Phys525:
Quantum Condensed Matter Physics: Quantum Criticality
Basics, Dynamics and Topological criticality

Episode Three:

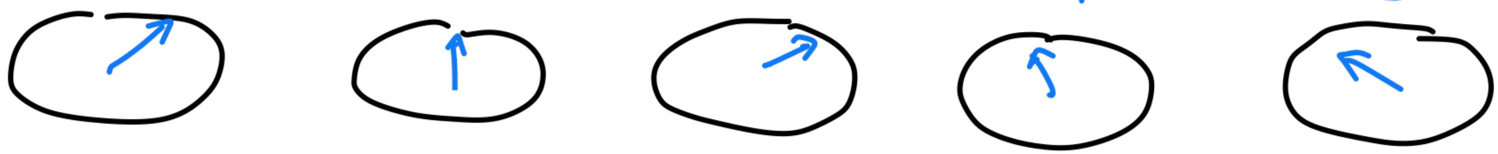
The concept of “coarse graining” II, entropy effect and quantum effects on symmetry restoring

Example II: Superconductors/Superfluids

$$\beta \mathcal{H}_{SC} = -\frac{J}{T} \sum_{\langle ij \rangle} \chi_i^* \cdot \chi_j + c.c. \qquad \chi_i^* \cdot \chi_i = 1, \quad \chi_i = e^{i\varphi_i}$$

planar vector

χ_1 χ_2 χ_3 χ_4 χ_5

...  ...

1 2 3 4 5

$$\Delta = \langle e^{i\varphi_i} \rangle$$

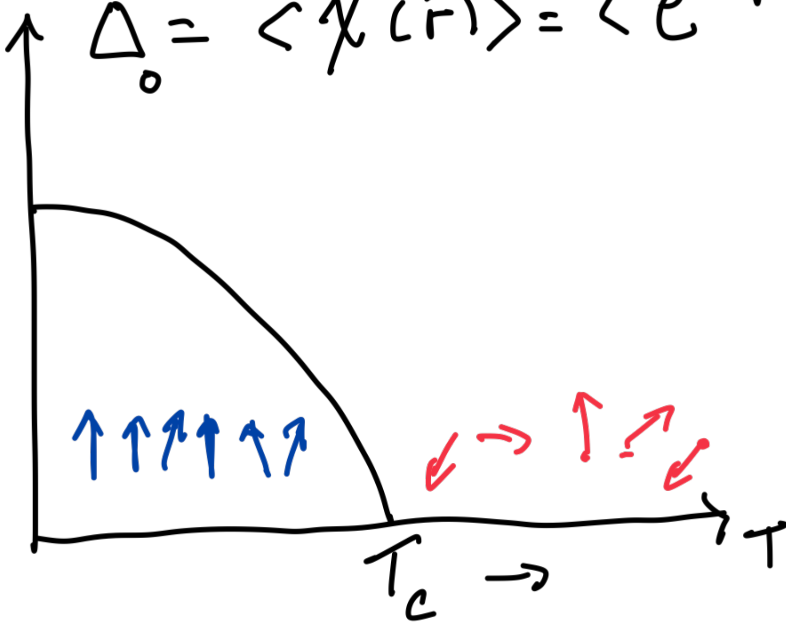
or $\beta \mathcal{H}_{SF} = -\frac{J}{T} \sum_{\langle ij \rangle} \hat{\chi}_i \cdot \hat{\chi}_j$, planar vector Rep.

