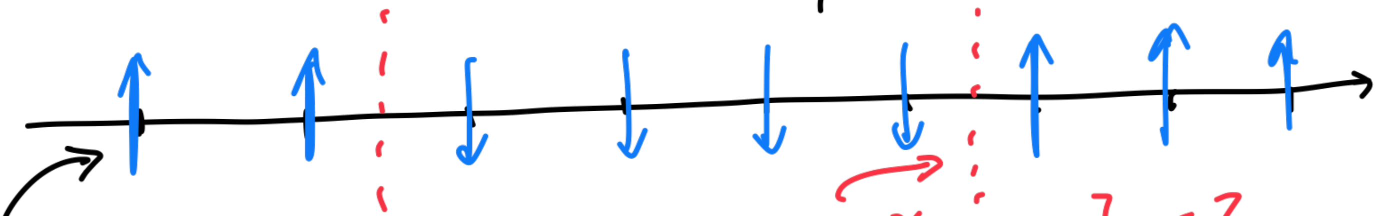
Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode Six: A brief comment on (1+1) D physics, Beyond Landau paradigm transitions and Failure of Mean fields

Kramer-Wannier duality (1+1)D



Site operator σ_i^z

σ_i^x

link operator

$T_i^x = \sigma_i^z \sigma_{i+1}^z$

$T_i^z = \prod_{j<i} \sigma_i^x$

$\prod$: "product ordered"

T_i^z : Domain Wall Creation

$$\mathcal{H}_{Ising} = -J \sum_i \sigma_i^z \sigma_{i+1}^z - I \sum_i \sigma_i^x$$

$$\mathcal{H}_{Ising}^{(dual)} = -J \sum_i \overset{\sigma_i^z \sigma_{i+1}^z}{T_i^x} - I \sum_i \overset{\sigma_{i+1}^x}{T_i^z T_{i+1}^z}$$

Kramer-Wannier duality (1+1)D

$$\mathcal{H}_{Ising} = -J\sum_i \sigma_i^z \sigma_{i+1}^z - I\sum_i \sigma_i^x$$

$$\mathcal{H}_{Ising}^{(dual)} = -J\sum_i \underbrace{T_i^x}_{\sigma_i^z\sigma_{i+1}^z} - I\sum_i \underbrace{T_i^z T_{i+1}^z}_{\sigma_{i+1}^x}$$

Self-duality $\rightarrow$ At QCP, $J/I = f_c = \frac{I}{J}$

Hence $J = I$ at QCP.

Order-Disorder quantum phase transitions (Best understood subset of QCPs !!)

1) “Order” usually involves condensation of bosonic fields

Ising model: Real scalar field;
Superfluids: Complex scalar field;
Nematic order: Real “Director” field;
Ferromagnetic order: Complex vector field
……

2) So transitions are usually described by Bosonic quantum field theories, the standard Landau paradigm.

- In Landau paradigm, only symmetry matters not representations of the symmetry groups. (Young Tableau stuff etc.)

$p=2$

$p=4$

"fundamental" of symmetry group SU(N)

Example: SU(2) symmetry, S= 1, 2, 3, ... Representations

Simple order-disorder transitions shall belong to the same class!

- beyond Landau paradigm quantum phase transitions:(usually can't make contact with statistical models)

- Transition between two ordered quantum states breaking the same symmetries but differing in Global topology. (Pfaffian FQHS Suggested by Read, Green, 2000;.... Part III based on Yang and Zhou, 2020, Topological QCPs with/without Global symmetries, CPT etc.)

