Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode Five:

Quantum coarse graining and A brief comment on Duality in (1+1) D:

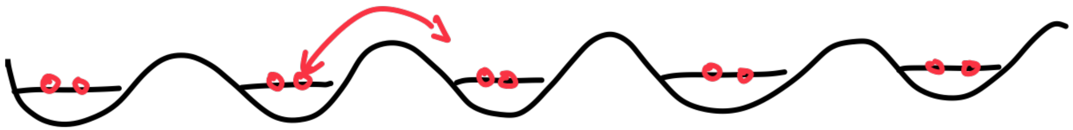
From Quantum Particles to quantum Fields

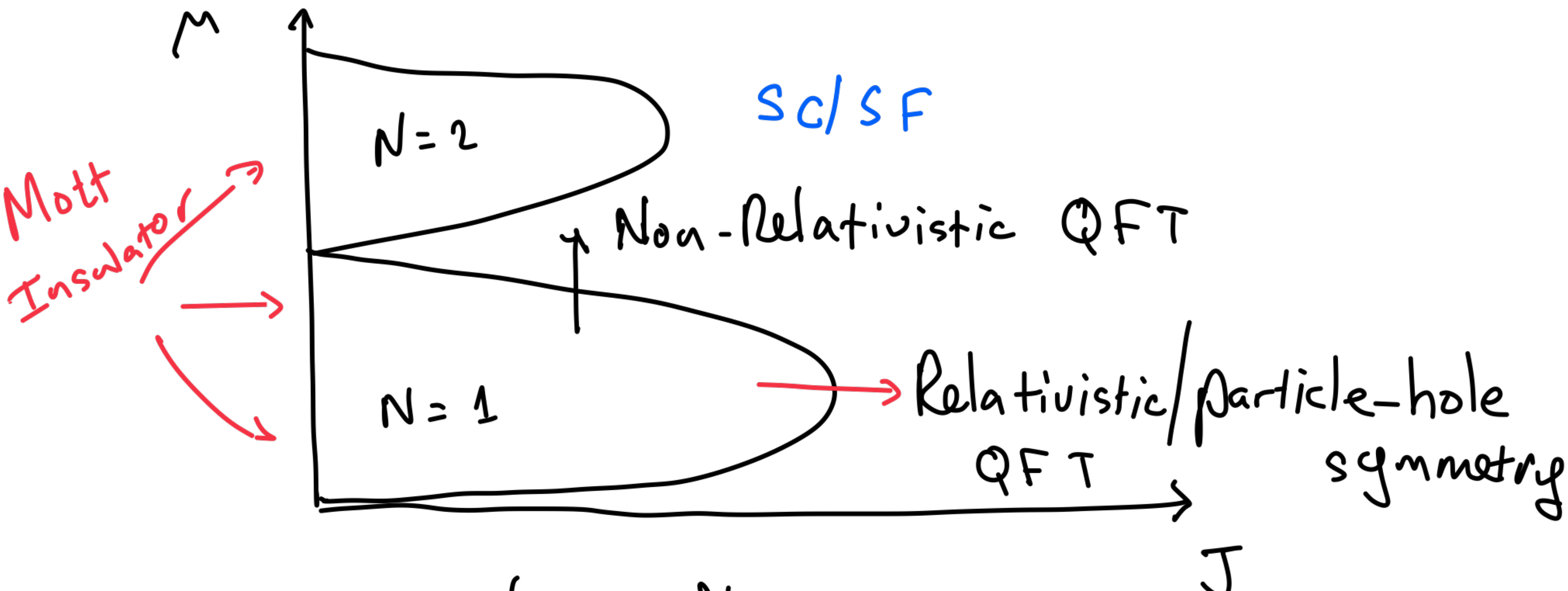
Quantum Model II : $[b_i, b_j] = 0, \quad [b_i, b_j^{+}] = \delta_{ij}$

$$\mathcal{H}_{BH} = \sum_i \frac{\hat{N}_i(\hat{N}_i - 1)}{2c} - \mu \hat{N}_i - J \sum_{\langle ij \rangle} b_i^{+} b_j + h.c.$$