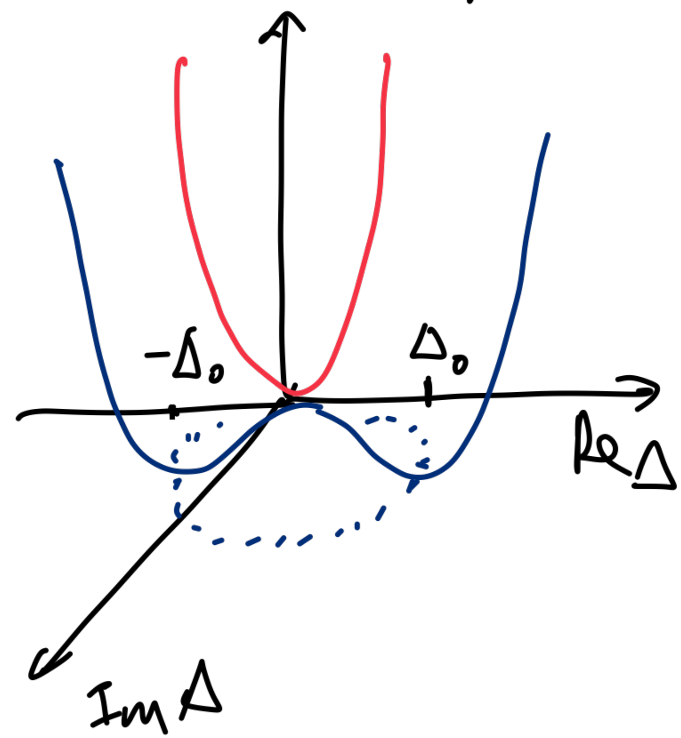
$$\hat{\chi} = (\mathrm{Re}\,\chi, \mathrm{Im}\,\chi)$$

Example II: Superconductors/Superfluids

$$\beta \mathcal{H}_{sc} = -\frac{J}{T}\sum_{\langle ij \rangle} \chi_i^* \cdot \chi_j + c.c.$$

