.\rangle$$

$\tau < 0$

$$\langle \underset{b_{\vec r}}{\phi}(\vec r,\tau)\,\phi^{*}(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\,b_0^{\dagger}\,\underbrace{e^{H\tau}\,b_{\vec r}\,e^{-H\tau}}_{b(\vec r,\tau)}|g.s.\rangle$$

Pull together,

$$\langle \phi(\vec r,\tau)\,\phi^{*}(0,0)\rangle_Z = \langle g.s.|\,\mathcal{T}\,b_{\vec r}(\tau)\,b_0^{(0)\dagger}|g.s\rangle$$

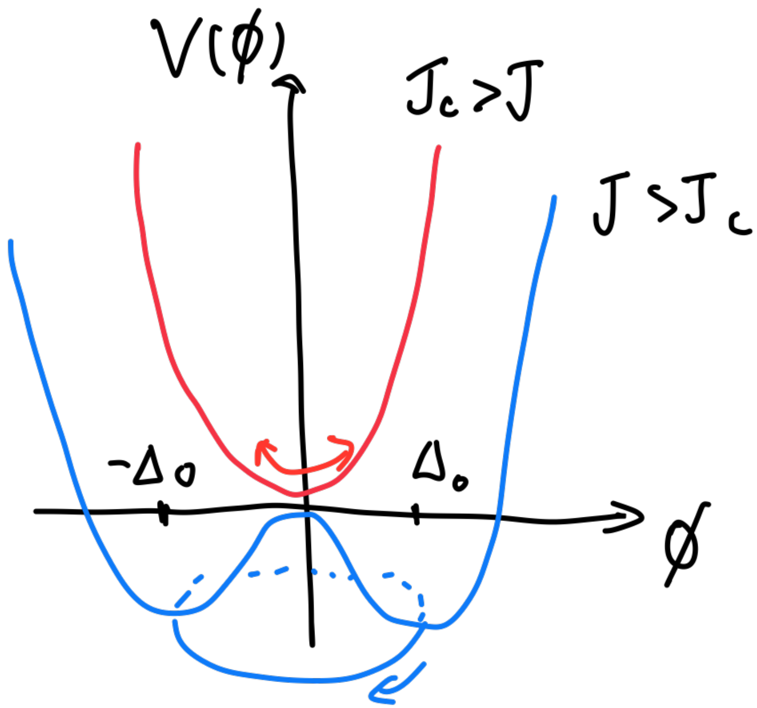
$$\mathcal{T}\,b_{\vec r}(\tau)\,b_0^{\dagger}(0) = \Theta(\tau)\,b_{\vec r}(\tau)\,b_0^{\dagger} + \Theta(-\tau)\,b_0^{\dagger}\,b_{\vec r}(\tau)$$