$$(\hat{N}_i = b_i^{+} b_i)$$

Bose-Hubbard Model



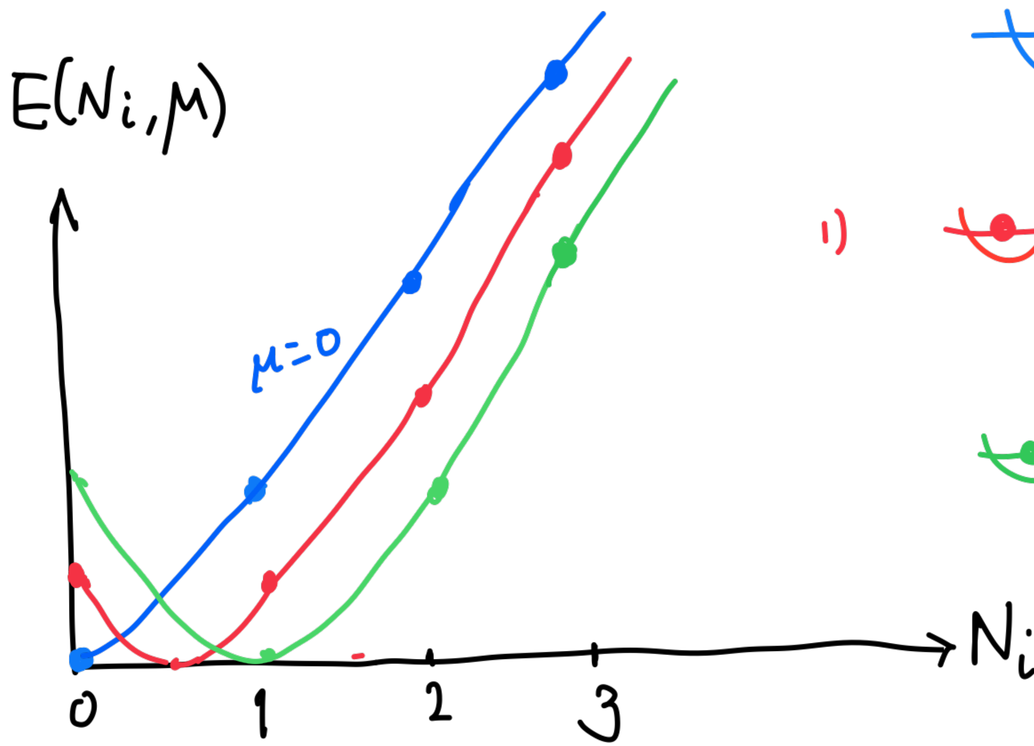


T = 0 phase diagram

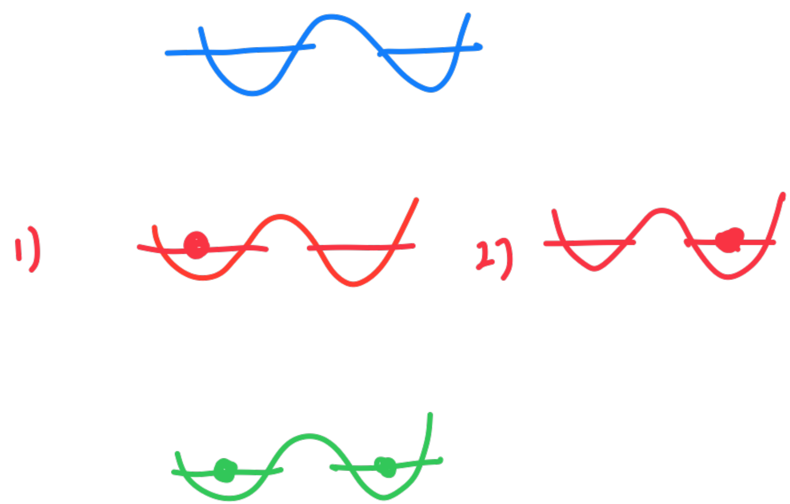
Quantum Model II:

$$\mathcal{H}_{BH} = \sum_i \frac{\hat{N}_i^2}{2c} - \mu \hat{N}_i + \overset{J}{\overset{\shortparallel}{0}}$$