$\Delta_0 = \langle \chi(\vec{r}) \rangle = \langle e^{i\varphi} \rangle$

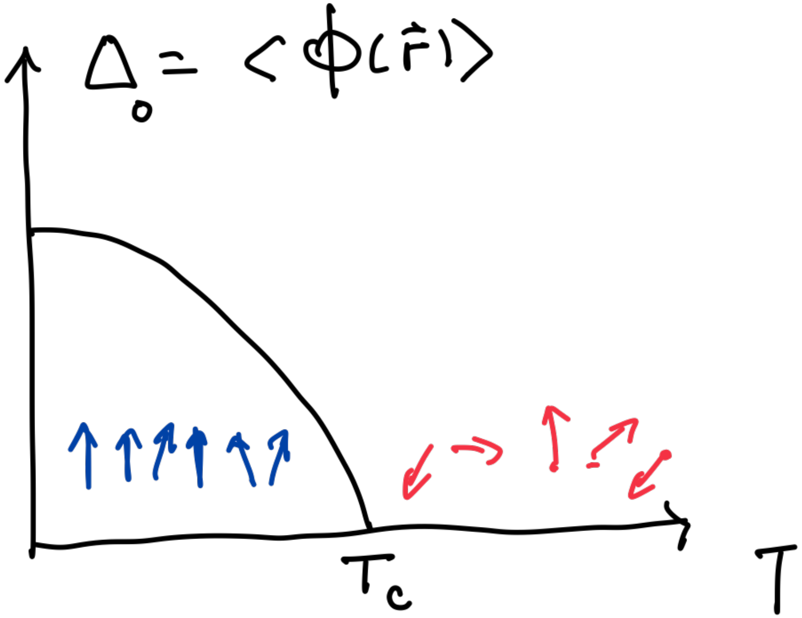


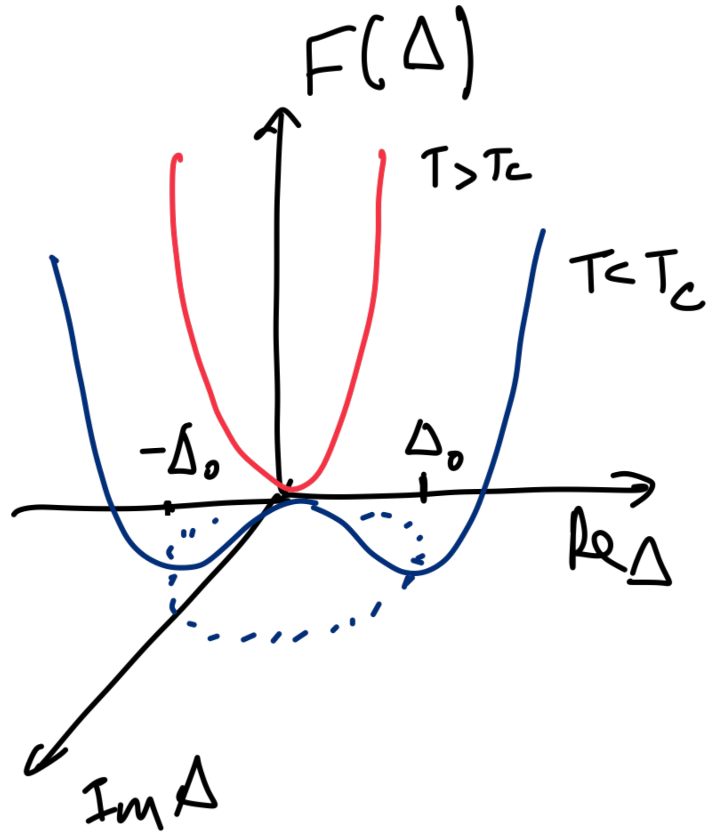


Proper field theory description (derived from BCS)

$$\beta f_{sc} = \int d^dx \, \frac{1}{2m} |\nabla \phi|^2 + a \, |\phi|^2 + b \, |\phi|^4 + \cdots$$

(ϕ : Complex field)

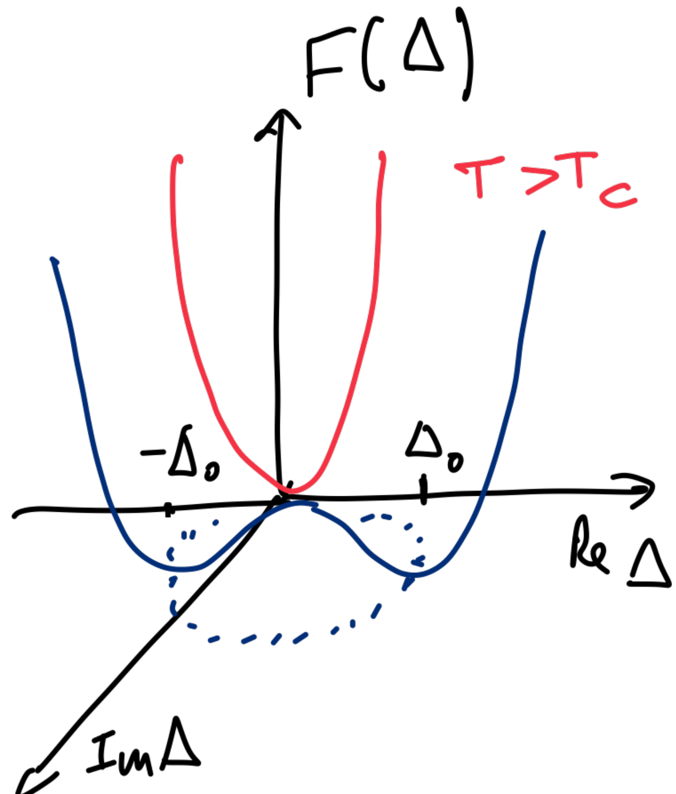




$a \sim T - T_c$

U(1) Gauge symmetry. $\mathcal{H}(\{\chi(\vec{r})\}) = \mathcal{H}(\{\chi(\vec{r})e^{i\varphi}\})$

"spontaneously broken"