Mean field and breakdown of Mean field

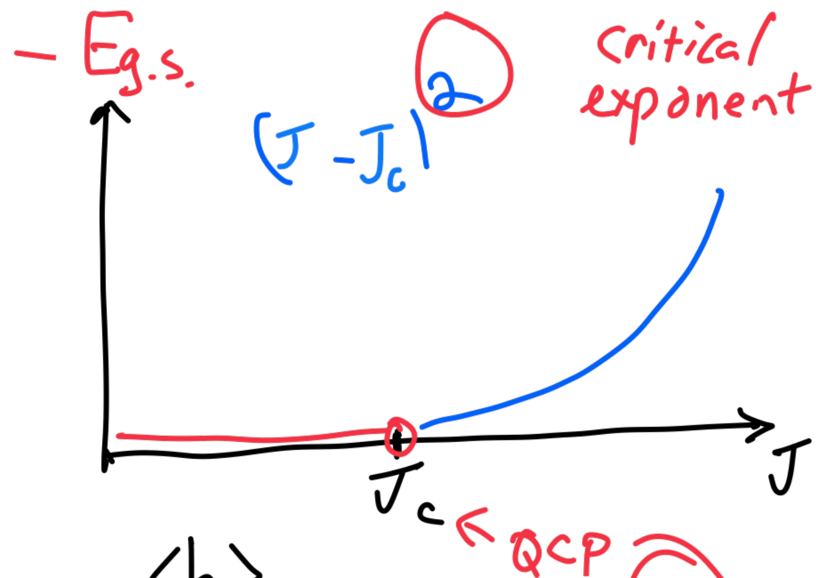
(d+1)D,

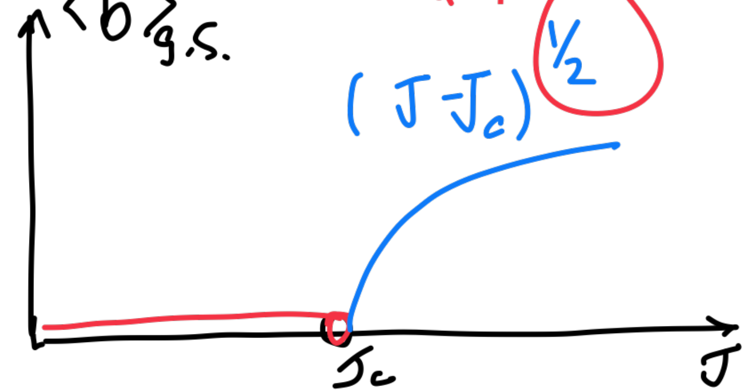
$$\mathcal{L}(\{\phi(\vec{r},\tau)\}) = |\partial_\tau \phi|^2 + |\nabla \phi|^2 + \alpha|\phi|^2 + \lambda|\phi|^4 + \dots$$

$$\alpha = A(J_c - J)$$



$$\langle \phi \rangle_{g.s.} = \Delta_0 e^{i\varphi}$$





- Towards time ordered Green's function **G**

$$G(r,\tau;0,0) = \langle g.s.|\, T\, b_{\vec r}(\tau)\, b_0^\dagger(0)\,|g.s.\rangle$$

or

$$G(q,\tau;q,0) = \langle g.s.|\, T\, b_q(\tau)\, b_q^\dagger(0)\,|g.s.\rangle$$

$$\overset{\tau>0}{=} \sum_{\vec q} \langle g.s.|\, \overset{e^{+H\tau} b_{\vec q}\, e^{-H\tau}}{b_{\vec q}(\tau)}\, |\vec q\rangle\langle\vec q|\, b_{\vec q}^\dagger(0)\,|g.s.\rangle$$

$\vec q$ ← all Many-body eigen states with Momentum $\vec q$

$$= \underbrace{|\langle g.s.|\, b_{\vec q}\, |\vec q\rangle^P|^2\, e^{-E^P_{\vec q}\tau}}_{\text{"Particle-like M.B. States"}} + \underbrace{\cdots\cdots\cdots}_{\text{Continuum}}$$

"Multiparticles" M.B. States

Fluctuations near $\langle\phi\rangle$, $\phi \to \langle\phi\rangle + \phi$

$$\mathcal{L}(\{\phi(\vec{r},\tau)\}) = |\partial_\tau \phi|^2 + |\nabla\phi|^2 + m^2|\phi|^2 + \ldots, \quad m^2 = 4|\alpha|$$

$$Z = \int D\phi_{\vec{r},\tau}\, e^{-\int d^{(d)}\vec{x}\, d\tau\, \mathcal{L}(\{\phi(\vec{r},\tau)\})}$$

$$\simeq \int D\phi_{\omega,q}\, e^{-\sum_\omega \sum_q \bar{\Phi}^*(\omega,\vec{q})\, G^{-1}(\omega,\vec{q})\, \Phi(\omega,\vec{q})}$$

$$= \prod_{\omega,q} Z_{\omega,\vec{q}}$$

$$G^{-1}(\omega,q) = \omega^2 + \vec{q}^{\,2} + m^2$$

MF

$$\langle \bar{\Phi}^*(\omega,\vec{q})\, \Phi(\omega,\vec{q}) \rangle \simeq X\, G(\omega,\vec{q})$$

$Z_{\omega,\vec{q}}$ HW Set II

Fluctuations near $\langle\phi\rangle$, $\quad \phi \to \langle\phi\rangle + \phi$

$$\mathcal{L}(\{\phi(\vec{r},\tau)\}) = |\partial_\tau \phi|^2 + |\nabla\phi|^2 + m^2 |\phi|^2 + \dots, \quad m^2 = 4|\alpha|$$

$$\langle \phi(\vec{r},\tau)\,\phi^*(0,0)\rangle_Z \simeq \int \frac{d^d k}{(2\pi)^d} \int \frac{d\omega}{2\pi}\, G(\omega,\vec{p})\, e^{i\vec{p}\cdot\vec{r} - i\omega\tau}$$