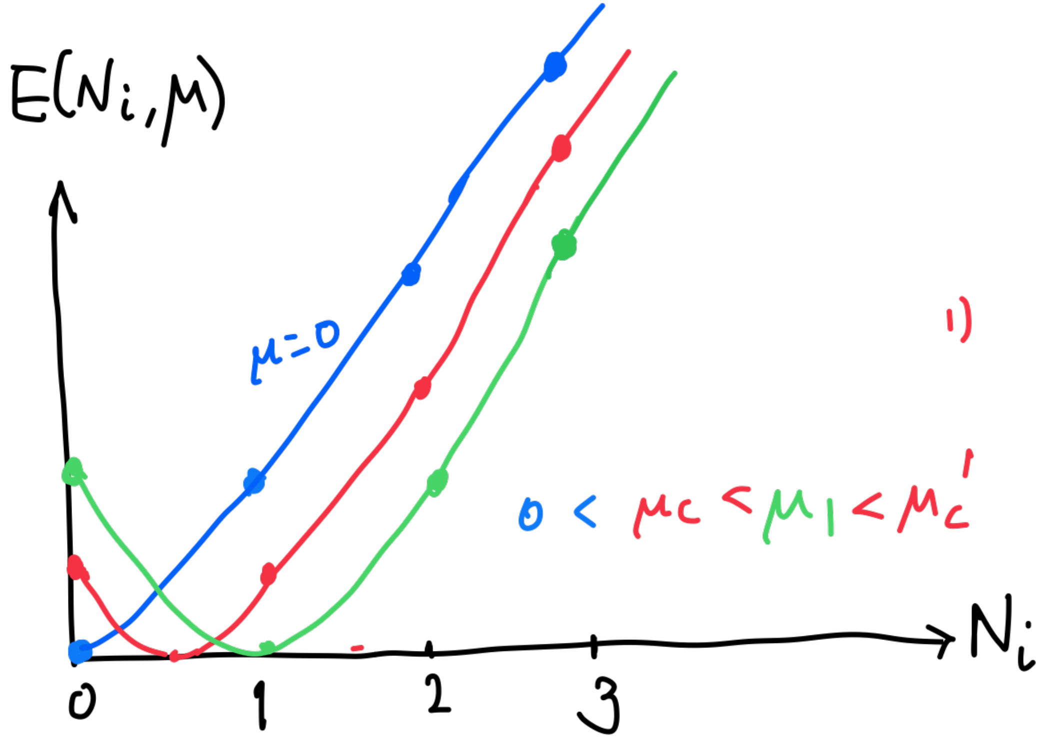
Bose-Hubbard Model

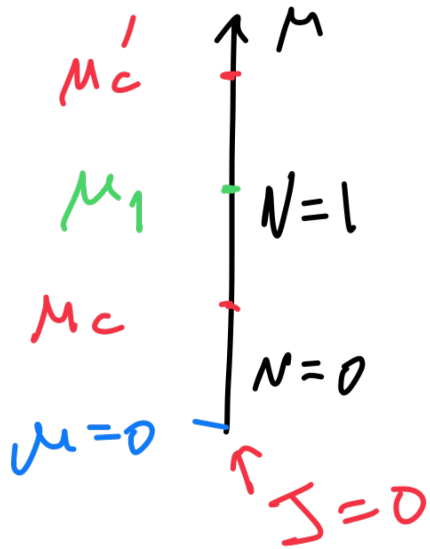


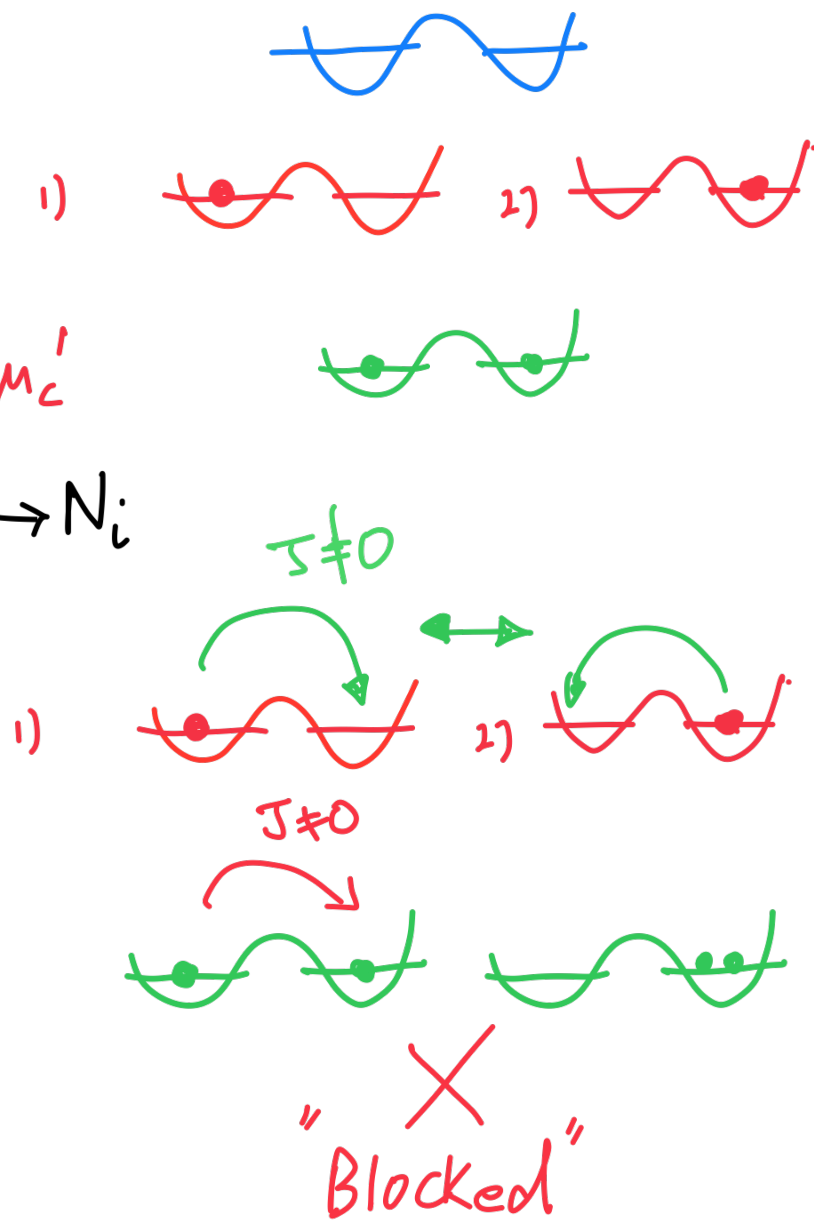
Two site physics

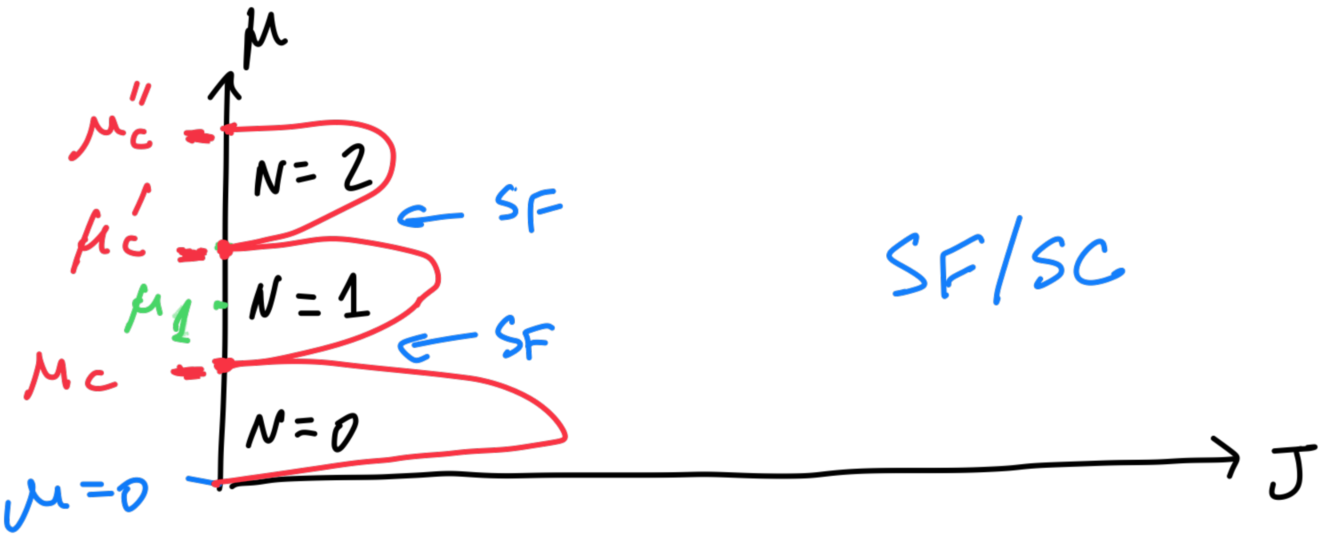


Two site physics