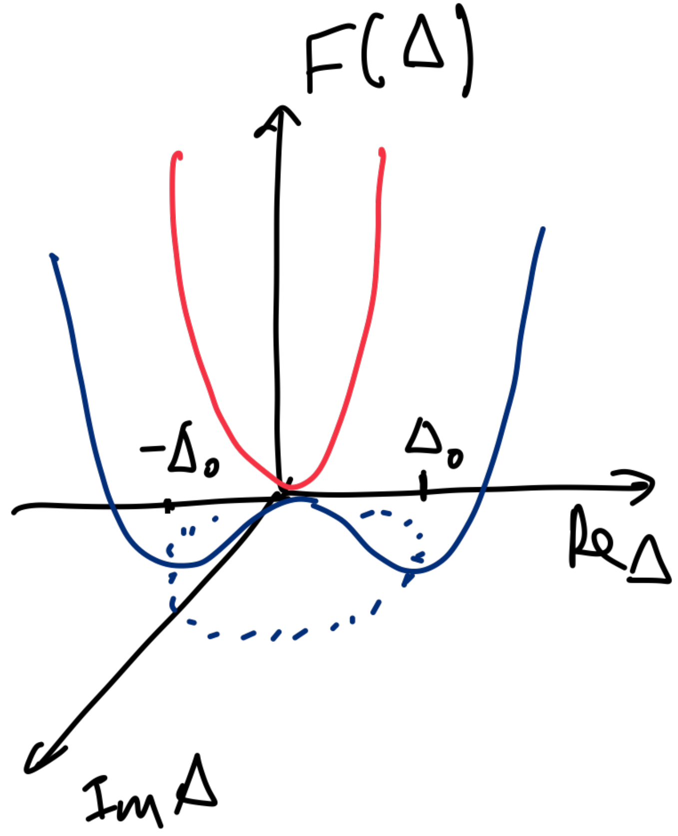
Ground state manifold $= S^1$ (unit circle)

breaks the Gauge symmetry !!

Are cubic terms always forbidden? "$\phi|\phi|^2$, $\phi^*|\phi|^2$"

$$\beta F_{sc} = \int d^d x \; \frac{1}{2m}|\nabla\phi|^2 - \mu|\phi|^2 + \lambda|\phi|^4 + \cdots$$

(ϕ : Complex field)

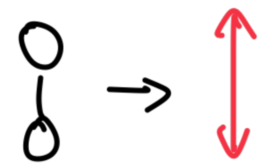


"$\phi|\phi|^2$, $\phi^*|\phi|^2$ are forbidden because of symmetries"

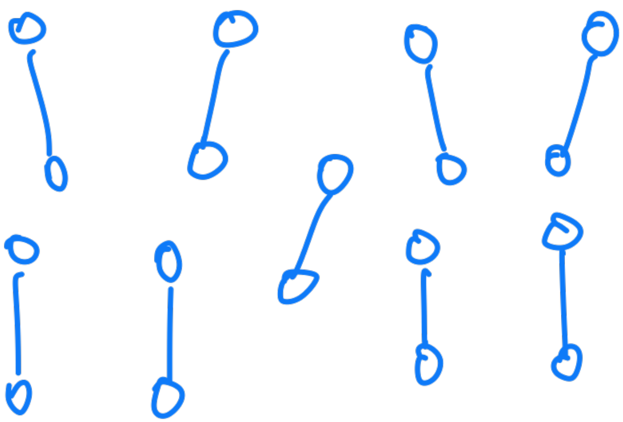
U(1) gauge symmetry

or $\mathbb{Z}_2$ Symmetry in Ising Model.

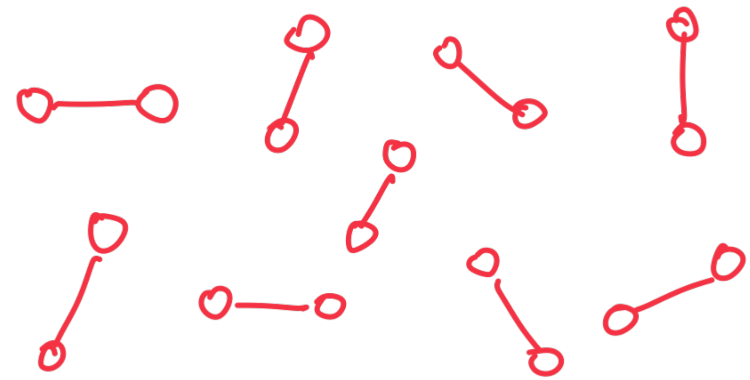
Nematic liquid Crystal



"director"



orientation order

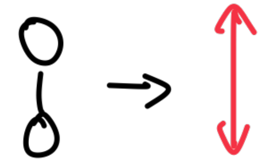


orientation disorder

Nematic Liquid Crystal:

breaks Rotation Symmetry but not translational.