$G(\vec{r},\tau)$

$= \langle g.s.|T\, b_{\vec{r}}(\tau)\, b_0^+(0)|g.s.\rangle$

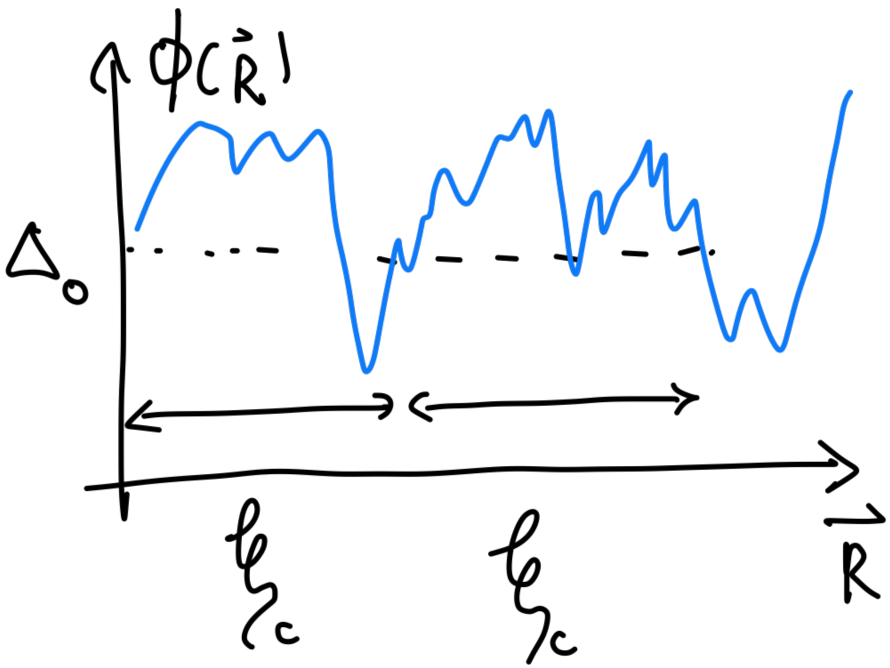
$$\sim \frac{1}{|\vec{R}|^{d-1}} e^{-\frac{|\vec{R}|}{\xi_c}}, \quad \underbrace{\vec{R} = (\vec{r},\tau)}_{(d+1)\text{ dimension}}$$

$$\xi_c \sim m^{-1} \sim \frac{1}{|J-J_c|^{1/2}}$$

$$(\,|\alpha| \propto J - J_c > 0\,)$$

$$\langle \Phi^*(\vec{R})\,\Phi(\vec{R}')\rangle \sim \frac{e^{-\frac{|\vec{R}-\vec{R}'|}{\xi_c}}}{|\vec{R}-\vec{R}'|^{d-1}}$$

Correlation length



$$\xi_c \sim \frac{1}{|J-J_c|^{1/2}}$$

"Correlated fluctuations"

$$G^{Q} = \frac{\left\langle \frac{1}{V_c}\int_{V_c} \tilde{\Phi}^*(\vec{R}')\, d^{(d+1)}\vec{R}' \cdot \frac{1}{V_c}\int_{V_c} \tilde{\Phi}(\vec{R})\, d^{(d+1)}\vec{R} \right\rangle}{\Delta_0^2}$$