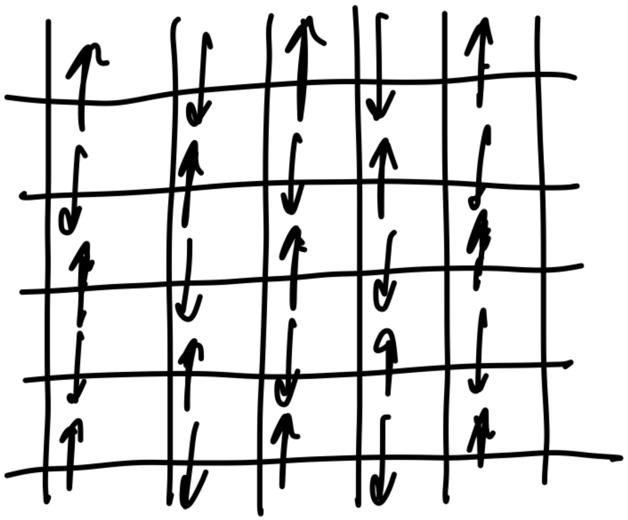
- Quantum systems with "quantum anomalies", SU(2), s=1/2, 3/2,... is purely quantum, not related to any classical models (Duncan Haldane, 80s (Noble prize 2016) Ian Affleck, 80s-now.). [related to p-form symmetries in Recent studies (2019 on)]

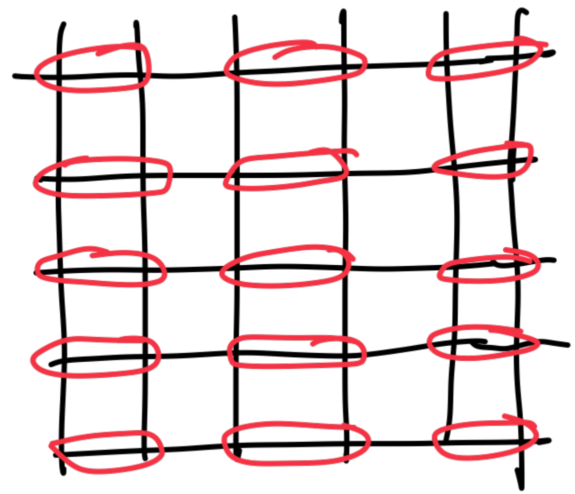
- Order A-Order B transitions, there can be emergent gauge symmetry at QCPs with elementary particles not appearing on either sides. De-confining QCPs. (Senthil, Vishwanath, Sachdev, Fisher, et al, 2010)

2D $S=\frac{1}{2}$, $\mathcal{H} = + J \sum'_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \dots$ Quantum fluctuations

Haldane Anomaly 88

Read, Sachdev 89





Spin ordered (breaks translation symmetry)

DQCP

(Senthil et.al, 04)

Spin-disordered (breaks translation symmetry and $\mathbb{Z}_4$)