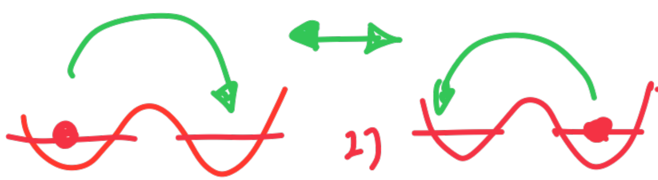




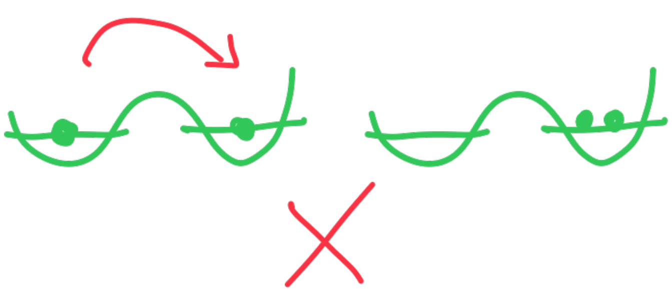
$\frac{1}{2C} = U \gg J \to 0$

$0 \leftarrow \frac{1}{2C} = U \ll J$

$J \neq 0$

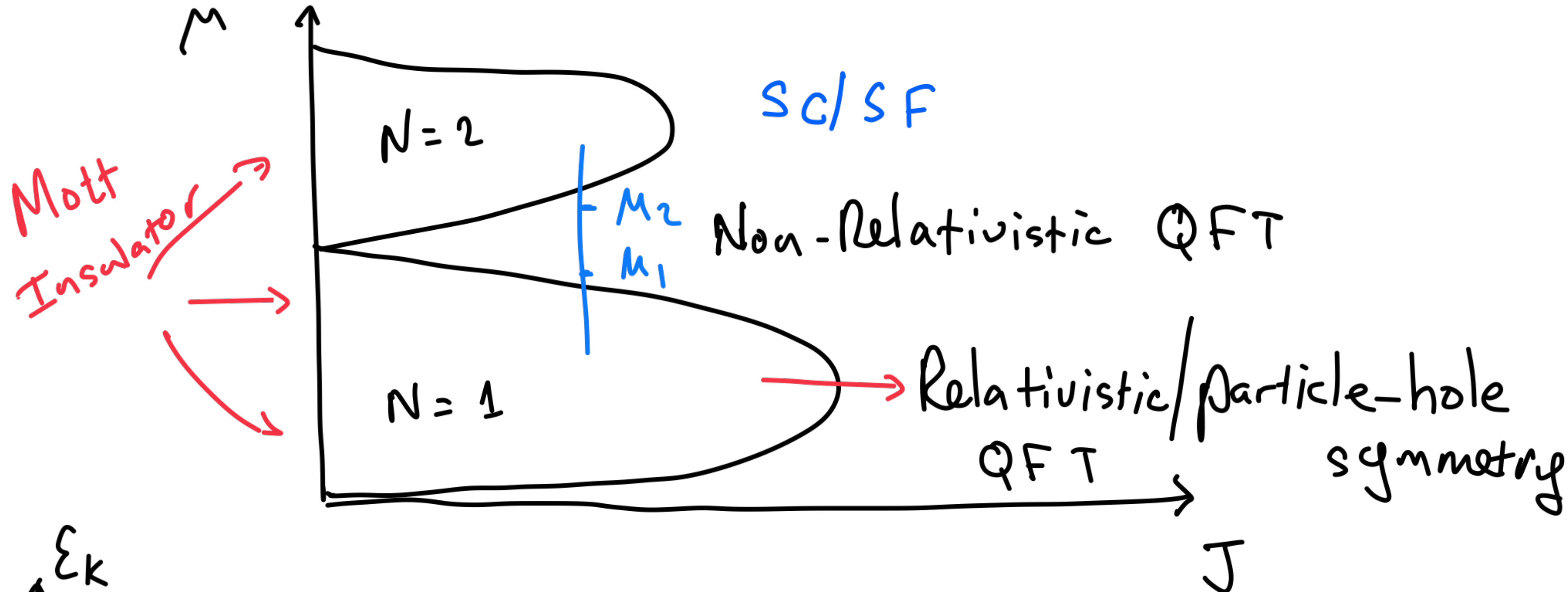
1)  2)