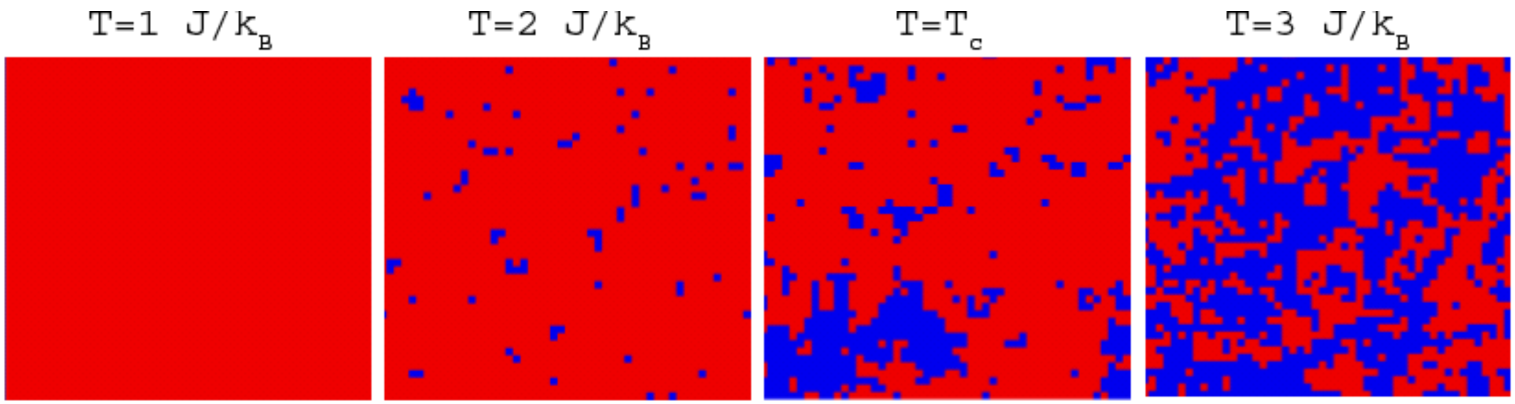
$$= \frac{1}{V_c^2 \Delta_0^2} \int_{V_c} \left\langle \tilde{\Phi}^*(\vec{R}')\, \tilde{\Phi}(\vec{R}) \right\rangle d^{(d+1)}\vec{R}'\, d^{(d+1)}\vec{R}$$

$$G(\vec{R}' - \vec{R}) \sim \frac{1}{|\vec{R}' - \vec{R}|^{d-1}}$$

$$\sim \frac{1}{V_c \Delta_0^2}\, \xi_c^2 \sim \xi_c^{3-d}$$

$$\left(V_c \sim \xi_c^{d+1}, \quad \Delta_0^2 \sim |\alpha| \sim \xi_c^{-2} \right)$$

Emergent Scale Symmetry at critical point of Ising Model (Kadanoff, 1960-70s)



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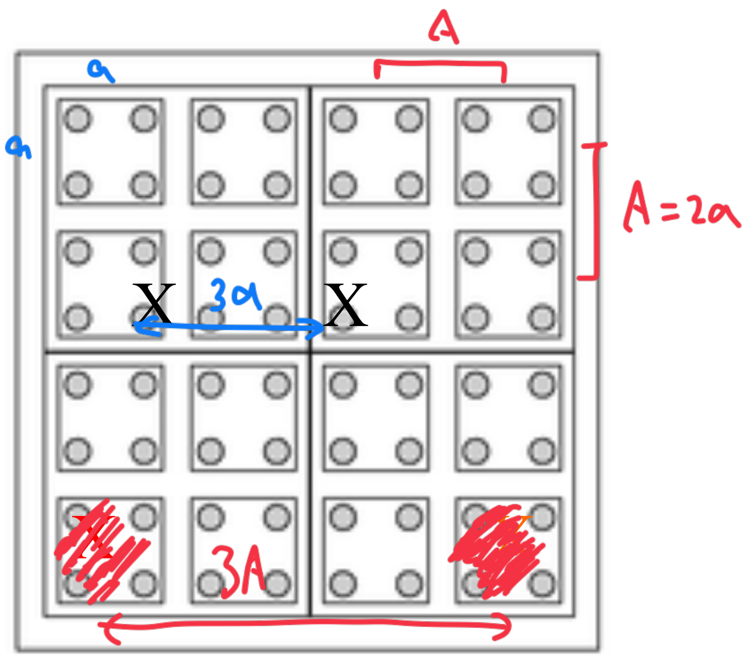
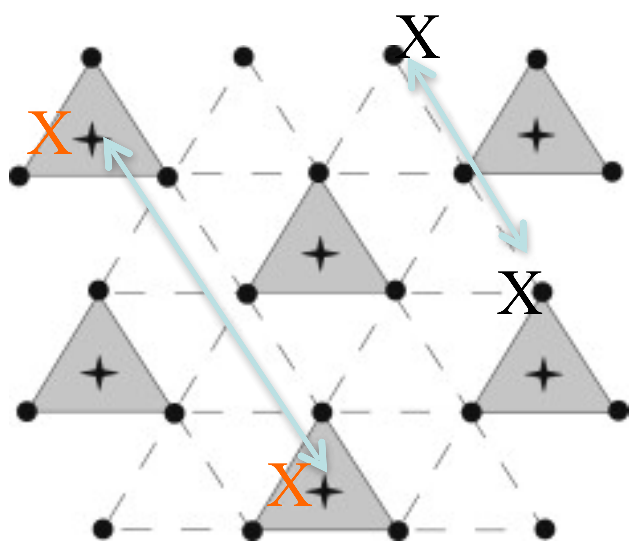