(believe to exist in HT_c systems !)

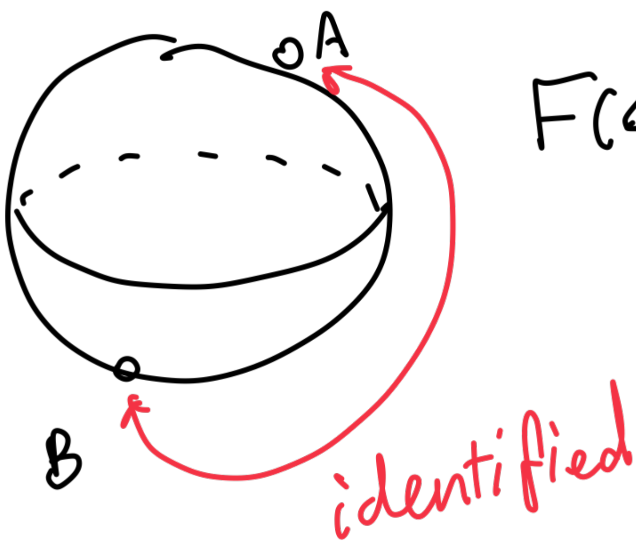
Nematic liquid Crystal  "director"

$$\Delta_{\alpha\beta} = \langle n_\alpha n_\beta \rangle - \frac{1}{3}\delta_{\alpha\beta} \mathrm{Tr}\langle n_\alpha n_\beta \rangle, \quad \mathrm{Tr}\,\Delta_{\alpha\beta} = 0$$

$\alpha = x, y, z$

<u>Real symmetric, traceless tensor</u>

manifold $\quad S^2/\mathbb{Z}_2$



$$F(\Delta) = a\,\mathrm{Tr}\,\Delta^2 + \overbrace{b\,\mathrm{Tr}\,\Delta^3}^{\text{cubic term allowed!}} + c\,\mathrm{Tr}\,\Delta^4 + \ldots$$

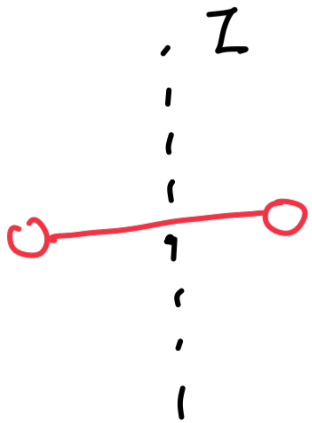
$$\Delta \to \Delta$$

if $\hat{n} \to -\hat{n}$.

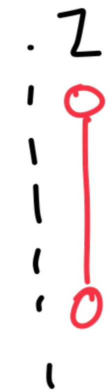
$$\Delta_1 = \begin{array}{c} \\ \end{array}\begin{array}{cc} & \begin{array}{ccc} x & y & z \end{array} \\ & \begin{bmatrix} 1 & & \\ & 1 & \\ & & -2 \end{bmatrix} \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

$$\Delta_2 = \begin{array}{cc} & \begin{array}{ccc} x & y & z \end{array} \\ & \begin{bmatrix} -1 & & \\ & -1 & \\ & & 2 \end{bmatrix} \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

$$= -\Delta_1$$



biaxial



uniaxial

Important feature for classical Order-disorder transition

A: The operator defines the order parameter communtes with $\mathcal{H}$.

$$[\sigma_{z_i}, \mathcal{H}_{Ising}] = [X_i, \mathcal{H}_{Ising}] = 0$$

B: At lower temperature is always ordered.