$J \neq 0$

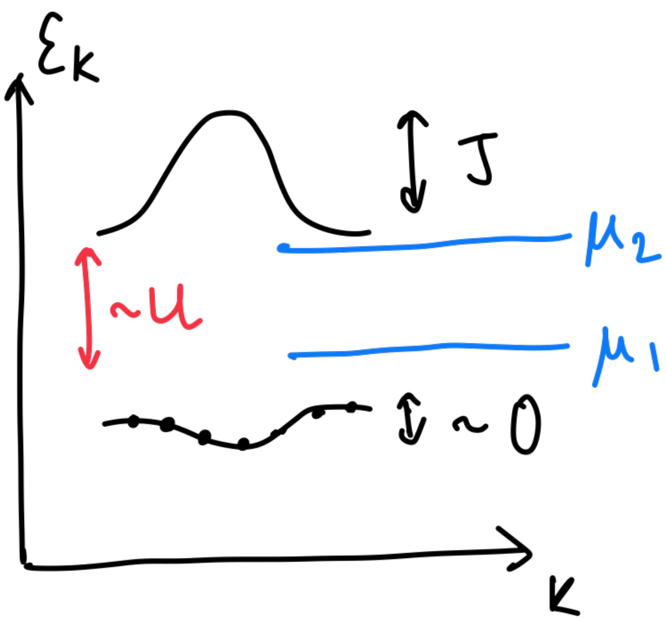


$$\mathcal{H} = -J \sum_{\langle ij \rangle} b_i^{+} b_j + h.c.$$

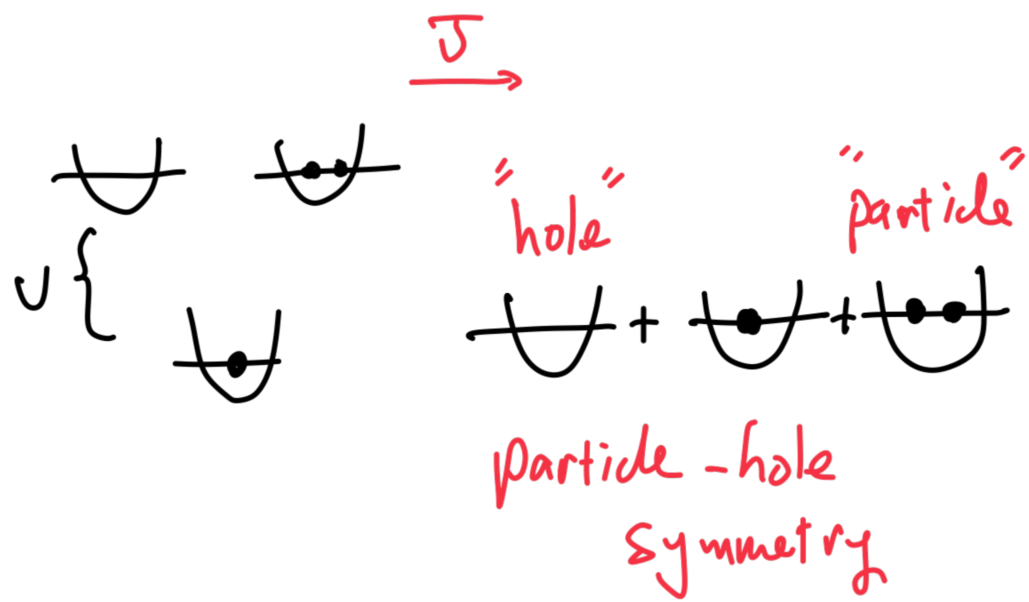
"free boson Model"



μ
J
$N=2$
$N=1$
Mott Insulator
SC/SF
μ_2
μ_1
Non-Relativistic QFT
Relativistic/particle-hole
QFT
symmetry

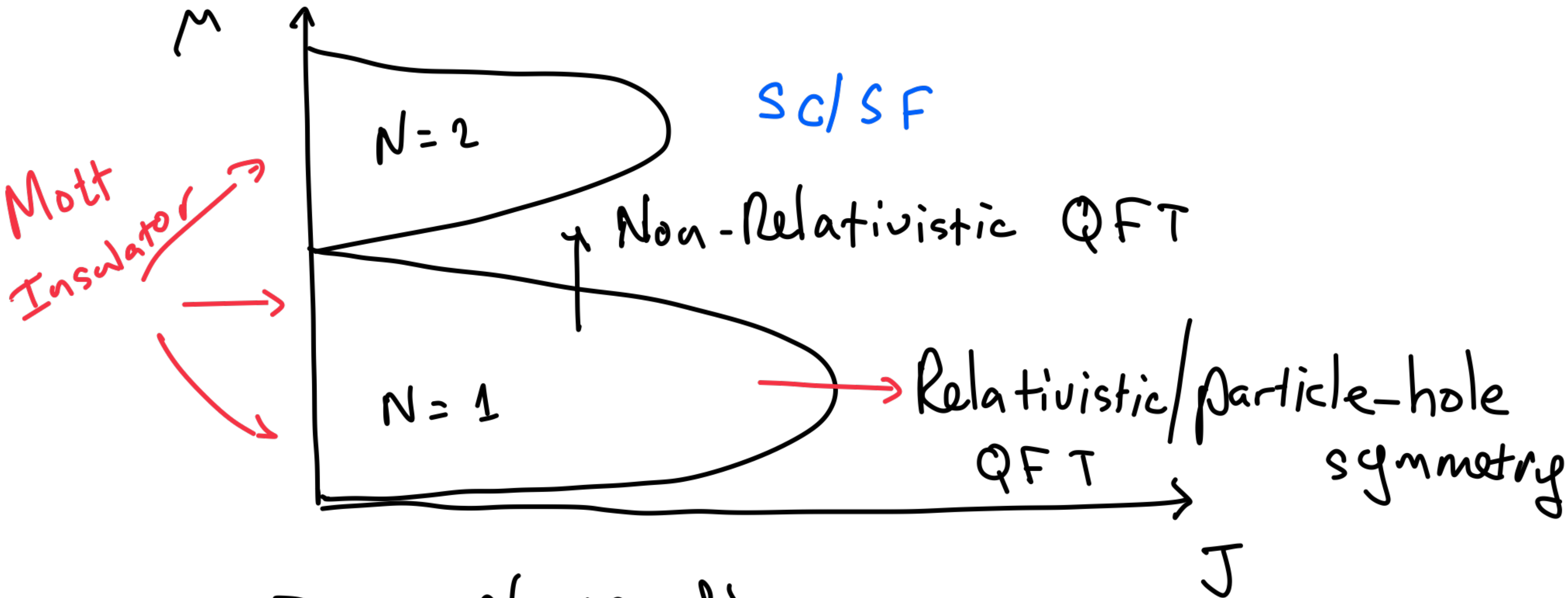


ε_k
k
J
$\sim U$
~ 0
μ_2
μ_1



U
J
"hole"
"particle"
particle-hole
symmetry

Quantum Model II :

