Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode seven: A brief comment On **Time Ordering** in "Z" Vs quantum Ground states:

Space-time temporal correlations. And scale symmetry at QCPs

Important Statements on fluctuations ← Quantum

$G^{Q} \sim \xi_c^{3-d}$ ($\xi_c \to \infty$ at critical points)

1) $d > 3$, G^{Q} approaches zero as $\xi_c \to \infty$.
fluctuations appear to be irrelevant.

2) $d < 3$, G^{Q} diverges as $\xi_c \to \infty$
fluctuations appear to be irrelevant
and MF can be qualitatively problematic.

$d = 3$ Upper critical dimension for Q.C.P. !!

$$Z = \langle 0 | e^{-HT} | 0 \rangle$$

$$= \int D\phi \, e^{-\underbrace{\int d^{(d)}\vec{r}\, d\tau \, \mathcal{L}(\{\phi(\vec{r},\tau)\})}_{S(\{\phi(\vec{r},\tau)\})}}$$

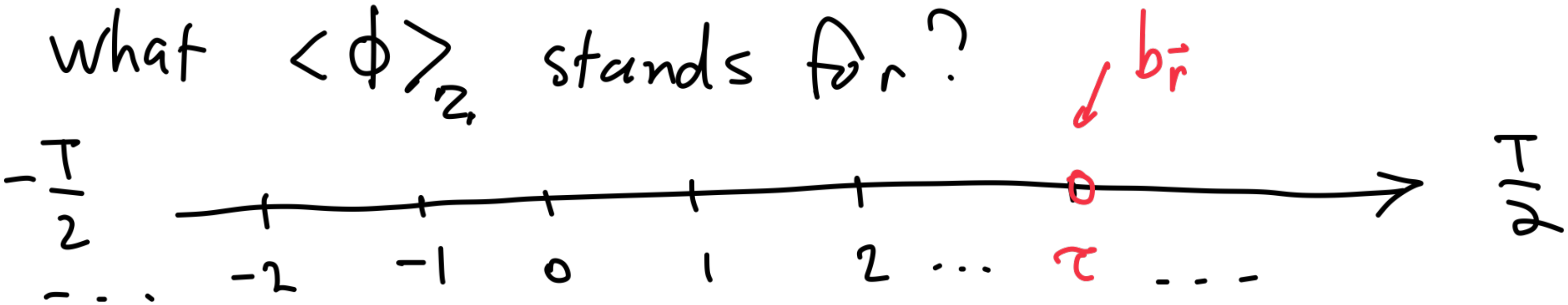
$$E_{g.s.} \overset{T\to\infty}{=} -\frac{1}{T} \ln Z$$

$$\langle \phi \rangle_Z = \frac{\int D\phi \; \phi(\vec{r},\tau)\, e^{-S(\{\phi(\vec{r},\tau)\})}}{\int D\phi \; e^{-S(\{\phi(\vec{r},\tau)\})}}$$

$$\langle \phi^*(\vec{r},\tau)\, \phi(0,0) \rangle_Z = \frac{\int D\phi \; \phi^*(\vec{r},\tau)\, \phi(0,0)\, e^{-S}}{\int D\phi \; e^{-S}}$$

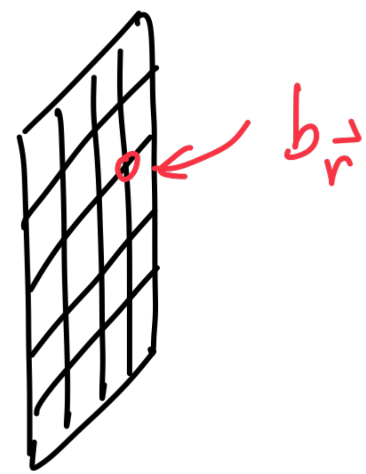
$$\langle \phi \rangle_Z = \frac{\int D\phi \;\; \phi(\vec{r},\tau)\, e^{-S(\{\phi(\vec{r},\tau)\})}}{\int D\phi \;\; e^{-S(\{\phi(\vec{r},\tau)\})}}$$

what $\langle \phi \rangle_Z$ stands for?