C. phase transition driven by the entropy effect.

Entropy effect: 1D Ising Model $\mathcal{H}_{Ising} = -J \sum_{\langle ij \rangle} \sigma_{z_i} \cdot \sigma_{z_j}$

$T=0$

$$\cdots \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \cdots$$

$$L_c = N_c \cdot a$$

$T \ll J$

$$\cdots \uparrow \uparrow \, \vdots \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \vdots \, \uparrow \cdots$$

Domain Walls

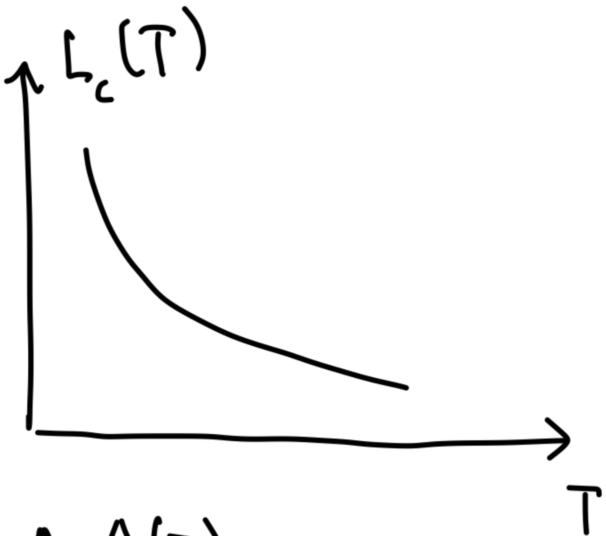
$$P_{DW}(L_c) \sim e^{-\beta F_{DW}}, \quad F_{DW} = 4J - T \ln \frac{N_c(N_c-1)}{2}$$

$$U - T S$$

Probability

$$L_c \sim e^{\frac{2J}{T}} \quad \text{when } P_{DW}(L_c) \sim O(1)$$

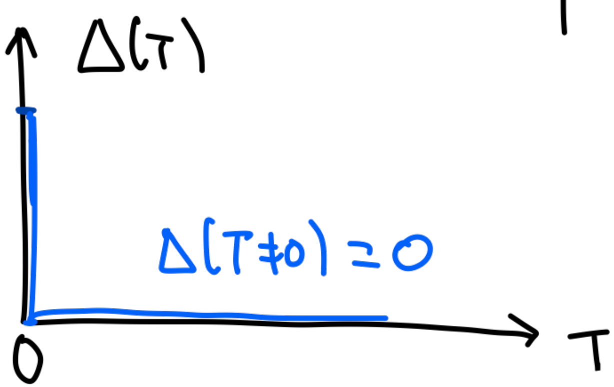
typical distance between DWs



$L_c(T) \to \infty$ when $T \to 0$

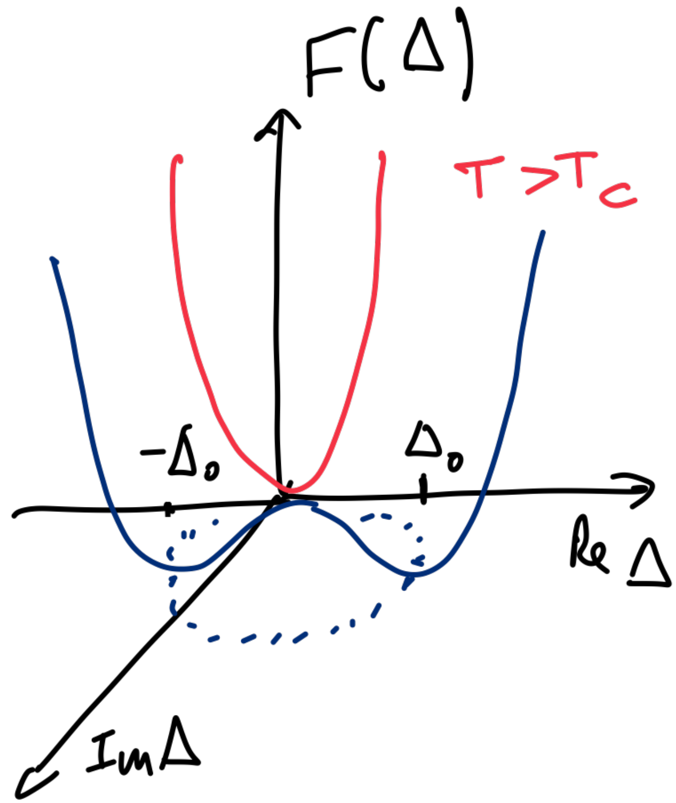
"A Dilute Gas of Domain Walls"