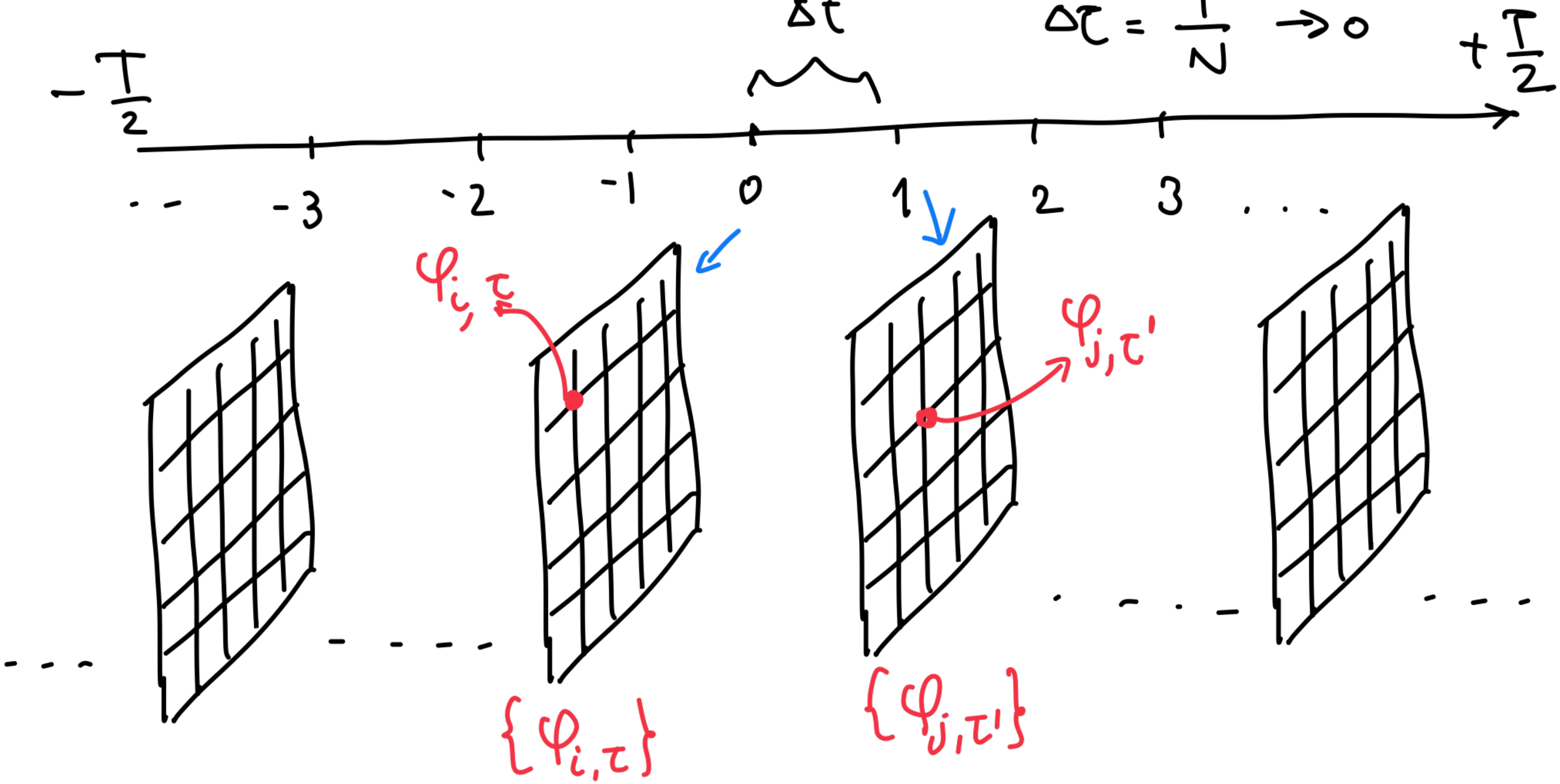


T = 0 phase diagram

Imaginary time evolution

$$Z = \langle 0 | e^{-HT} | 0 \rangle$$

$$\Delta\tau = \frac{T}{N} \to 0$$

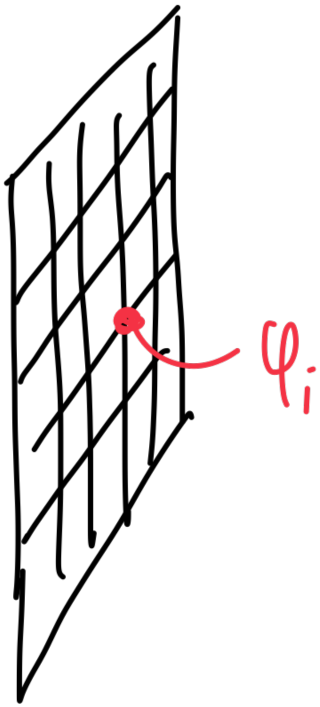


$$Z = \sum_{\dots \{\varphi_{i,\tau}\}} \dots \sum_{\{\varphi_{i,0}\}} \sum_{\{\varphi_{j,1}\}} \dots \sum_{\{\varphi_{i,\tau}\}} \dots \langle \{\varphi_{j,1}\} | e^{-H\Delta\tau} | \{\varphi_{i,0}\} \rangle \dots$$