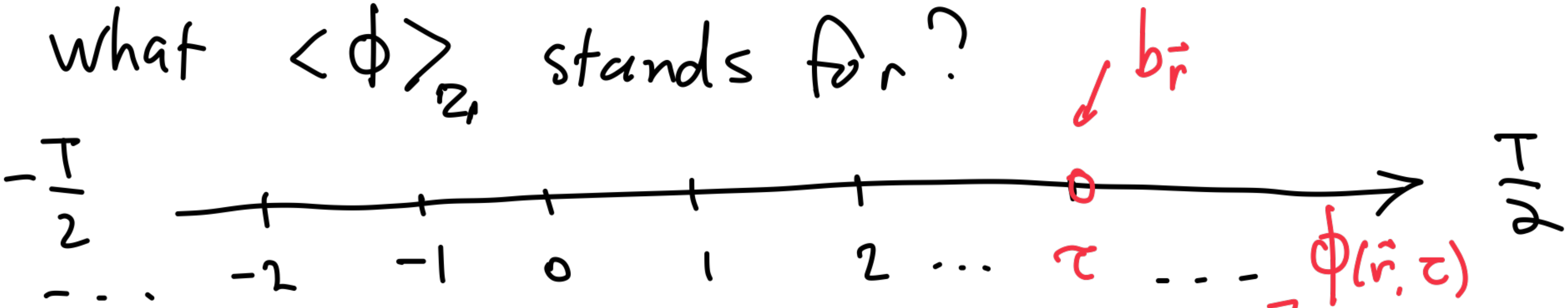
$$b_{\vec{r}} \,|\phi_{\vec{r}}\rangle = \phi(\vec{r})\,|\phi_{\vec{r}}\rangle$$

"Coarse grained state"



$$\langle \phi \rangle_Z = \frac{\int \Delta\phi \;\; \phi(\vec r,\tau)\, e^{-S(\{\phi(\vec r,\tau)\})}}{\int \Delta\phi \;\; e^{-S(\{\phi(\vec r,\tau)\})}}$$

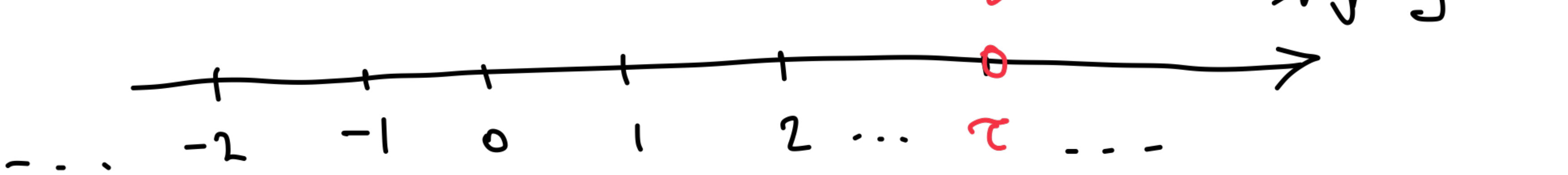
what $\langle \phi \rangle_Z$ stands for?



$$Z_{b_{\vec r}} = \cdots \sum_{\{\phi(\vec r,\tau+1)\}} \; \sum_{\{\phi(\vec r,\tau)\}} \cdots \langle\{\phi(\tau+1)\}|\, e^{-H\Delta\tau}\, b_{\vec r}\, |\{\phi(\tau)\}\rangle \cdots$$

$$= \langle 0|\, \underbrace{e^{-H\left(\frac{T}{2}-\tau\right)}}_{\text{After}}\; b_{\vec r}\; \underbrace{e^{-H\left(\tau+\frac{T}{2}\right)}}_{\text{before}}\, |0\rangle$$

what $\langle \phi \rangle_2$ stands for?