= Ising Model Ground state at $T \neq 0$



U(1) Gauge symmetry. $H(\{\chi(\vec{r})\}) = H(\{\chi(\vec{r})e^{i\varphi}\})$

"spontaneously broken"

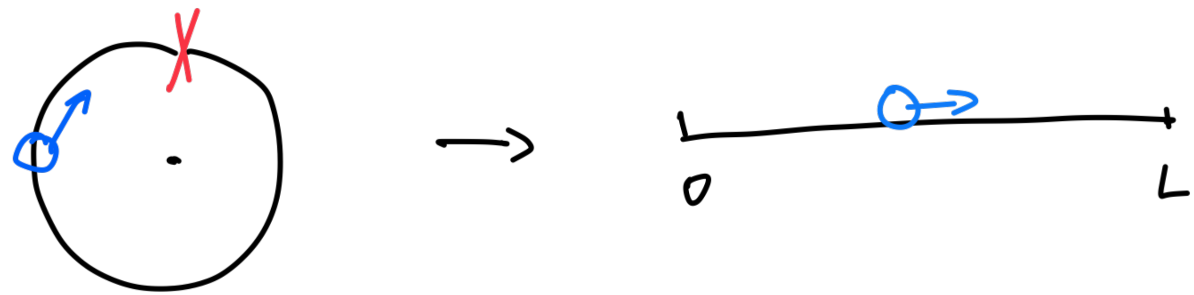


Ground state manifold = S^1 (unit circle)

- Quantum Physics and Quantum Phase transitions

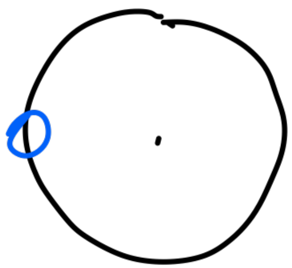
More on $U(1)$ Symmetry breaking : Quantum effects

Recall:

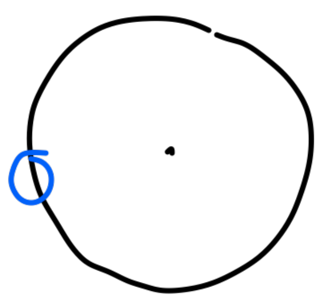


Classical Dynamics

$t=0$

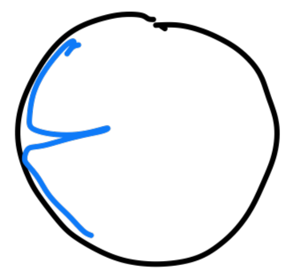


$t \neq 0$



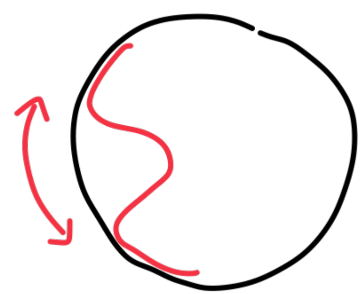
Quantum Dynamics

$t=0$



$$\langle \Delta X^2 \rangle \simeq \frac{1}{\alpha} \to 0$$

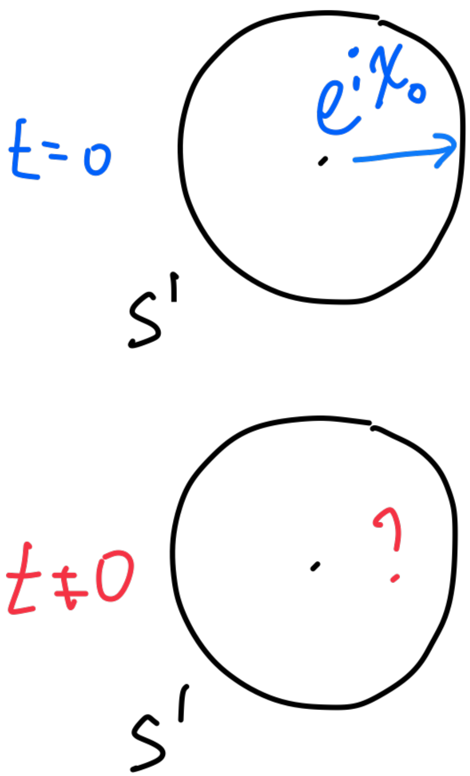
$t \neq 0$



$$\langle \Delta X^2(t) \rangle \simeq \frac{\alpha t^2}{m^2}$$

More on U(1) Symmetry breaking : Quantum effects

How long it takes to Restore U(1) symmetry for a system of "N" particles, quantum mechanically?