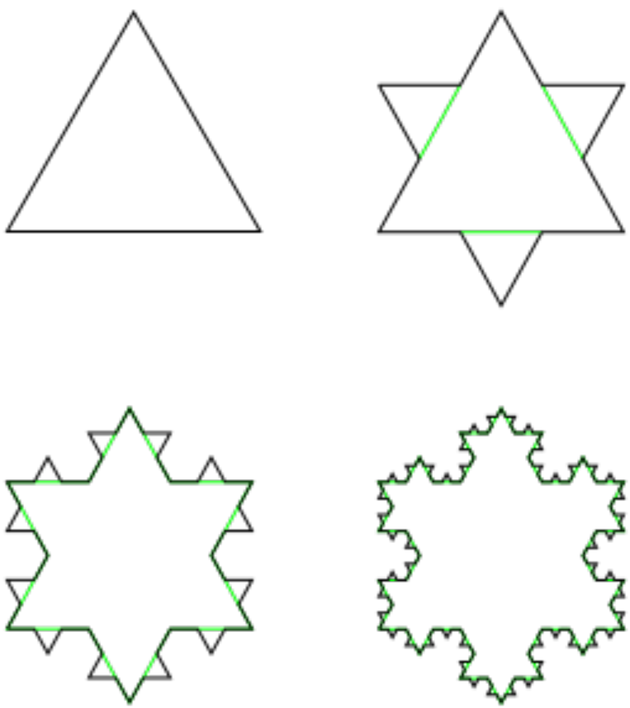


X: spin; X: block spin

Scale symmetries in nature

The correlation of BLOCK spins (b X b) at distance bL away
= The the correlations of Microscopic spins at distance L away.

-->The correlation of “rescaled fields” at distance Lb
=The correlation of original fields at distance L



Slightly formally speaking, if $a \to \lambda a$ (d+1) D space-time

A) $\vec{x} \longrightarrow \vec{y} = \frac{\vec{x}}{\lambda}$ or $\vec{x} = \lambda \vec{y}$ $\in R^{d+1}$

$$\psi(\vec{x}) \longrightarrow \psi'(\vec{y}) = \lambda^{\eta} \psi(\vec{x} = \lambda \vec{y}),$$

B) $G'(\vec{y}', \vec{y}) = \langle \psi'(\vec{y}') \psi'(\vec{y}) \rangle$

$$= G(\vec{y}', \vec{y})$$

$$= \langle \psi(\vec{y}') \psi(\vec{y}) \rangle$$

then

The system has a scale symmetry

$$\psi(\vec{x}) \longrightarrow \psi'(\vec{y}) = \lambda^{\eta}\,\psi(\vec{x} = \lambda\vec{y})$$

$$G'(\vec{y}', \vec{y}) = \langle \psi'(\vec{y}')\,\psi'(\vec{y})\rangle$$

$$= G(\vec{y}', \vec{y})$$

Example: A QCP ($\xi_c \to \infty$), $G(\vec{y}', \vec{y}) = \dfrac{1}{|\vec{y}' - \vec{y}|^{\beta}}$

$$G'(\vec{y}', \vec{y}) = \lambda^{2\eta}\,\langle \psi(\lambda\vec{y}')\,\psi(\lambda\vec{y})\rangle$$

$$= \lambda^{2\eta}\,\frac{1}{\lambda^{\beta}\,|\vec{y}' - \vec{y}|^{\beta}} = G(\vec{y}', \vec{y})$$

if $2\eta = \beta$ ($\beta = d-1$ in MF)