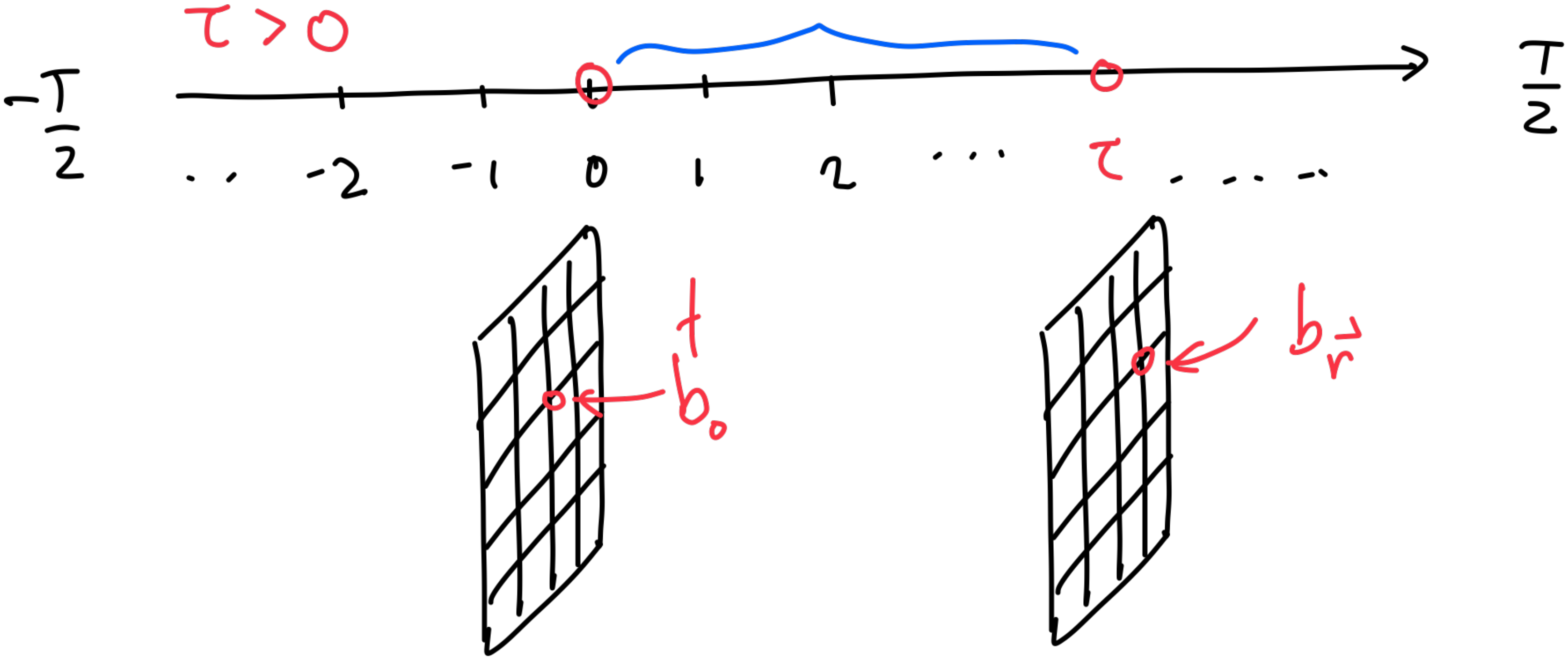


$$\langle \phi \rangle_2 = \frac{Z(b_{\vec r}, \tau)}{Z} = \frac{\langle 0 | e^{-(\frac{T}{2}-\tau)H} \, b_{\vec r} \, e^{-(\tau+\frac{T}{2})H} | 0 \rangle}{\langle 0 | e^{-TH} | 0 \rangle}$$

$$\overset{T\to\infty}{\simeq} \frac{|\langle 0 | g.s \rangle|^2 e^{-E_{g.s.} T}}{|\langle 0 | g.s. \rangle|^2 e^{-E_{g.s.} T}} \cdot \langle g.s. | b_{\vec r} | g.s. \rangle$$

g.S. Quantum Average!

Averge over "Z" Vs Quantum State averge



$$\langle \phi(\vec r,\tau)\, \phi^*(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\, \underbrace{e^{H\tau}\, b_{\vec r}\, e^{-H\tau}}_{b(\vec r,\tau)}\, b_0^\dagger(0)\, |g.s.\rangle$$

(Heisenberg Rep.)