Emergent U(1) Gauge Symmetry/fields

mean field, fluctuations and beyond

MF

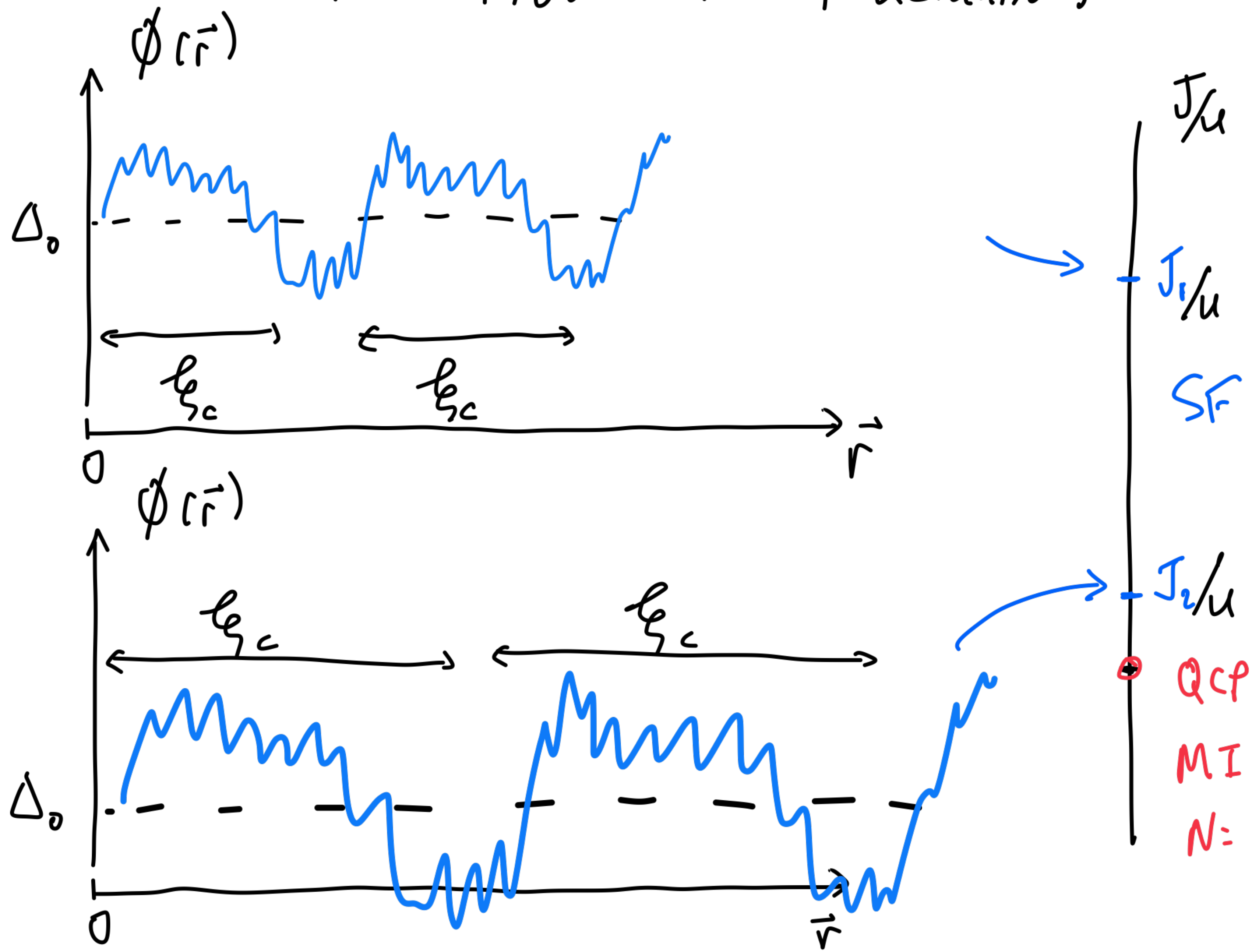
Reality

Blue $-$ Black $=$ Fluctuations

$$\phi(\vec{r}) = \Delta_0 + \delta\phi(\vec{r})$$

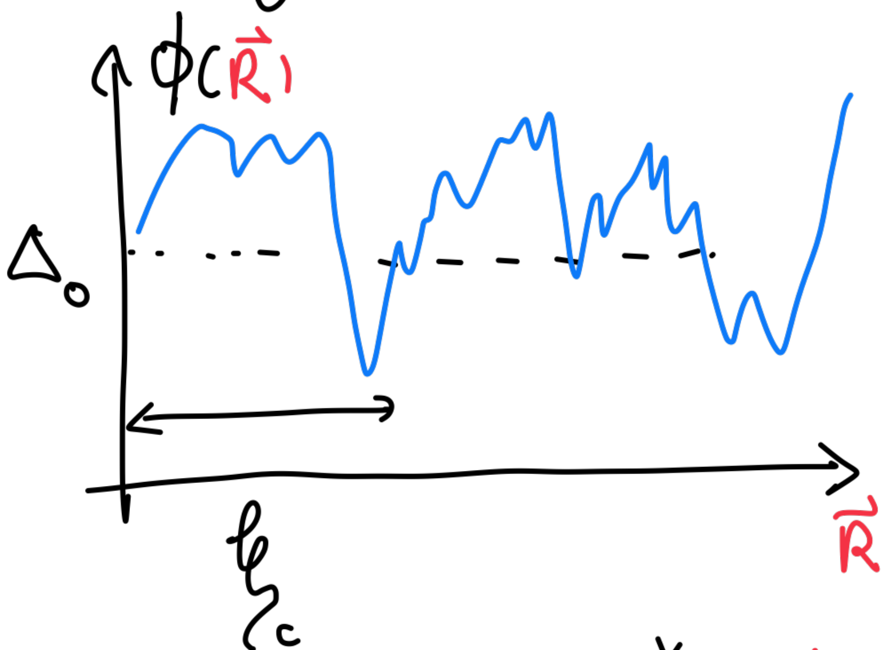
In disorder phase $\Delta_0 = 0$, $\phi(\vec{r}) = \delta\phi(\vec{r})$.