Important :

$$\Psi_{N\text{-Boson}} \to \Psi_{N\text{-Boson}}\, e^{i\varphi \cdot N}$$

if $e^{i\chi_0} \to e^{i\chi_0 + i\varphi}$

$$[\varphi, \hat{N}] = i, \qquad \hat{N} = \frac{\partial}{i\,\partial\varphi}$$

$\hat{N}$ and φ are conjugate variables!

More on U(1) Symmetry breaking: Quantum effects

$$\hat{N} = \frac{1}{i}\frac{\partial}{\partial \varphi}, \quad [\varphi, \hat{N}] = i$$

With short range interactions, $\mathcal{H}_{eff} = g \int \rho^2(r)\, d^d\vec{r} \sim \frac{N^2}{2C}$

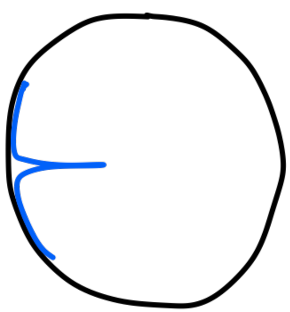
$$C = \frac{\Omega}{g}$$

Ω ← Vol.

g ← interaction

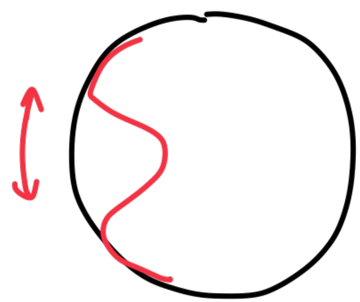
$t = 0$

$\langle \Delta\varphi^2 \rangle \sim \frac{1}{\alpha}$



S^1 ← internal space

$t \neq 0$



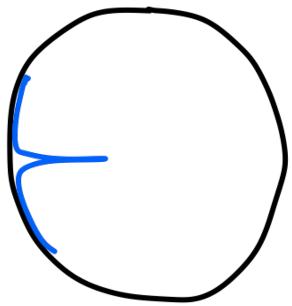
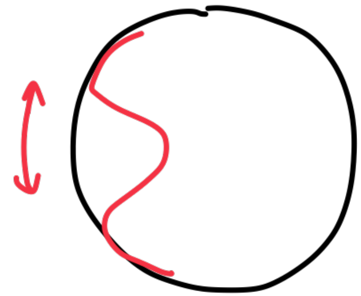
$\langle \Delta\varphi^2(t) \rangle \sim \frac{\alpha t^2}{C^2}$

S^1

$(C \to \infty \text{ if } g \to 0)$

More on U(1) Symmetry breaking : Quantum effects

$t = 0$ $\qquad\qquad\qquad\qquad$ $t \neq 0$

$\langle \Delta\varphi^2 \rangle \sim \frac{1}{\alpha}$   $\langle \Delta\varphi^2(t) \rangle \sim \frac{\alpha t^2}{c^2}$

S^1 ← internal space $\qquad$ ↗ S^1 $\qquad$ $(c \to \infty \text{ if } g \to 0)$

time it takes for $\langle \Delta\varphi^2(t) \rangle \sim O(1)$,

$$t_{O(1)} \sim \frac{1}{\sqrt{\alpha}} \cdot c = \frac{1}{\sqrt{\alpha}} \cdot \frac{\Omega}{g} \longrightarrow \infty \text{ if } \Omega \to \infty .$$

Ω ← Vol.

g ← interaction

Spontaneous Symmetry Breaking Vs Symmetry Restoring