$\tau > 0$

$$\langle \phi(\vec{r},\tau)\,\phi^{*}(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\, \underbrace{e^{H\tau}\, b_{\vec{r}}\, e^{-H\tau}}_{b^{\dagger}(\vec{r},\tau)}\, b_0^{\dagger}(0)\,|g.s.\rangle$$

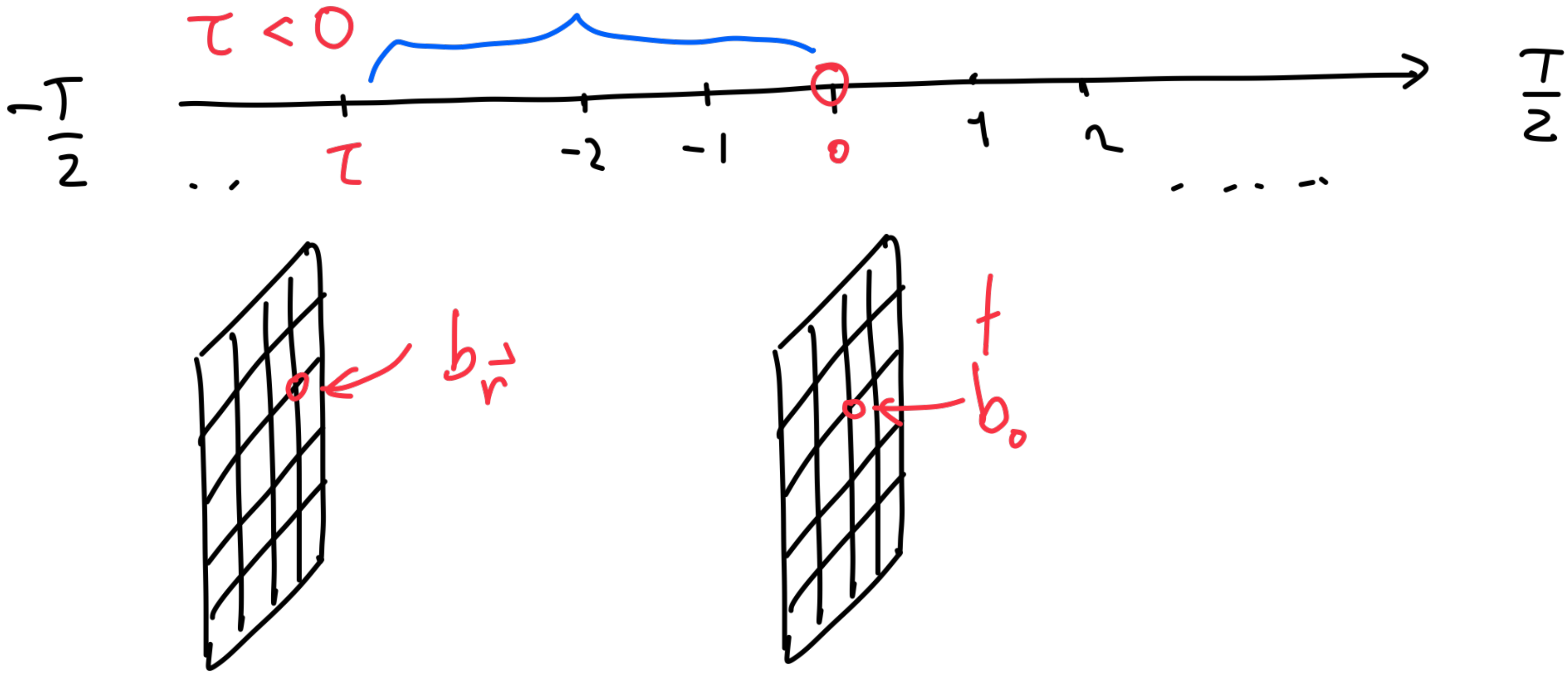
(Heisenberg Rep.)

$$\langle g.s.|\, b(\vec{r},\tau)\, b_0^{\dagger}\,|g.s.\rangle$$

All excitations

$$= \sum_n \langle g.s.|\, b(\vec{r},\tau)\,|n\rangle\langle n|\, b_0^{\dagger}\,|g.s.\rangle$$

Dynamic Correlation function $\longleftrightarrow$ excitations



$$\langle \phi(\vec r,\tau)\,\phi^*(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\, b_0^\dagger\, e^{-H(0-\tau)}\, \underbrace{b_{\vec r}\, e^{-H\tau}}_{b(\vec r,\tau)} |g.s.\rangle$$

$\tau < 0$

(Heisenberg Rep.)