Reality = Mean field + fluctuations



Quantum Spatial-Temporal

Ginsburg Criterion – Measurement of fluctuations



$\vec{R} = (\vec{r}, \tau)$

$$V_c = \xi_c^{\,d+1}$$

Vol. of Correlation length

$$G^Q = \frac{\left\langle \frac{1}{V_c}\int_{V_c} \bar{\Phi}^*(\vec{R}')\, d^{(d+1)}\vec{R} \cdot \frac{1}{V_c}\int_{V_c} \bar{\Phi}(\vec{R})\, d^{(d+1)}\vec{R} \right\rangle_Z}{\Delta_0^2}$$

$$= \frac{1}{V_c^2 \Delta_0^2} \int_{V_c} \left\langle \tilde{\Phi}^*(\vec{R}')\, \bar{\Phi}(\vec{R}) \right\rangle_Z d^{(d+1)}\vec{R}'\, d^{(d+1)}\vec{R}$$

$$G^Q = \frac{\left\langle \frac{1}{V_c}\int_{V_c} \tilde{\Phi}^*(\vec{R}')\, d^{(d+1)}\vec{R}' \cdot \frac{1}{V_c}\int_{V_c} \tilde{\Phi}(\vec{R})\, d^{(d+1)}\vec{R} \right\rangle}{\Delta_0^2}$$

$$= \frac{1}{V_c^2 \Delta_0^2} \int_{V_c} \langle \tilde{\Phi}^*(\vec{R}')\, \tilde{\Phi}(\vec{R}) \rangle\, d^{(d+1)}\vec{R}'\, d^{(d+1)}\vec{R}$$

$$\rightarrow G(\vec{R}'-\vec{R}) \sim \frac{1}{|\vec{R}'-\vec{R}|^{d-1}}$$

$$\sim \frac{1}{V_c \Delta_0^2}\, \xi_c^2 \sim \xi_c^{3-d}, \qquad \xi_c \sim \frac{1}{|J-J_c|^{1/2}}$$

$$\left(V_c \sim \xi_c^{d+1}, \quad \Delta_0^2 \sim |\alpha| \sim \xi_c^{-2} \right) \qquad |\alpha| = A|J-J_c|$$

MF input

Important Statements on fluctuations ← Quantum

$G^{Q} \sim \xi_c^{3-d}$ ($\xi_c \to \infty$ at critical points)

1) $d > 3$, G^{Q} approaches zero as $\xi_c \to \infty$.
fluctuations appear to be irrelevant.

2) $d < 3$, G^{Q} diverges as $\xi_c \to \infty$
fluctuations appear to be irrelevant
and MF can be qualitatively problematic.

$d = 3$ Upper critical dimension for Q.C.P. !!

Scale invariance and Phase Transitions

1) Effective QFT or EFT constructed out of symmetry and other general considerations.

1) Critical point is identified as a scale invariant QFT (or CFT if $z=1$) or a fixed point Hamiltonian understand scale transformation.

3) Microscopic information (which appears in EFT) wiped out in the course of renormalization leading to universalities.