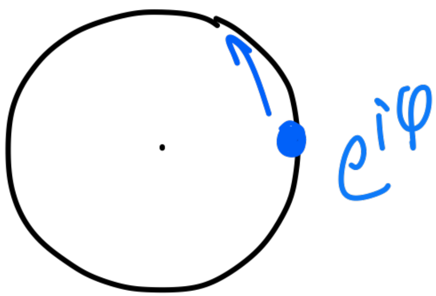
"Complete Set for Quantum Coarse graining with Large $\bar{N}$"

$$|\{\varphi_{i,\tau}\}\rangle = |\varphi_1,\tau\rangle \otimes |\varphi_2,\tau\rangle \otimes |\varphi_3,\tau\rangle \dots \otimes |\varphi_N,\tau\rangle$$

$$\begin{cases} \int D\phi_i \, |\{\varphi_{i,\tau}\}\rangle\langle\{\varphi_{i,\tau}\}| = \mathbb{1} \\ D\phi_i = \prod_{i=1}^{N} d\phi_i \end{cases}$$



"τth slice"

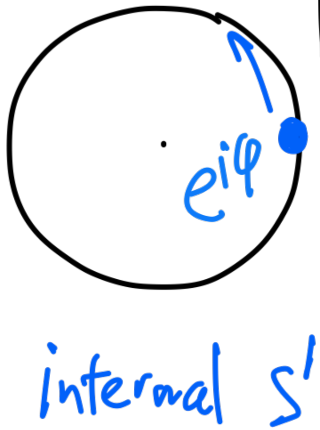


internal S^1 for site i

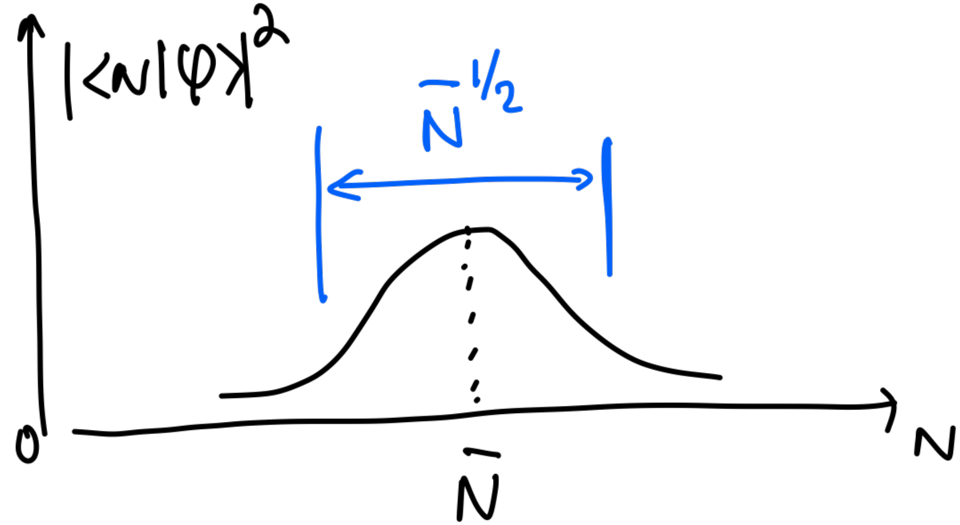
$$\begin{cases} \langle\varphi|\varphi'\rangle \simeq \delta_{\varphi,\varphi'} \\ b|\varphi\rangle = \bar{N}^{1/2} e^{i\varphi}|\varphi\rangle \\ \langle\varphi|b^{\dagger} b|\varphi\rangle = \bar{N} \gg 1 \end{cases}$$

$$|\varphi\rangle \sim e^{\bar{N}^{1/2} e^{i\varphi} b^{\dagger}} |\mathrm{vac}\rangle$$

"Complete Set for Quantum Coarse graining with Larg $\bar{N}$"



$$|\varphi\rangle \sim e^{\bar{N}^{1/2} e^{i\varphi} b^{\dagger}}|vac\rangle, \quad |N\rangle \sim b^{\dagger N}|vac\rangle$$



a) $b|\varphi\rangle = \bar{N}^{1/2} e^{i\varphi}|\varphi\rangle$

$$\langle\varphi|b^{\dagger} b|\varphi\rangle = \bar{N} \gg 1$$

$$|N\rangle \sim \int d\phi \, e^{-iN\phi} |\phi\rangle$$

b) $\hat{N}|N\rangle = N|N\rangle$

Fock states

" $\hat{N}, \varphi$ are Conjugate."

$\cdots \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad \cdots$ $\qquad b|\varphi\rangle \sim e^{i\varphi}|\varphi\rangle$

$$Z = \sum_{\{\varphi_{i,\tau}\}} \cdots \sum_{\{\varphi_{i,0}\}} \sum_{\{\varphi_{i,1}\}} \cdots \sum_{\{\varphi_{i,\tau'}\}} \cdots \underbrace{\langle\{\varphi_{i,1}\}|e^{-H\Delta\tau}|\{\varphi_{i,0}\}\rangle}_{T_{1,0}} \cdots$$

$$H = H_1 + H_2, \qquad H_1 = \sum_i \frac{\delta\hat{N}_i^2}{2C}, \qquad H_2 = -J\sum_{\langle ij\rangle} \underset{e^{-i\varphi_{i,\tau}}}{b_i^+}\, \underset{e^{i\varphi_{j,\tau}}}{b_j} + h.c.$$

$$T_{1,0} = \underbrace{\langle\{\varphi_{i,1}\}|e^{-H_1\Delta\tau}|\{\varphi_{i,0}\}\rangle}_{e^{-\left(\frac{\varphi_{i,1}-\varphi_{i,0}}{\Delta\tau}\right)^2\Delta\tau \cdots}} \; e^{-H_2(\{\varphi_{i,0}\})\Delta\tau}$$