TIME ORDERING

$\tau > 0$

$$\langle \phi(\vec{r},\tau)\,\phi^{*}(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\,\underbrace{e^{H\tau}\, b_{\vec{r}}\, e^{-H\tau}}_{b(\vec{r},\tau)}\, b_0^{\dagger(0)}|g.s.\rangle$$

$\tau < 0$

$$\langle \phi(\vec{r},\tau)\,\phi^{*}(0,0)\rangle_Z \overset{T\to\infty}{=} \langle g.s.|\, b_0^{\dagger}\, \underbrace{e^{H\tau}\, b_{\vec{r}}\, e^{-H\tau}}_{b(\vec{r},\tau)}|g.s.\rangle$$

Pull together,

$$\langle \phi(\vec{r},\tau)\,\phi^{*}(0,0)\rangle_Z = \langle g.s.|\,\mathcal{T}\, b_{\vec{r}}(\tau)\, b_0^{\dagger}(0)|\,g.s\rangle$$

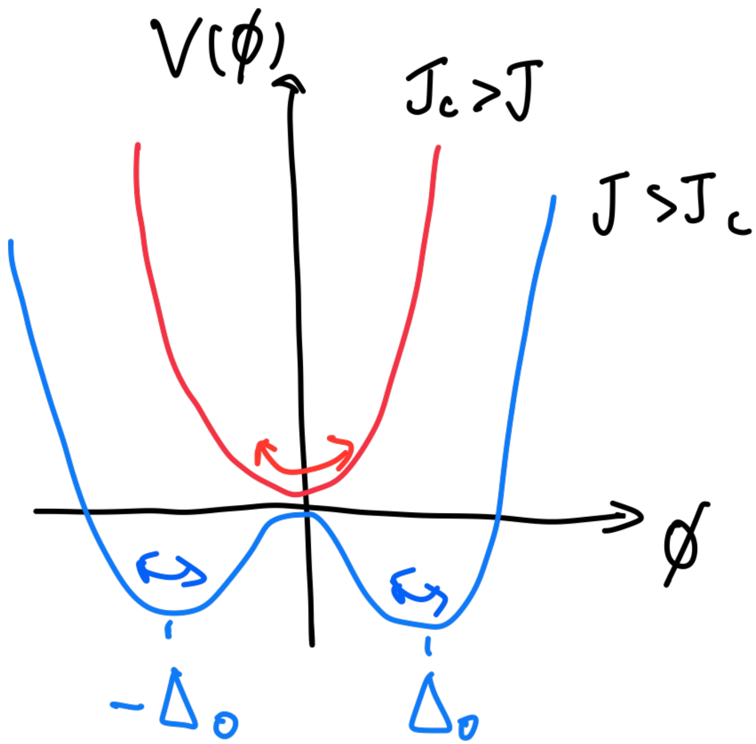
$$\mathcal{T}\, b_{\vec{r}}(\tau)\, b_0^{\dagger}(0) = \Theta(\tau)\, b_{\vec{r}}(\tau)\, b_0^{\dagger} + \Theta(-\tau)\, b_0^{\dagger}\, b_{\vec{r}}(\tau)$$

Mean field and break down of Mean field

(d+1)D,

$$\mathcal{L}(\{\phi(\vec{r},\tau)\}) = |\partial_\tau \phi|^2 + |\nabla\phi|^2 + \alpha|\phi|^2 + \lambda|\phi|^4 + \dots$$

$$\alpha = A(J - J_c)$$



M.F. $\mathcal{L}_{MF} = \alpha|\phi|^2 + \lambda|\phi|^4$.

$$(\nabla\phi = \partial_\tau \phi = 0)$$

$$e^{-S} = e^{-\Omega_d \cdot T \cdot \mathcal{L}_{MF}}$$

$$E_{g.s.} = \Omega_d \cdot \mathcal{L}_{MF}(|\phi| = \Delta_0)$$

Mean field and break down of Mean field

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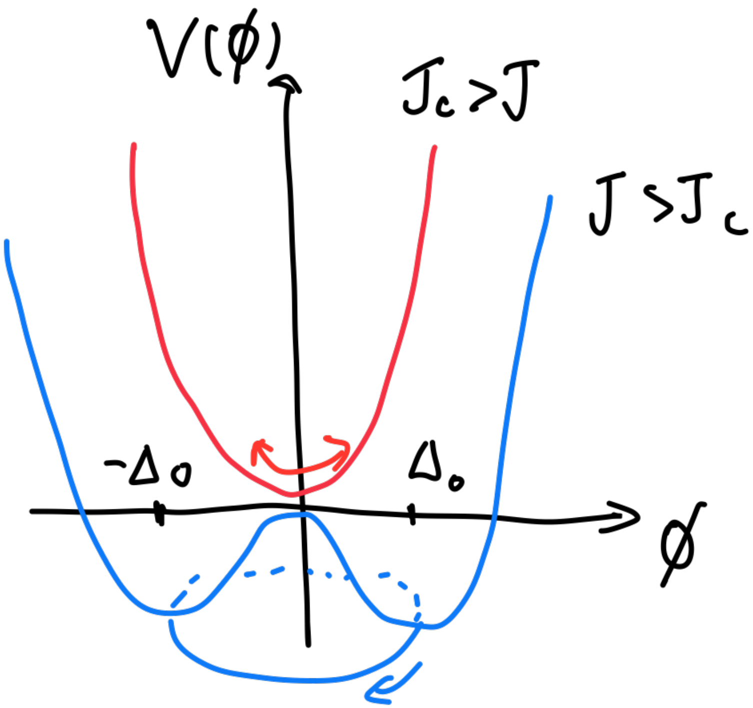
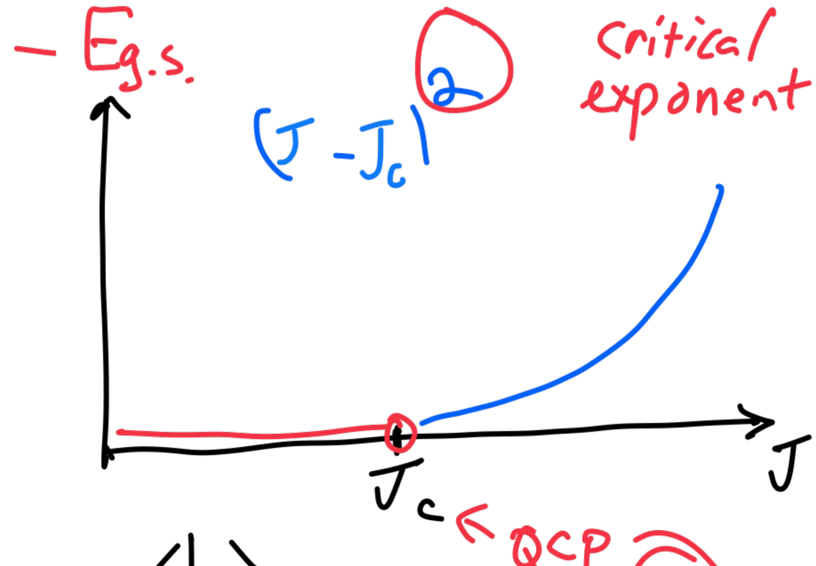
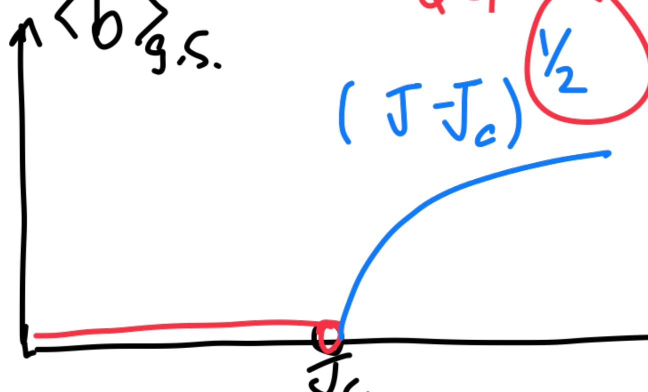
$$\mathcal{L}(\{\phi(\vec{r},\tau)\}) = |\partial_\tau \phi|^2 + |\nabla \phi|^2 + \alpha |\phi|^2 + \lambda |\phi|^4 + \dots$$

$$\alpha = A(J_c - J)$$



$$\langle \phi \rangle_{g.s.} = \Delta_0 e^{i\varphi}$$





Fluctuations near $\langle\phi\rangle$, $\quad \phi \to \langle\phi\rangle + \phi$