Hint:

$$\sum_{\delta N} \langle\varphi_1|e^{-\delta\hat{N}^2\Delta\tau}|\delta N\rangle\langle\delta N|\varphi_0\rangle$$

approximate with $\int \delta N$

Imaginary time evolution

$$Z = \sum_{\{\varphi_{i,\tau}\}} \cdots \sum_{\{\varphi_{i,0}\}} \sum_{\{\varphi_{j,1}\}} \cdots \sum_{\{\varphi_{i,\tau}\}} \cdots \langle \{\varphi_{j,1}\} | e^{-H\Delta\tau} | \{\varphi_{i,0}\} \rangle \cdots$$

$$= \int D\varphi \, e^{-S(\{\varphi(\vec{x},\tau)\})}, \qquad S = \int d^d\vec{x} \int d\tau \, \mathcal{L}(\{\varphi(\vec{x},\tau)\})$$

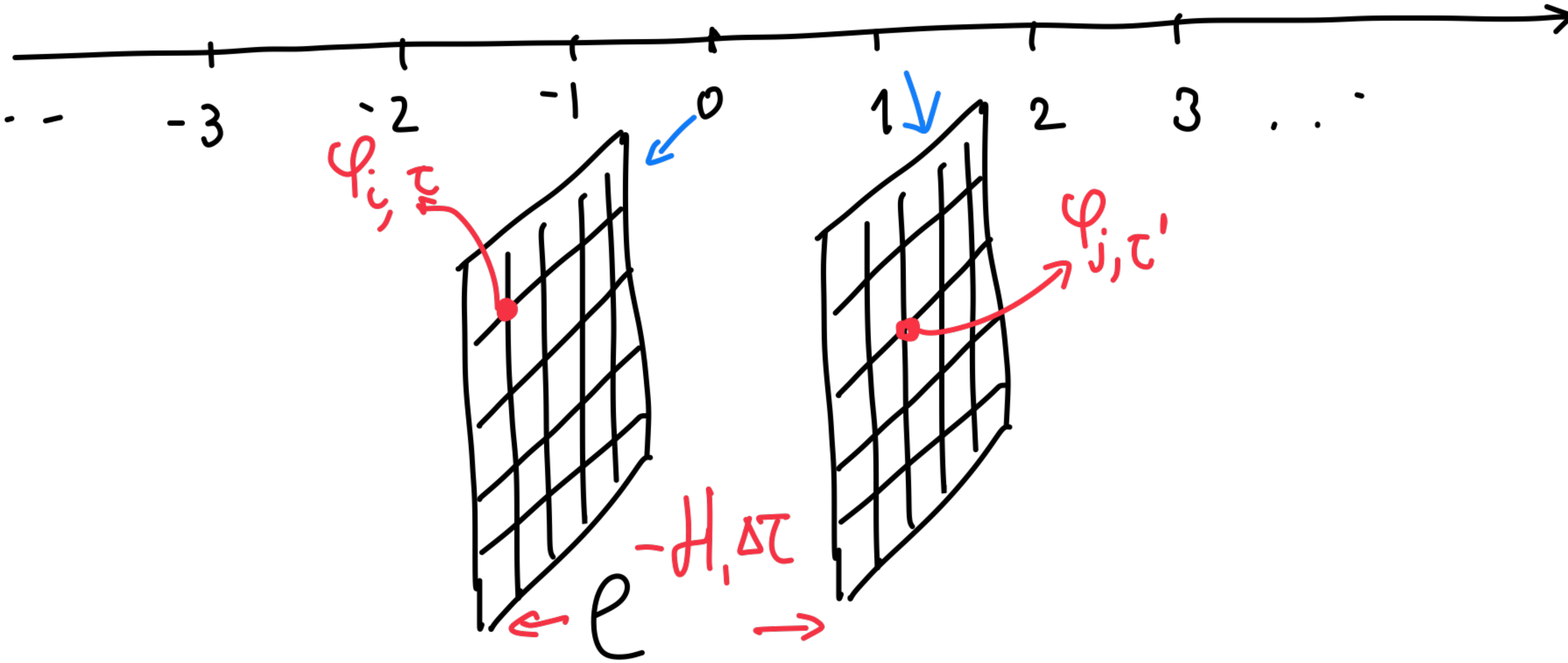
$\varphi(x,\tau)$ Action

$D\varphi$: functional integral

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_1 = -\tilde{J}_\tau \cos(\varphi(\vec{x},\tau) - \varphi(\vec{x},\tau+\Delta\tau))$$

$$\mathcal{L}_2 = -\tilde{J}_{\vec{x}} \cos(\varphi(\vec{x}_i,\tau) - \varphi(\vec{x}_j,\tau))$$



$$S = \int d^{d-1}\vec{x} \int d\tau \, \mathcal{L}\left(\{\varphi(\vec{x},\tau)\}\right)$$

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$$

$$\mathcal{L}_1 = -\tilde{J}_\tau \cos\left(\varphi(\vec{x},\tau) - \varphi(\vec{x},\tau+\Delta\tau)\right)$$

$$\mathcal{L}_2 = -\tilde{J}_{\vec{x}} \cos\left(\varphi(\vec{x}_i,\tau) - \varphi(\vec{x}_j,\tau)\right)$$

Order-Disorder quantum phase transitions

1) "Order" usually involves condensation of bosonic fields

Ising model: Real scalar field;
Superfluids: Complex scalar field;
Nematic order: Real "Director" field;
Ferromagnetic order: Complex vector field
……

2) So transitions are usually described by Bosonic quantum field theories.