$$\mathcal{L}(\{\phi(\vec{r},\tau)\}) = |\partial_\tau \phi|^2 + |\nabla\phi|^2 + m^2|\phi|^2 + \dots, \quad m^2 = 4|\alpha|$$

$$Z = \int \mathcal{D}\phi_{\vec{r},\tau}\, e^{-\int d^{(d)}\vec{x}\, d\tau\, \mathcal{L}(\{\phi(\vec{r},\tau)\})}$$

$$\cong \int \mathcal{D}\phi_{\omega,q}\, e^{-\sum_\omega \sum_q \bar{\Phi}^*(\omega,\vec{q})\, G^{-1}(\omega,\vec{q})\, \Phi(\omega,\vec{q})}$$

$$= \prod_{\omega,q} Z_{\omega,\vec{q}}$$

$$G^{-1}(\omega,q) = \omega^2 + \vec{q}^{\,2} + m^2$$

MF

$$\underbrace{\langle \bar{\Phi}^*(\omega,\vec{q})\, \Phi(\omega,\vec{q})\rangle}_{Z_{\omega,\vec{q}}} \cong \underbrace{\cancel{}}_{\text{HW Set II}}\, G(\omega,\vec{q})$$

Fluctuations near $\langle\phi\rangle$, $\quad \phi \to \langle\phi\rangle + \phi$

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$$\langle \phi(\vec r,\tau)\,\phi(0,0)\rangle \simeq \int \frac{d^d k}{(2\pi)^d}\int\frac{d\omega}{2\pi}\, G(\omega,\vec p)\, e^{i\vec p\cdot\vec r - i\omega\tau}$$

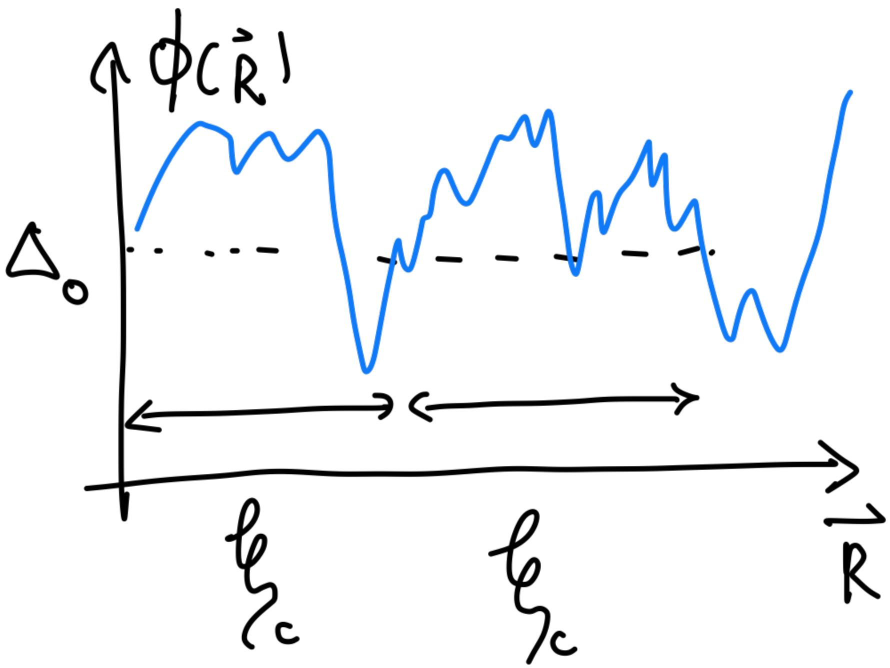
$G(\vec r,\tau)$

$$\sim \frac{1}{|\vec R|^{d-1}}\, e^{-\frac{|\vec R|}{\xi_c}}, \quad \underbrace{\vec R = (\vec r,\tau)}_{(d+1)\text{ dimension}}$$

$$\xi_c \sim m^{-1} \sim \frac{1}{|J-J_c|^{1/2}} \qquad (\,|\alpha| \propto J - J_c > 0\,)$$

$$\langle \Phi^*(\vec{R})\, \Phi(\vec{R}') \rangle \sim \frac{e^{-\frac{|\vec{R}-\vec{R}'|}{\xi_c}}}{|\vec{R}-\vec{R}'|^{d-1}}$$

Correlation length

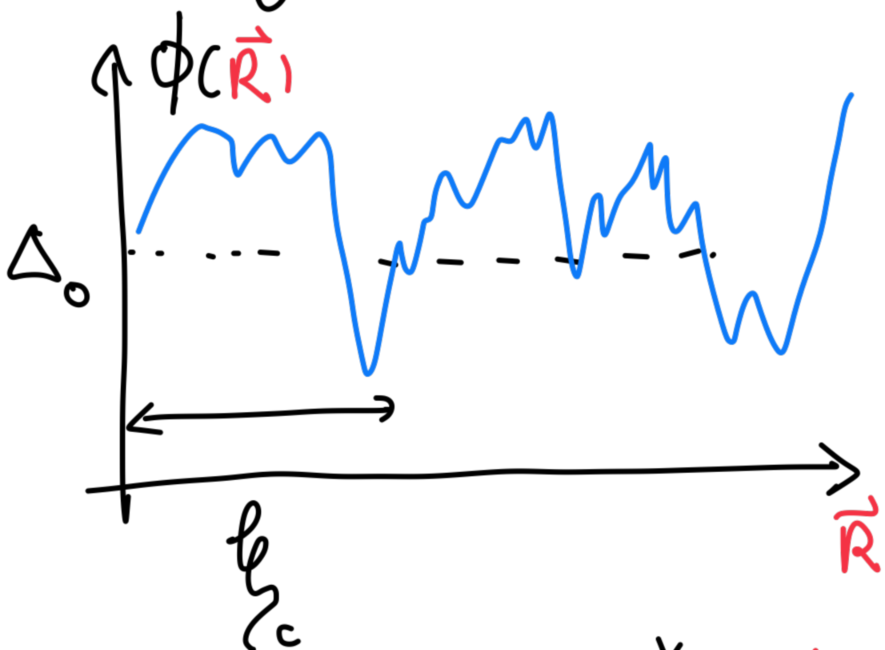


$$\xi_c \sim \frac{1}{|J-J_c|^{1/2}}$$

"Correlated fluctuations"

Quantum Spatial-Temporal

Ginsburg Criterion — Measurement of fluctuations



$\vec{R} = (\vec{r}, \tau)$

$V_c = \xi_c^{\,d+1}$

Vol. of Correlation length

$$G^Q = \frac{\left\langle \frac{1}{V_c}\int_{V_c} \tilde{\Phi}^*(\vec{R}')\, d^{(d+1)}\vec{R} \cdot \frac{1}{V_c}\int_{V_c} \tilde{\Phi}(\vec{R})\, d^{(d+1)}\vec{R} \right\rangle}{\Delta_0^2}$$

$$= \frac{1}{V_c^2 \Delta_0^2} \int_{V_c} \langle \tilde{\Phi}^*(\vec{R}')\, \tilde{\Phi}(\vec{R}) \rangle\, d^{(d+1)}\vec{R}'\, d^{(d+1)}\vec{R}$$

$$G^{Q} = \frac{\left\langle \frac{1}{V_c} \int_{V_c} \tilde{\Phi}^*(\vec{R}')\, d^{(d+1)}\vec{R} \cdot \frac{1}{V_c} \int_{V_c} \tilde{\Phi}(\vec{R})\, d^{(d+1)}\vec{R} \right\rangle}{\Delta_0^2}$$

$$= \frac{1}{V_c^2 \Delta_0^2} \int_{V_c} \left\langle \tilde{\Phi}^*(\vec{R}')\, \tilde{\Phi}(\vec{R}) \right\rangle d^{(d+1)}\vec{R}'\, d^{(d+1)}\vec{R}$$

$$\rightarrow G(\vec{R}' - \vec{R}) \sim \frac{1}{|\vec{R}' - \vec{R}|^{d-1}}$$

$$\sim \frac{1}{V_c \Delta_0^2} \xi_c^2 \sim \xi_c^{3-d}$$

$$\left(V_c \sim \xi_c^{d+1}, \quad \Delta_0^2 \sim |\alpha| \sim \xi_c^{-2} \right)$$

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